KINESIOLOGY – PT617
SOLUTION 1
Introduction: Vectors and Forces

1) Each of the following vectors is described by its magnitude ($|\underline{V}|$) and its direction (θ) relative to the x-axis (positive angles are counter-clockwise). Resolve each vector into its components along the x- and y-axes (*i.e.*, find Vx and Vy).
   a) $|\underline{V}| = 7$, $\theta = 30°$
   b) $|\underline{V}| = 5$, $\theta = 110°$
   c) $|\underline{V}| = 2$, $\theta = -160°$
   d) $|\underline{V}| = 3$, $\theta = -75°$

Solution

a) $Vx = |\underline{V}| \cos\theta$ = 7 x cos 30° = 7 x 0.866 = **6.062**
   $Vy = |\underline{V}| \sin\theta$ = 7 x sin 30° = 7 x 0.5 = **3.5**

b) $Vx = |\underline{V}| \cos\theta$ = 5 x cos 110° = 5 x -0.342 = **-1.71**
   $Vy = |\underline{V}| \sin\theta$ = 5 x sin 110° = 5 x 0.939 = **4.698**

c) $Vx = |\underline{V}| \cos\theta$ = 2 x cos (-160)° = 2 x cos (160)° = 2 x -0.939 = **-1.878**
   $Vy = |\underline{V}| \sin\theta$ = 2 x sin (-160)° = 2 x (-sin (160)°)= 2 x –0.342 = **-0.684**

d) $Vx = |\underline{V}| \cos\theta$ = 3 x cos (-75)° = 3 x cos (75)° = 3 x 0.258 = **0.78**
   $Vy = |\underline{V}| \sin\theta$ = 3 x sin (-75)° = 3 x (-sin (75)°) = 3 x –0.966 = **-2.898**

2) Each of the following vectors is described by its components along the x- and y-axes (Vx and Vy, respectively). For each vector, find its magnitude and its direction relative to the x-axis (positive angles are counter-clockwise) (i.e., find $|\underline{V}|$ and θ).
   a) Vx = 7, Vy = -5
   b) Vx = -2, Vy = -3
   c) Vx = 1, Vy = 5
   d) Vx = -3, Vy = 3

Solution

a) $|\underline{V}| = \sqrt{Vx^2 + Vy^2} = \sqrt{7^2 + (-5)^2}$ = **8.6**

$\theta = \tan^{-1}\left(\frac{Vy}{Vx}\right) = \tan^{-1}\left(\frac{-5}{7}\right) = \tan^{-1}(-0.7) =$ **324.5°**

b) $|\underline{V}| = \sqrt{Vx^2 + Vy^2} = \sqrt{(-2)^2 + (-3)^2}$ = **3.61**

$\theta = \tan^{-1}\left(\frac{Vy}{Vx}\right) = \tan^{-1}\left(\frac{-3}{-2}\right) = \tan^{-1}(1.5) =$ **236.3°**

c) $|\underline{V}| = \sqrt{Vx^2 + Vy^2} = \sqrt{1^2 + 5^2}$ = **5.099**

$\theta = \tan^{-1}\left(\frac{Vy}{Vx}\right) = \tan^{-1}(\frac{5}{1}) = \tan^{-1}(5) =$ **78.69°**

d) $|\underline{V}| = \sqrt{Vx^2 + Vy^2} = \sqrt{(-3)^2 + (3)^2}$ = **4.24**

$\theta = \tan^{-1}\left(\frac{Vy}{Vx}\right) = \tan^{-1}(\frac{3}{-3}) = \tan^{-1}(-1) =$ **135°**

3) Given the following three vectors, specified by their magnitude and their direction relative to the x-axis (positive angles are counter-clockwise):

$\underline{V}_1$: $|\underline{V}_1|$ = 1.0 cm, θ_1 = 90°
$\underline{V}_2$: $|\underline{V}_2|$ = 3.61 cm, θ_2 = -56.3°
$\underline{V}_3$: $|\underline{V}_3|$ = 3.16 cm, θ_3 = -161.6°

Use the quantitative method to find the magnitude and direction of:

a) $\underline{V}_1 + \underline{V}_2$
b) $\underline{V}_2 - \underline{V}_3$
c) $\underline{V}_2 + 3\,\underline{V}_1$
d) $\underline{V}_3 - \underline{V}_2 + \underline{V}_1$

Perform a rough check of your results using the graphical method.

Solution

a) $\underline{V}_1x = |\underline{V}_1| \cos\theta$
= 1 x cos 90°
= 1 x 0 = 0

$\underline{V}_1y = |\underline{V}_1| \sin\theta$
= 1 x sin90°
= 1 x 1 = 0

$\underline{V}_2x = |\underline{V}_2| \cos\theta$
= 3.61 x cos (-56.3°)
= 3.61 x cos 56.3°
= 3.61 x 0.555
= 2.00

$\underline{V}_2y = |\underline{V}_2| \sin\theta$
= 3.61 x sin (-56.3)°
= 3.61 x –sin 56.3°
= 3.61 x –0.832
= -3

$V_{1+2}x = V_1x + V_2x = 0 + 2 = 2$
$V_{1+2}y = V_1y + V_2y = 1 + (-3) = -2$
$|\underline{V}_{1+2}| = \sqrt{2^2 + (-2)^2} = \mathbf{2.83}$

$\theta_{1+2} = \tan^{-1}\left(\frac{-2}{2}\right) = \mathbf{-45°}$

b) $\underline{V}_3x = |\underline{V}_3| \cos\theta$
$= 3.16 \text{ x } \cos(-161.6°)$
$= 3.16 \text{ x } \cos 161.6°$
$= 3.16 \text{ x } -0.949$
$= -2.99$

$\underline{V}_3y = |\underline{V}_3| \sin\theta$
$= 3.16 \text{ x } \sin(-161.6)°$
$= 3.16 \text{ x } -\sin 161.6°$
$= 3.16 \text{ x } -0.316$
$= -0.999$

$V_{2-3}x = V_2x - V_3x = 2 - (-2.999) = 4.999$
$V_{2-3}y = V_2y - V_3y = 3 - 0.999 = 2.001$
$|\underline{V}_{2-3}| = \sqrt{4.999^2 + 2.001^2} = \mathbf{5.38}$

$\theta_{2-3} = \tan^{-1}\left(\frac{2.001}{4.999}\right) = \mathbf{21.82°}$

c) Vector multiplication by a scalar
Let $\underline{V}_4 = 3 \text{ x } |\underline{V}_3|$
Then, $\underline{V}_4x = 3 \text{ x } \underline{V}_3x$
$= 3 \text{ x } |\underline{V}_3| \cos\theta$
$= 3 \text{ x } 0$
$= 0$
$\underline{V}_4y = 3 \text{ x } \underline{V}_3y$
$= 3 \text{ x } |\underline{V}_3| \sin\theta$
$= 3 \text{ x } 1$
$= 3$
$V_{2x} + 3V_3x = 2 + 0 = 2$
$V_{2y} + 3V_3y = 3 + 3 = 6$
$|\underline{V}_{2+}\underline{V}_3| = \sqrt{2^2 + 6^2} = \mathbf{6.32}$

$\theta = \tan^{-1}\left(\frac{6}{2}\right) = \mathbf{71.57°}$

d) $V_{3-2+1}x = V_3x - V_2x + V_1x = -2.999 - 2 + 0 = -4.999$
$V_{3-2+1}y = V_3y - V_2y + V_1y = 0.999 - 3 + 1 = -1.001$
$|\underline{V}_{3-2+1}| = \sqrt{(-4.999)^2 + (-1.001)^2} = \mathbf{5.098}$

$\theta_{2-3} = \tan^{-1}\left(\frac{-1.001}{-4.999}\right) = 21.82°$