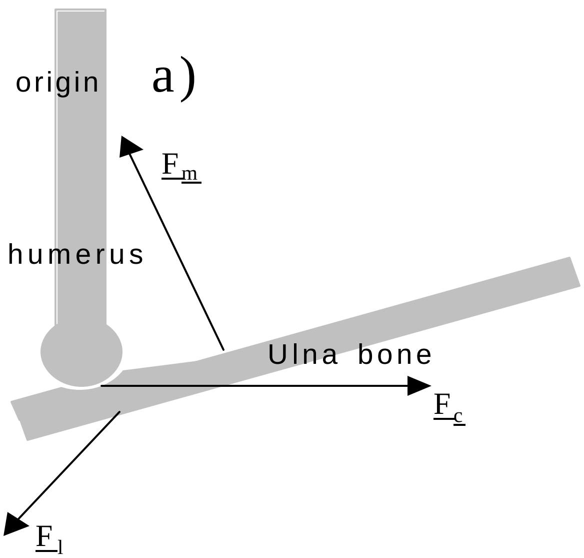
KINESIOLOGY – PT617
HOMEWORK
Forces acting on Humans

1) There are three forces acting on the ulna, biceps muscle force $|\underline{F}_m|$ = 1000 N, θ_m = 120°, posterior ligament force $|\underline{F}_l|$ = 1000, θ_l = 240°, & contact force with humerus $|\underline{F}_c|$ = 1000 N, θ_c = 360°. Note the angles made by the forces are relative to the horizontal coordinate axis.

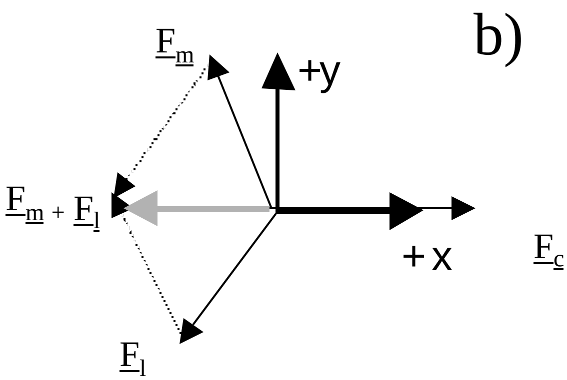
a) Draw these three vectors on the schematic diagram of the elbow joint (figure a).

Solution



b) Draw these three vectors relative to the coordinate axis drawn in figure b. Graphically represent the resultant force vector using parallelogram rule. (refer to lecture one notes).

Solution



The resultant force vector is zero as seen in the diagram. In the first step, using the parallelogram rule we add $\underline{F}_m$ & $\underline{F}_l$. This vector is shown as a thick gray arrow in the figure. Note that the vector $|\underline{F}_m + \underline{F}_l|$ is equal in magnitude and opposite in direction to $\underline{F}_c$. Thus there is no resultant force acting on the system.

c) Determine the sum of forces around the elbow joint using the numerical method find the direction and magnitude of the resultant joint vector. Would you expect the system to have any linear motion?

Solution

$Fx = |\underline{F}_m| \cos \theta_m + |\underline{F}_l| \cos \theta_t + |\underline{F}_c| \cos \theta_c$

$= 1000 \times \cos 120° + 1000 \times \cos 240° + 1000 \times \cos 360°$

$= -500 - 500 + 1000 = 0$ N

$Fy = |\underline{F}_m| \sin \theta_m + |\underline{F}_l| \sin \theta_t + |\underline{F}_c| \sin \theta_c$

$= 1000 \times \sin 120° + 1000 \times \sin 240° + 1000 \times \sin 360°$

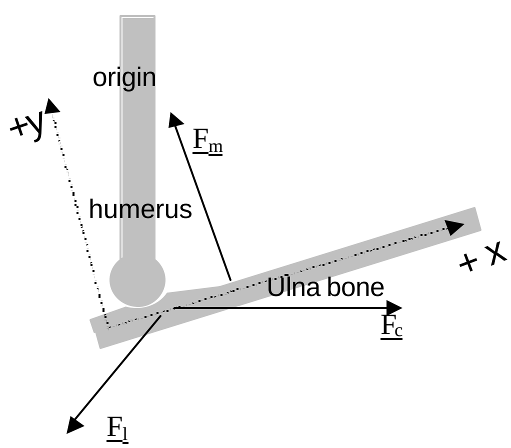
$= 866 - 866 + 0 = 0$ N

$$|\underline{F}| = \sqrt{Fx^2 + Fy^2} = \sqrt{0^2 + 0^2} = 0 \text{ N}$$

Since the sum of the forces in both x & y directions is zero, we would not expect the system to have any linear motion. Recall Newton's second law. The system will have motion only if there is a resultant force acting on it.

d) If we change the coordinate axes, such that the x-axis is along the ulna and the y axis is perpendicular to the ulna, orientation of the three force vectors relative to the new coordinate axis change to: $\theta_m = 90°$ $\theta_l = 210°$ $\theta_c = 330°$. If the magnitude of the forces remain the same, i.e. $|\underline{F}_m| = 1000$ N, $|\underline{F}_l| = 1000$ N & $|\underline{F}_c| = 1000$, using the numerical method find the magnitude and direction of the resultant joint vector. Is there a change in the magnitude and direction of the resultant force vector (from that calculated in 1c) by changing the coordinate axes?

Solution



$Fx = |\underline{F}_m| \cos \theta_m + |\underline{F}_l| \cos \theta_t + |\underline{F}_c| \cos \theta_c$
$= 1000 \times \cos 90° + 1000 \times \cos 210° + 1000 \times \cos 330°$
$= 0 - 866 + 866 = 0$ N

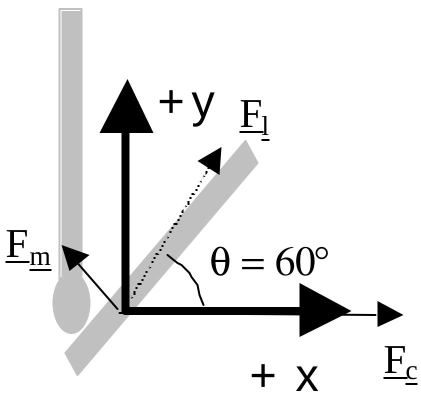
$Fy = |\underline{F}_m| \sin \theta_m + |\underline{F}_l| \sin \theta_t + |\underline{F}_c| \sin \theta_c$
$= 1000 \times \sin 90° + 1000 \times \sin 210° + 1000 \times \sin 330°$
$= 1000 - 500 + 500 = 0$

$$|\underline{F}| = \sqrt{Fx^2 + Fy^2} = \sqrt{0^2 + 0^2} = 0 \text{ N}$$

The magnitude of the resultant force vector is zero. Thus by changing the reference coordinate system, we do not change the resultant force acting on the system.

e) Suppose the elbow remains in a static condition in the horizontal direction (x) but changes its rotational position from 30° to 60°, the orientation of the biceps muscle force (θ_m) relative to the horizontal axis changes to 150° from 120° & but the orientation of the contact force vector (θ_c) remains at 360°. What would be the θ_l at this new elbow position if $|\underline{F}_m|$ = 1500 N, θ, $|\underline{F}_c|$ = 1000 N, and $|\underline{F}_l|$ =1000 N.

Solution



Static condition in horizontal direction means Fx = 0

$Fx = |\underline{F}_m| \cos \theta_m + |\underline{F}_l| \cos \theta_l + |\underline{F}_c| \cos \theta_c$
$= 1500 \times \cos 150° + 1500 \times \cos \theta_l + 1000 \times \cos 360°$.
$= -1300 + 1500 \times \cos \theta_l + 1000 = 0$
$\theta_l = \cos^{-1}(300/1500)$
$\theta_l = 78.46°$

Thus the ligament force, $\underline{F}_l$ will be directed at 78.46° from the horizontal as shown by the dotted line in the figure above.

2) A person weighing 80kg is walking with a horizontal velocity of 0.8 m/s and with a vertical acceleration of 1.5 m/s^2.

a) Find the total vertical force acting on the person.

Solution

From Newton's second law of motion:

$$F = m \times a$$
$$\Sigma F_y = m \times a_y$$
$$\Sigma F_y = 80 \times 1.5$$
$$= 120\ kgm/s^2$$
$$= \mathbf{120\ N}$$

Thus the total vertical force acting on the subject is 120N.

b) Find the horizontal momentum of the person.

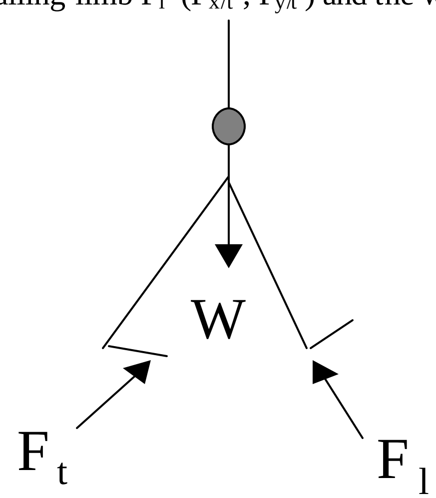
Solution

Horizontal momentum (M_x)= mass x velocity
$M_x = 80 \times 0.8$
M_x = **64 kgm/s**

c) The person receives a vertical ground reaction force of 450 N on the leading foot at heel strike. Find the vertical ground reaction force under the trailing foot. g = 10 m/s^2

Solution

There are a total of three forces acting on the person.
The contact force under the leading limb F_l ($F_{x/l}$, $F_{y/l}$), the contact force under the trailing limb F_l ($F_{x/t}$, $F_{y/t}$) and the weight (W) of the person as shown in the figure.



The forces acting in the vertical direction are:
$\Sigma F_y = F_{y/l} + F_{y/t} - W$
Substituting ΣF_y from part a) we get:

$120 = 450 + F_{y/t} - (80 X10)$ Note: $W = m \times g = 80 \times 10$
$F_{y/t} = 120 - 450 + 800$
$\mathbf{F_{y/t} = 470\ N}$

Thus the vertical ground reaction force under the trailing foot is 470 N.

d) If the person receives a posterior ground reaction force of -30 N under the leading foot and a 20 N force under trailing foot upon heel strike. What is the horizontal acceleration of the person? Is the person speeding up or slowing down?

Solution

There are only two forces acting on the person in the horizontal direction, i.e. the two contact forces ($F_{x/l} + F_{x/t}$).

Therefore, $\Sigma F_x = F_{x/l} + F_{x/t}$
$F_{x/l} + F_{x/t} = m \times a$
$-30 + 20 = 80 \times a$
$a = -10/80$
$\mathbf{a = -0.125\ m/s^2}$

The person is therefore slowing down upon initial ground contact of the leading limb.

e) The velocity of the body center of mass decreases from 0.8 m/s to 0.6 m/s from heel strike (t1) to foot flat (t2), and the sum of horizontal ground reaction force remains constant at -160N. Find the horizontal momentum of the person at time of foot flat, and the impulse from time t1 to t2.

Solution

$G(t2) = 80 \times 0.6$
$= \mathbf{48\ kgm/s}$

Impulse = Change in momentum

$= m(v2 - v1)$

= 80 (0.6 – 0.8)

= -16 kgm/s

Alternative way

Impulse = $\int F dt$

$= F \int_{t1}^{t2} dt$

= -160 (t2 – t1)

= -160 x .1

= -16 Ns