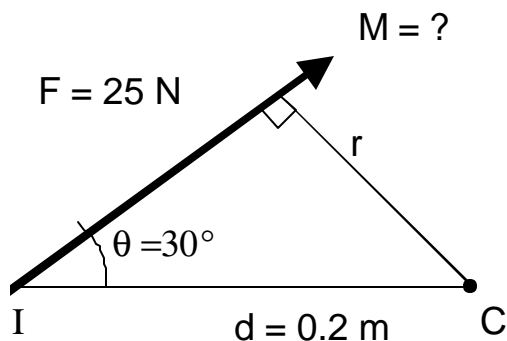


KINESIOLOGY – PT617
Moment of Force and Rotation
Homework Solution

- Muscle force $|\mathbf{F}| = 25 \text{ N}$ has an insertion at I, $\theta = 30^\circ$, and the distance between insertion and joint center C is $d = 0.2 \text{ m}$. Sketch a diagram to represent the muscle force, joint center and the moment arm from the joint center to the force. Find the direction and the magnitude of moment M about C using method 1 described in your class notes.



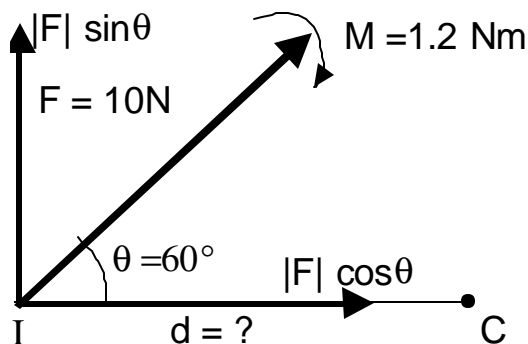
$$r = d \sin \theta = 0.2 \text{ m} \times \sin \theta = 0.2 \times \sin 30^\circ$$

$$M = |\mathbf{F}| \times r = 25 \text{ N} \times 0.2 \times \sin 30^\circ = 2.5 \text{ Nm}$$

The moment should be in clockwise in direction.

- Muscle force $|\mathbf{F}| = 10 \text{ N}$ has an insertion at I, $\theta = 60^\circ$, and a moment arm d between insertion and joint center. If the muscle moment is 1.2 Nm calculate the moment arm d for this muscle.

Solution:



Using method 2 compute moment arm d as:

$$|M| = d \times |F| \sin \theta$$

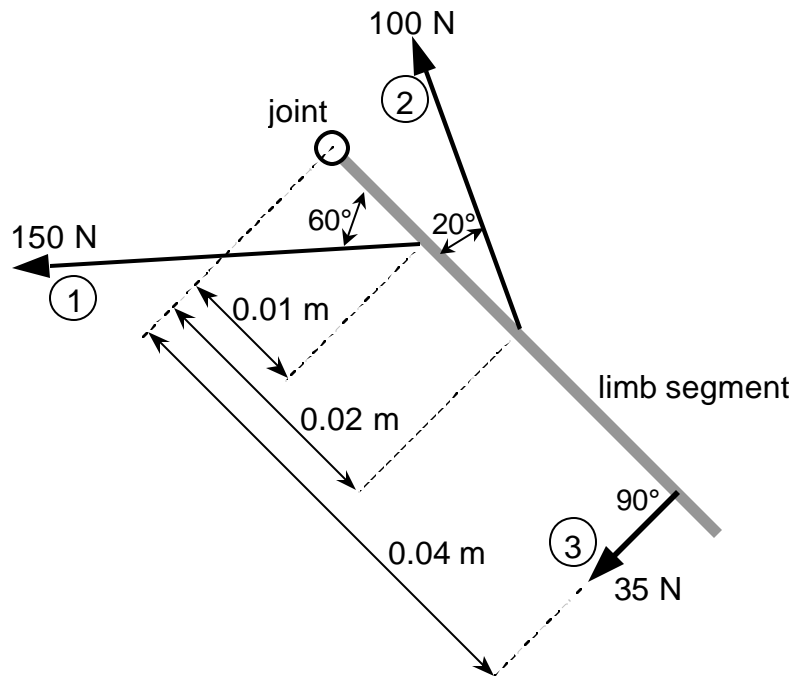
$$d = |M| / |F| \sin \theta$$

$$d = 1.2 / 10 \sin 60^\circ$$

$$d = 1.2 / 8.66$$

$$d = 0.1385\text{m}$$

3. Three muscles (labeled 1-3) act about the proximal joint of a limb segment. The magnitude, direction, and point of application of the force generated by each muscle is shown in the figure below:



- a) Which muscle generates the largest force? The smallest force?
 You can read the force generated by each muscle directly from the above diagram. Muscle 1 generates the largest force and muscle 3 generates the smallest force.
- b) Which muscle has the largest moment arm? The smallest moment arm?
 Find the moment arm using method 2.

Steps

- Make a table as shown in the examples solved in class.
- find the tangential (F_t) & normal (F_n) components of the force

-moment arm (r) = distance of F_n from the axis. Thus, $r = d$ for all the muscles.

Table 1

Muscle	Force	F_t	F_n	Moment arm
A	F N	$ F \cos \theta$	$ F \sin \theta$	$r = d$
1	150 N	$150 \times \cos 60^\circ$ $= 75 \text{ N}$	$150 \times \sin 60^\circ$ $= 130 \text{ N}$	$r_1 = 0.01$
2	100 N	$100 \times \cos 20^\circ$ $= 94 \text{ N}$	$100 \times \sin 20^\circ$ $= 34 \text{ N}$	$r_2 = 0.02 \text{ m}$
3	35 N	0	$35 \times \sin 90^\circ$ $= 35 \text{ N}$	$r_3 = 0.04 \text{ m}$

< smallest

< largest

- c) Which muscle generates the joint moment with the largest magnitude? The joint moment with the smallest magnitude?

-We calculated the magnitude of F_n & the moment arm r in table 1.

-Compute the magnitude ($|M|$) of the moment by multiplying r times F_n .

-Use the right hand rule to determine whether the moment is positive or negative.

-The magnitude of the joint moment is the absolute value of the moment.

Muscle	r	F_n	Moment	Magnitude
1	0.01 m	130 N	-1.30 Nm	1.30 Nm
2	0.02 m	34 N	0.68 Nm	0.7 Nm
3	0.04 m	35 N	-1.4 Nm	1.4 Nm

< smallest

< largest

- d) Which muscle produces the largest compressive force at the joint? The smallest compressive force?

To find the compressive or distractive force produced by a muscle at a joint:

Step 1: Find the magnitude of the tangential force vector that points along the line between the joint axis and muscle's point of attachment.

$$F_t = |F| \cos \theta.$$

Step 2: Identify whether the muscle produces a compressive or a distractive force, based on the direction of the tangential force. A tangential force that points towards the joint axis produces a compressive force; a tangential force that points away from the joint axis produces a distractive force.

Note: If a muscle force points perpendicular to the line between the joint axis and the muscle's point of attachment, there will be no tangential force vector! In this case, $F_t = 0$; the muscle produces neither a compressive nor a distractive force.

We calculated the tangential component of the muscle forces in table 1.
Thus:

Muscle	F_t	F_t	Direction
1	$150 \times \cos 60^\circ$ $= 75 \text{ N}$	75 N	compressive
2	$100 \times \cos 20^\circ$ $= 94 \text{ N}$	94 N	compressive
3	no tangential force vector	0 N	none

< largest

< smallest

e) What is the magnitude and direction of the resultant joint moment?

By definition, the resultant joint moment (RJM) is the composition (sum) of all of the moments about a joint that are produced by forces that (i) act across the joint and (ii) originate from inside the body.

In this problem, all four muscles act across the joint and their forces originate from inside the body, so the moment generated by each muscle gets included in the RJM.

Using the moments computed in (c):

$$\text{RJM} = (-1.35 \text{ Nm}) + (0.7 \text{ Nm}) + (-1.4 \text{ Nm}) = \mathbf{-2.05 \text{ Nm}}$$

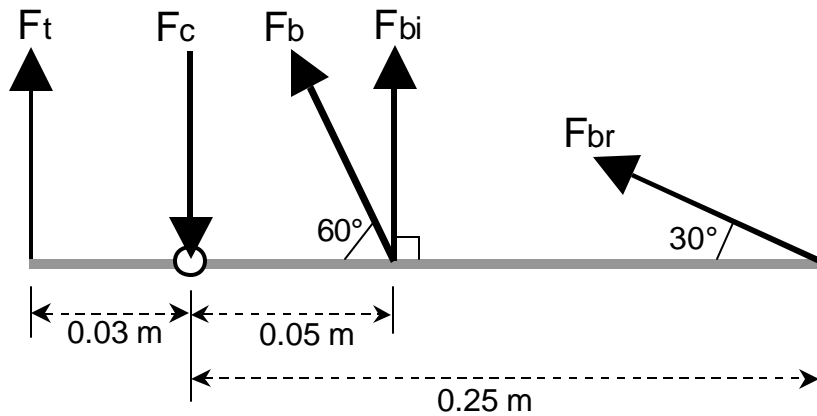
Because the resultant joint moment is negative, **it acts clockwise about the joint.**

Note: Some things to consider:

- Muscle 1 produced the greatest force, but did not produce the greatest moment because its moment arm was small.
- Muscle 3 produced the smallest force, but did not produce the smallest moment because its moment arm was large.
- Muscle 1 attached to the limb segment closest to the joint, but did not have the smallest moment arm because of the muscle's direction.
- Muscle 2 produced the greatest compressive force, even though it did not produce the greatest total force. Again, this is because of the direction of the muscle.
- Muscles directed more perpendicular to the limb segment and less along the segment (e.g. muscle 3) produce primarily rotation, with little compression at the joint.
- Muscles directed more along the limb segment and less perpendicular to the segment (e.g. muscle 2) produce primarily compression at the joint, with less rotation.

The effect of a muscle is clearly determined by a number of different factors.

4. A forearm is acted on by three flexor muscles and one extensor muscle. The diagram below shows the point of application and direction of the force generated by the triceps (F_t), the brachialis (F_b), the biceps brachii (F_{bi}), and the brachioradialis (F_{br}). Also shown is the joint contact force (F_c) acting at the joint center.



- a) Derive an expression for the resultant joint moment (RJM) about the joint center, in terms of F_t , F_b , F_{bi} , F_{br} , and F_c .

As in Problem 2, the resultant joint moment (RJM) is the composition (sum) of all moments about a joint that result from the forces originating from within the body and acting across the joint. In this problem, all four muscle forces and the joint contact force act across the joint and originate from within the body, so the moment generated by each force gets included in the resultant joint moment.

Step 1: Using the method in Problem 2, find the moment arm (r) of each force that will be included in the RJM.

Force	Moment Arm Equation	Moment Arm
F_t	$r_t = 0.03 \text{ m}$	0.03 m
F_b	$r_b = (0.05 \text{ m}) \sin 60^\circ$	0.04 m
F_{bi}	$r_{bi} = 0.05 \text{ m}$	0.05 m
F_{br}	$r_{br} = (0.25 \text{ m}) \sin 30^\circ$	0.125 m
F_c	$r_c = 0$	0 m

The moment arm of the contact force is 0 because the force acts directly into the joint center.

Step 2: Compute the magnitude of the moment (M) about the joint center due to each force by multiplying the moment arm (r) times the force (F).

Step 3: Use the right hand rule to determine whether each moment is positive or negative.

Force	r	F	M	M
Ft	0.03 m	Ft	(0.03 m) Ft	– (0.03 m) Ft
Fb	0.04 m	Fb	(0.04 m) Fb	(0.04 m) Fb
Fbi	0.05 m	Fbi	(0.05 m) Fbi	(0.05 m) Fbi
Fbr	0.125 m	Fbr	(0.125 m) Fbr	(0.125 m) Fbr
Fc	0 m	Fc	0	0

For now, since we don't know the magnitudes of any of the forces, we substitute the variable name for F in the equation for |M| and, when determining the sign of the moment, we assume that the force acts in the direction shown in the figure.

Step 4: Compute the resultant joint moment as the sum of the moments generated about the joint center by each of the forces we've identified as contributing to the RJM:

$$\mathbf{RJM} = - (0.03 \text{ m}) \mathbf{Ft} + (0.04 \text{ m}) \mathbf{Fb} + (0.05 \text{ m}) \mathbf{Fbi} + (0.125 \text{ m}) \mathbf{Fbr}$$

b) Use the expression from (a) to compute the resultant joint moment for each of the following combination of forces:

Solve by substituting the numbers in each row into the above equation...

Ft	Fb	Fbi	Fbr	Fc	RJM
100 N	0 N	0 N	0 N	100 N	-3 Nm
100 N	20N	0 N	0 N	117 N	-2.2 Nm
100 N	30N	20 N	6.4 N	149 N	0
100 N	150 N	0 N	0 N	230 N	3 Nm

Note: Some things to consider:

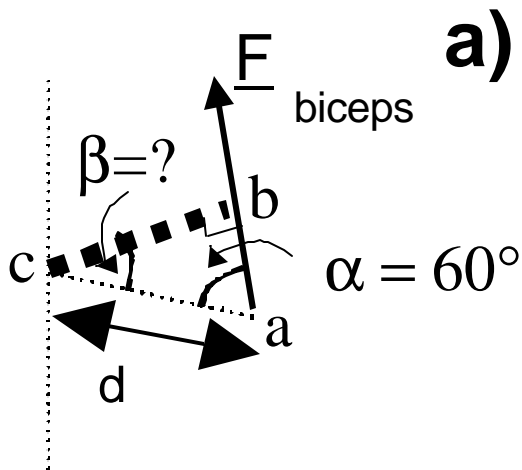
- The fact that an extension (negative) RJM was observed did not mean that the flexor muscles (b, bi, br) were inactive (i.e. producing no force).
- A flexion (positive) RJM did not mean that the extensor muscle (t) was inactive.
- An RJM of 0 did not mean that the muscles acting about the joint were inactive.
- The same RJM could result from different combinations of force produced by the 3 flexor muscles.
- RJM's of different magnitude and direction were generated in the first 5 examples, even though the triceps force remained constant.

Thus, the RJM tells you which muscle group is producing the greatest moment about the joint, but does not tell you how much force each individual muscle is generating.

5. Drawn below are three positions of the elbow joint: a) $\theta = 120^\circ$ b) $\theta = 75^\circ$ c) $\theta = 60^\circ$. Moment arms are indicated by dark dotted lines. Also drawn on the figures are the

angles (α) made by the biceps muscle with the ulna and some additional information that you can use.

- a) If the biceps muscle is inserted on the ulna at a distance (d) of 3 cm from the elbow joint, and the force exerted by the biceps for all the three positions is 400N, calculate the magnitude of the moment created by the biceps muscle about the elbow joint for each of the three positions. **Hint:** Use your concepts about laws for triangles.



Solution

For figure a): elbow at $\theta = 120^\circ$

Step 1:

Find the angle β formed in the triangle abc.

Since sum of angles of triangles = 180°

$$\beta + \alpha + 90^\circ = 180^\circ$$

$$\beta + 60^\circ + 90^\circ = 180^\circ$$

$$\beta = 30^\circ$$

Step 2

The moment arm is formed by side cb (black dotted line) of the triangle abc.

If moment arm cb = r

$$r = d \times \cos\beta$$

$$r = 0.03 \text{ m} \times \cos 30^\circ$$

$$r = 0.0259 \text{ m}$$

$$r = 2.6 \text{ cm}$$

Step 3

$$M = |\underline{F}| \times r$$

$$= 400 \times 0.026$$

$$= \mathbf{10.4 \text{ Nm}}$$

For figure b): elbow at $\theta = 75^\circ$, follow similar steps as for a).

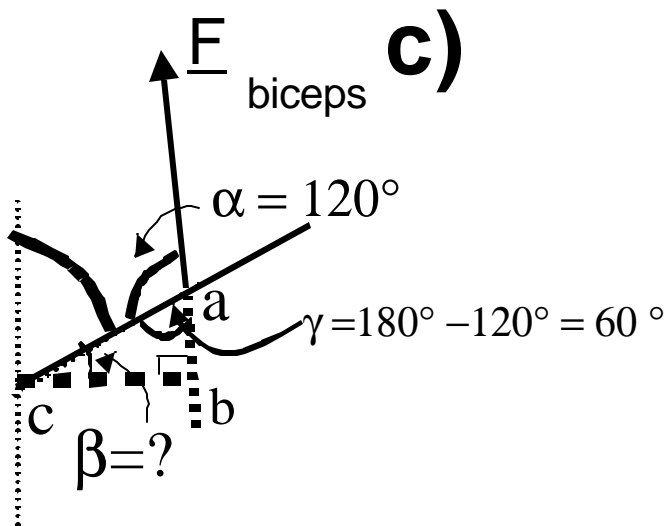
$$\begin{aligned}\beta + \alpha + 90^\circ &= 180^\circ \\ \beta + 75^\circ + 90^\circ &= 180^\circ \\ \beta &= 15^\circ\end{aligned}$$

$$\begin{aligned}r &= d \times \cos\beta \\ r &= 0.03 \text{ m} \times \cos 15^\circ \\ r &= 0.0289 \text{ m} \\ r &= 2.9 \text{ cm} \\ M &= |\underline{F}| \times r \\ &= 400 \times 0.029 \\ &= \mathbf{11.6 \text{ Nm}}\end{aligned}$$

For figure c): elbow at $\theta = 120^\circ$, follow similar steps as for a).

Note that when we draw the moment arm as the perpendicular distance from the joint center to the line of action of the biceps force to form triangle abc, the angle α formed by the biceps insertion doesn't form one of the angles of triangle abc.

Thus we need to perform an additional step to find γ before we can find β (see figure below).



$$\begin{aligned}\gamma &= 180^\circ - \alpha \\ &= 180^\circ - 120^\circ \\ \gamma &= 60^\circ\end{aligned}$$

Since sum of angles of triangles = 180°

$$\begin{aligned}\beta + \gamma + 90^\circ &= 180^\circ \\ \beta + 60^\circ + 90^\circ &= 180^\circ\end{aligned}$$

$$\beta = 30^\circ$$

Step 2

The moment arm is formed by side cb (black dotted line) of the triangle abc.

If moment arm cb = r

$$r = d \times \cos\beta$$

$$r = 0.03 \text{ m} \times \cos 30^\circ$$

$$r = 0.0259 \text{ m}$$

$$r = 2.6 \text{ cm}$$

Step 3

$$M = |\underline{F}| \times r$$

$$= 400 \times 0.026$$

$$= \mathbf{10.4 \text{ Nm}}$$

- b) Suppose the force exerted by the biceps is different for the three positions, however the moment created by the biceps about the elbow joint remains the same for all the positions. In which of the three positions will the person have to generate minimal biceps force to hold the position. Justify your answer. **Hint:** Biceps forms a second class lever at the elbow joint.

Answer

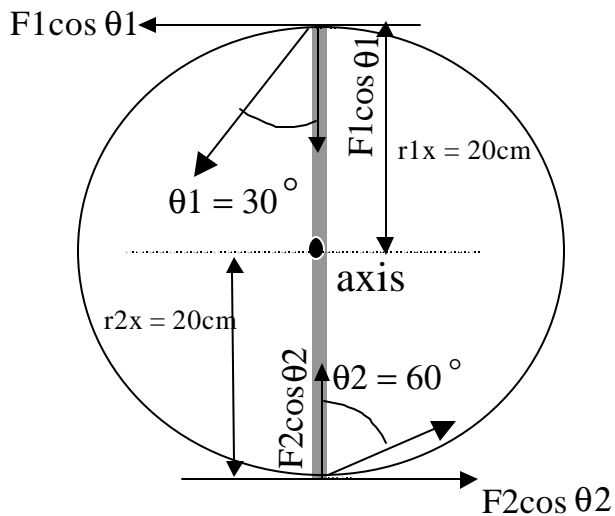
The person will have to generate minimal biceps force in position b), i.e. when the elbow is at 75° .

Justification

The biceps forms a second class lever at the elbow joint with the effort arm of the lever being the moment arm of the biceps muscle. Thus greater moment arm, greater is the effort arm and lesser is the effort (force) required to be generated by the biceps muscle to hold the ulna in that position. In position b) the moment arm of the biceps muscle is greatest compared to positions a) & c). Thus the person will have to generate minimal biceps force to maintain position b) compared to a) & c).

6. There are two forces $|F_1| = 300\text{N}$ & $|F_2| = 300\text{N}$ acting at angles of 30° & 60° respectively at the two ends of a vertical spike attached to a circular wheel along its diameter. The diameter of the wheel is 40 cm and the axis of rotation of the wheel is at the center of the circle.
- a) Resolve the forces to find the sum of forces in the x & y direction.
- b) Find the magnitude of the resultant moment about the axis of the wheel.

Solution



Components for F1

$$F1x = |F1| \cos \theta = 300 \times \cos 30^\circ = 300 \times 0.866 = -259.8 \text{ N}$$

$$F1y = |F1| \sin \theta = 300 \times \sin 30^\circ = 300 \times 0.5 = -150 \text{ N}$$

$$F2x = |F2| \cos \theta = 300 \times \cos 60^\circ = 300 \times 0.5 = 150 \text{ N}$$

$$F2y = |F2| \sin \theta = 300 \times \sin 60^\circ = 300 \times 0.866 = 259.8 \text{ N}$$

Force	x	rx	Moment x	y	ry	Moment	Direction
$ F_1 $	-259.8 N	0	0	-150 N	.2m	30 Nm	CCW
$ F_2 $	150 N	0	0	259.8 N	.2 m	51.96 Nm	CCW
Sum	-109.8 N		0	109.8 N		81.89 Nm	CCW

The sum of forces in the x direction = -109.8 N

The sum of forces in the y direction = 109.8 N

The magnitude of the resultant joint moment about the axis of the wheel = 81.89 Nm.

- c) Which direction will the wheel rotate as a result of the application of the forces to the spike? Do these two forces form a force couple? Justify your answer.

Answer: The resultant joint moment acts in the counter clockwise direction, thus the wheel will rotate in the counter clockwise direction as a result of the application of the forces to the spike.

No, these two forces do not form a force couple.

Justification:

A force couple must satisfy the following conditions:

- produce a resultant moment about an axis located midway between the points of application of the two forces.
- produce no resultant force on the object (i.e., the forces sum to zero).

thus:

- a force couple produces rotation without translation.

Since the sum of forces produced by the two forces does not equal zero, these two forces do not form a force couple. The wheel will translate as well as rotate.

- d) Why do the two forces F_1 and F_2 of equal magnitude produce moments of different magnitudes about the axis of the wheel?

Answer: Though the forces have equal magnitudes, they are acting at different angles relative to the spike and thus produce different moments.

Note: If these two forces F_1 & F_2 of equal magnitude were acting at equal angles relative to spike they would:

- produce moments of equal magnitudes about the axis of the wheel
- form as force couple, i.e. Sum of forces in x & y direction would = 0.