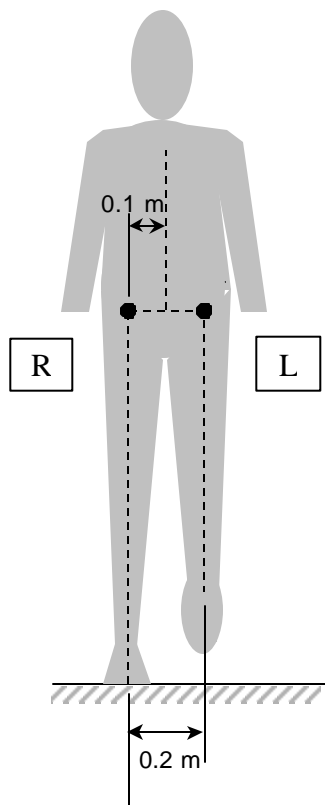


KINESIOLOGY – PT617
HOMEWORK

Mechanical Principles : Free Body Diagrams and Equilibrium

- 1) A man is standing still on his right leg, as shown in the figure. The person experiences only a normal reaction force between the ground and the right foot (fg) which acts along the line joining the center of left hip and the right foot (dotted line). The center of mass of the right lower limb passes through the line same line.



mass of the man: 50 kg.
 mass of each lower limb: 10 kg.
 mass of the cane : 0 kg

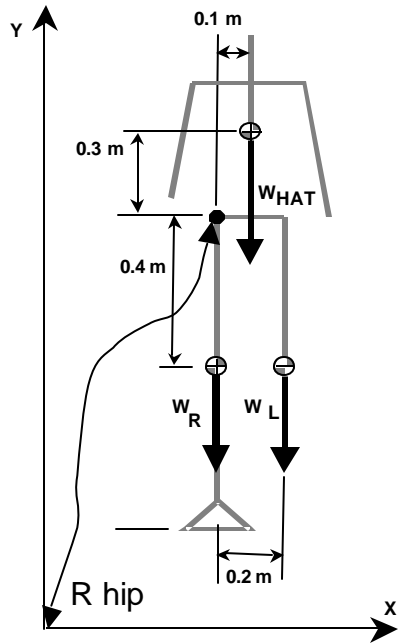
center of mass of the head-arms-torso (HAT): 0.3 m above the hip midpoint
 center of mass of the lower limb:
 0.4 m below the hip

- 1) Find the location of the center of mass of the person (total body) with respect to the R hip joint.

Solution: we need to use the following equations.

$$x_{com} = \frac{\sum x_i m_i}{\sum m_i}, \quad y_{com} = \frac{\sum y_i m_i}{\sum m_i}$$

- We are given the mass of the lower extremity and the mass of the person. Thus, the mass of the HAT is $50\text{kg} - 10\text{kg} - 10\text{kg} = 30\text{kg}$.
- Find the position coordinates of the center of mass of each segment in relation to the hip joint from the given information.



Mass m_i	x_i	Y_i	$x_i m_i$	$y_i m_i$	
M_{HAT}	30kg	0.1m	0.3m	$0.1 \times 30 = 3$	$0.3 \times 30 = 9$
M_R	10kg	0	-0.4m	0	$-0.4 \times 10 = -4$
M_L	10kg	0.2m	-0.4m	$0.2 \times 10 = 2$	$-0.4 \times 10 = -4$
Σ	50	0.1m	0.02m	5	1

- 2) Find the location of the center of mass of the HAT+L-limb with respect to the R hip joint.

Solution:

Mass m_i	x_i	Y_i	$x_i m_i$	$y_i m_i$	
M_{HAT}	30kg	0.1m	0.3m	$0.1 \times 30 = 3$	$0.3 \times 30 = 9$
M_L	10kg	0.2m	-0.4m	$0.2 \times 10 = 2$	$-0.4 \times 10 = -4$
Σ	40	0.125m	0.125m	5	5

- 3) Suppose the person were to hold a cane in the L hand, as shown in the figure below and press straight downward on the cane, such that there is only a normal

force (f_c) between the cane and the ground 0.3 m from R hip in horizontal. Find the reaction force on the cane from the ground. (Hint, use total body for your FBD, and (Hint, use total body for your FBD, and sum of moment about the point of application of f_g is zero.)

Solution

Step 1: Identify the basic approach that will be used to solve the problem.

Since we're told that sum of moments about R hip = 0

$$\sum (\underline{r} \times \underline{F}_{\text{external}}) + \sum \underline{M}_{\text{external}} = 0$$

We can therefore construct this equation and solve it to find the ground reaction force.

Step 2: Draw the Free Body Diagram.

a) Identify the system

Since all the forces we know and all the forces we want to find act on the outside of the total body+cane, we choose the total body+cane as the system.

b) Identify the external forces and moments

The external forces acting on our system are then:

- the force f_c between the cane and the ground
- weight of the body+cane (we can use the total body mass since the weight of the cane = 0)
- the force f_g between the right foot and the ground.

Note: there is not shear component of the ground reaction force as f_g is normal to the ground.

There are no external moments acting on our system.

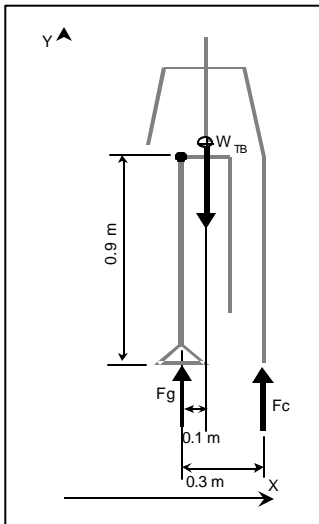
c) Find the point of application and direction of each external force and moment

- We know that f_c acts at the tip of the cane and we were told that f_c is normal to the ground
- f_g acts along the line joining the center of the hip joint and the center of the foot and is normal to the ground.
- We know that the body+cane's weight acts downward from the body+cane at a distance 0.1m from the R hip. (obtained from question 1)

d) Add a reference frame

Since all our forces point vertically or horizontally, choose x to point right, y to point upward (90° counterclockwise of the x-axis).

The resulting free body diagram is shown on the left on the next page.



Free Body Diagram

Step 3: Compute the magnitude of the total body weight appearing in the free body diagram:

$$|\text{weight}| = (\text{mass}) * (\text{acceleration due to gravity})$$

$$|W_{TB}| = 50 * 10 = 500 \text{ N (acting downwards, therefore } -500 \text{ N)}$$

Step 4: Construct the equations of equilibrium for our system.

Since we are given the hint that sum of moments about the point of application of $f_g = 0$, and we construct our equation as:

$$\Sigma M = 0:$$

There are two unknowns' f_c & f_g , thus typically we would need two equations of motion to solve for these. However, since we are taking the moments about f_g , the moment about $f_g = 0$ and we eliminate f_g from the equation as shown below.

$$f_g \times 0 - 500 \text{ N} \times 0.1\text{m} + f_c \times 0.3\text{m} = 0$$

Step 5: Solve the equation of equilibrium to find the unknown value.

$$f_c = (500\text{N} \times 0.1\text{m}) / 0.3\text{m}$$

$$f_c = \mathbf{166.7 \text{ N}}$$

- 4) Find the magnitude of f_g acting on the R limb. (Hint, use total body for your FBD, and sum of vertical forces is zero.)

Solution

Follow same steps as in question 3) up to step 4.

Step 4: Construct the equations of equilibrium for our system.

Since we are given the hint that sum of forces in the vertical direction = 0 and we know all the forces except f_g in the vertical direction, we use the equation:

$$\Sigma F_y = 0$$

$$f_c + W_{TB} + f_g = 0$$

Force	Fy
WTB	-500 N
fc	166.7 N
fg	? N

Step 5: Solve the equation of equilibrium to find the unknown value.

$$167.7\text{N} - 500\text{N} + fg = 0$$

$$fg = 500 - 166.7$$

$$\mathbf{fg = 333.3\text{ N}}$$

- 5) Find the resultant joint force (RJF) and resultant joint moment (RJM) at the hip of the right limb. (Hint, use HAT+L-limb for your FBD, and sum of moment about R hip is zero.)

Solution

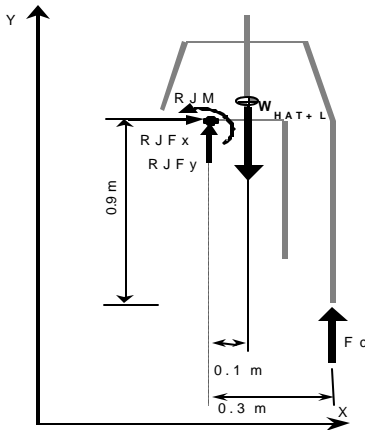
Step 1: Identify the basic approach that will be used to solve the problem.

We are given that the sum of moments about the hip joint = 0 and that the sum of vertical forces = 0. Thus we can construct our equations of motion and solve the problem

$$\Sigma M = 0$$

$$\Sigma F = 0$$

Step 2: Draw the Free Body Diagram using HAT + L limb as the system



Step 3: Compute the magnitude of the weight if the HAT + L limb appearing in the free body diagram.

$$|\text{weight}| = (\text{mass}) * (\text{acceleration due to gravity})$$

$$|W_{TB}| = 40 \times 10 = 400\text{ N (acting downwards, therefore } -400\text{ N)}$$

Step 4: Construct the equations of equilibrium for our system.

a) Find the amount of each force in the x and y directions

Force	F _x	F _y
WHAT +L	0	-400N
RJF _x	RJF _x	0
RJF _y	0	RJF _y
f _c	0	167.7 N

b) Choose an axis of rotation to compute the moments about.

Since we are given that sum of moments about the R hip = 0, we commute the moments with respect to the R hip.

c) Compute the moment produced by each force and external moment about the axis of rotation chosen.

Force	F	moment arm (r)	M = r F	M
WHAT +L	400 N	0.125 m	50 Nm	- 50 Nm
RJF _x	RJF _x	0	0	0
RJF _y	RJF _y	0	0	0
RJM	-	-	RJM	+ RJM
f _c	166.7	0.3 m	50 Nm	+50 Nm

d) Create the equations of equilibrium by adding the corresponding table column

$$\Sigma F_x = 0: \quad RJF_x = 0$$

$$\Sigma F_y = 0: \quad - 400 \text{ N} + 166.7 \text{ N} + RJF_y = 0$$

$$\Sigma M = 0: \quad - 50 \text{ Nm} + 50 \text{ Nm} + RJM = 0$$

Step 5: Solve the equations of equilibrium to find the unknown values.

$$RJF_x = 0 \text{ N}$$

$$RJF_y = 233.3 \text{ N} \quad (\text{i.e. } RJF = 233.3 \text{ N upward})$$

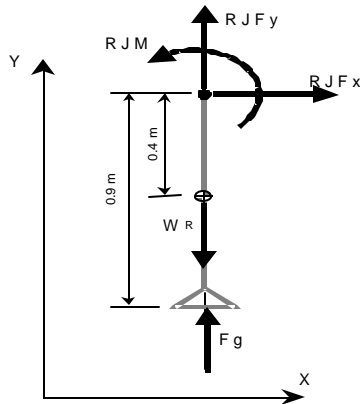
$$RJM = 0 \text{ Nm}$$

Thus the resultant joint force is upward as shown in our free body diagram and the RJM about the hip joint = 0.

6) Find the resultant joint force (RJF) and resultant joint moment (RJM) at the hip of the right limb. Use R-limb for your FBD.

Step 1: Similar to 5)

Step 2: Draw the Free Body Diagram using R limb as the system



Step 3: Compute the magnitude of the weight if the HAT + L limb appearing in the free body diagram.

$$|\text{weight}| = (\text{mass}) * (\text{acceleration due to gravity})$$

$$|W_R| = 10 * 10 = 100 \text{ N (acting downwards, therefore -100 N)}$$

Step 4: Construct the equations of equilibrium for our system.

a) Find the amount of each force in the x and y directions

Force	F _x	F _y
W _R	0	-100 N
R _J F _x	R _J F _x	0
R _J F _y	0	R _J F _y
f _g	0	333.3 N (from question 4)

b) Choose an axis of rotation to compute the moments about.

Since we are given that sum of moments about the R hip = 0, we compute the moments with respect to the R hip.

c) Compute the moment produced by each force and external moment about the axis of rotation chosen.

Force	moment arm (r)	F	M = r F	M
W _R	0	-100 N	0	0
R _J F _x	0	R _J F _x	0	0
R _J F _y	0	R _J F _y	0	0
F _g	0	333.3 N	0	0
R _J M	-	-	R _J M	+ R _J M

d) Create the equations of equilibrium by adding the corresponding table column

$$\Sigma F_x = 0: \quad R_{JF_x} = 0$$

$$\Sigma F_y = 0: \quad -100 \text{ N} + R_{JF_y} + 333.3 \text{ N} = 0$$

$$\Sigma M = 0: \quad 0 + R_{JM} = 0$$

Step 5: Solve the equations of equilibrium to find the unknown values.

$$R_{JF_x} = 0 \text{ N}$$

$$R_{JF_y} = -233.3 \text{ N} \quad (\text{i.e. } R_{JF} = 233.3 \text{ N downward})$$

$$R_{JM} = 0 \text{ Nm}$$

Note: Using different systems for the FBD in 5) vs 6) gives the same answer !!!