

Transferability, Finality, and Debt Settlement*

Charles M. Kahn
Department of Finance
University of Illinois
c-kahn@uiuc.edu

William Roberds
Research Department
Federal Reserve Bank of Atlanta
william.roberds@atl.frb.org

March 16, 2005

Abstract

The process of payment is fundamental to exchange in a decentralized economy. In production economies, payments often take the form of transfers of inside money, i.e., specialized forms of debt. Associated with each type of inside money is a set of rules that governs both the legitimacy of such transfers as means of extinguishing other debts, and the allocation of the ensuing risks.

In this paper we develop a model of debt as inside money. In a simple mechanism design framework we show the advantages of transferable debt over simple chains of credit.

*The authors are grateful to Roberto Chang for helpful comments, as well as to participants in seminars at the Bank of England, the European Central Bank, the Federal Reserve Banks of Atlanta, Cleveland, Kansas City, New York, and Richmond, the University of Iowa, the London School of Economics, Pennsylvania State University, Purdue University, Stanford University, the Swiss National Bank, Texas Christian University and Tokyo University. Any remaining errors are the authors' responsibility. The views expressed here are the authors' and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

1 Introduction

Since at least Wicksell (1935), economists have focused on money as a means of overcoming the double-coincidence problem. Before fiat money, goods were commonly traded for gold and gold for goods. But there have long been means of overcoming this problem other than the exchange of gold. Debt, in particular, ties up fewer resources than does the use of precious metal. Nonetheless, not all debt serves as money: only debt which is passed from hand to hand feels like money—mere chains of debt obligations do not. Two characteristics are crucial to the money-ness of debt. The first is that the enforceability of the debt should not diminish when the debt is transferred. The second is that the transfer should be “final” in some sense, i.e., debt can only be considered a good substitute for commodity money when it can be used to discharge other debts.

The transfer of debt has served as a fundamental building block of both historical and contemporary financial systems. The special role of certain types of debt in providing final payment, e.g., bank debt, is firmly entrenched into the laws and business practices of developed economies. In the U.S., for example, a cashier’s check (a check drawn by a bank on itself) can serve much the same function as legal tender in terms of its capacity to discharge an obligation.¹

In this paper we develop a simple story of transferable debt. The story is explicitly dynamic: The agents find it desirable to extend credit at one time and then to extinguish that credit through a payment—that is, by transfer of someone else’s debt.² We begin with an introductory account of the model and then give a more formal account in a mechanism design framework. In order to show the significance of transferable debt, it is necessary to compare it with non-transferable alternatives. We introduce a variety of debt instruments and show how transferable debt provides efficiency gains relative to other instruments.³ Our mechanism design framework points out two noteworthy advantages of transferable debt: First and fundamentally, it allows for finality—it makes it possible for less-than-reliable agents to

¹Despite its legal ubiquity, the concept of a final debt transfer might seem odd from the standpoint of contract theory. Having recourse to debtors in certain eventualities would generally seem to be advantageous from the point of view of risk sharing and useful in generating optimal incentives. Cutting off this flexibility by making a debt transfer final at least requires some justification.

²Thus, unlike previous models of inside money, ours accounts for settlement of *debt*, not simply for exchange of inside money for goods.

³An earlier version of the paper, available from the authors, then uses a similar framework to examine a historically prevalent set of rules for effecting payments.

be removed from a credit chain in a timely fashion. Additionally, it allows agents in a bilateral transaction to receive information concerning the actions of third parties.

1.1 Introduction to the model

In our model, trade occurs through a sequence of bilateral meetings. Preferences, endowments, and trading opportunities are such that within a period there is never a double coincidence of wants, ruling out barter exchanges. Nor is fiat money available to facilitate exchange. Instead, trade must be carried out on the basis of promises to pay. We assume that enforcement is in the hands of a centralized court. The court cannot observe the trades themselves, but it can carry out limited rewards and punishments, based on reports made by the parties subsequent to the trades.

We suppose there are three agents, A , B , and C . In period 1, A can produce a good (“flour”). In period 2, B can produce a finished good (“bread”) that *sometimes* requires A ’s good as an input. The finished good is always desired by C , *provided he can acquire it in period 2*. At a later date (period 4), C is endowed with a good (“gold”) that is desired by both A and B .

The specific sequence of meetings is depicted in figure 1. A and B meet in period 1, and B and C meet in period 2. Following the arrival of C ’s endowment in period 4, C meets again with B (period 4) and C meets directly with A (period 6).

We allow two additional meetings between A and B (periods 3 and 5); these meetings will become significant when we introduce transferable debt. Meetings other than these are not feasible, and in particular there is never a meeting of all three agents.

If either party to a trade makes a report, then the court examines the available documentation and may decide to penalize the nonperforming counterparty. The court has a technology available for transferring part of a penalty to other parties. The court’s enforcement technology has two limitations: there is a limit to the size of a feasible punishment, and the transfer of resources to other participants involves wastage. In addition it is costly for agents to access the court.

First let us focus only on B and C , by considering the special case where B *never* needs A ’s flour to deliver bread to C in period 2. If access to the court were costless, B could deliver bread to C in period 2, with the understanding that C would reciprocate by delivering gold to B in period 5. Failure by C to reciprocate would result in B making a protest to the court, which would result in the court punishing C . As long as B receives no

reward for his announcement he has nothing to gain by lying to the court. Hence, under the threat of a sufficiently large penalty, C would voluntarily transfer his gold to B in period 5.

Such an arrangement breaks down, however, if making reports to the court is costly. Let us imagine that B faces a variable cost (disutility) of making announcements. Thus, B will no longer make such announcements, unless the court is willing to transfer resources from C to compensate B for the costs of making reports. If the court offers a low amount of compensation to B in such cases, then B will come forward only when his cost of making announcements is low. If his cost is high, B will make no report to the court. As a result, C will be tempted sometimes to fail to deliver gold to B . On the other hand, if the court offers a high amount of compensation to B , the frequency of announcements may make the arrangement prohibitively expensive.

In this case, trade can be sustained by the introduction of physical evidence. Suppose C provides an IOU to B in period 2, acknowledging delivery of the bread and, in effect, promising delivery of gold in period 5, with the IOU to be returned by B to C in period 5, against delivery of the promised gold. If C does not perform, B can make an announcement of nonperformance to the planner. If the complaint is backed by an IOU, the court can punish C , and reward B . If no IOU is forthcoming from B , then the court can punish B for making a frivolous complaint.⁴

Next, let us consider the special case where B *always* needs A 's good to produce his own good. In this case a system of IOU's—a "credit chain"—can be effective: A produces and delivers his good in period 1 to B in return for B 's IOU and B delivers bread in period 2 to C in return for C 's IOU. C redeems his IOU with a delivery of gold to B in period 4, and B extinguishes his IOU by transferring gold to A in period 5. If an agent makes a complaint to the court, appropriate punishments can be meted out based on an examination of unredeemed IOU's.

For a credit chain to be effective, the punishments available must be severe enough to ensure that B is not tempted to "take the money and run." In other words, a weak point in the enforcement arrangement is ensuring that when B receives gold from C , he is willing to pass part of the gold along to A . If C knew that such indebtedness existed, trade could be sustained by

⁴The holder is willing to redeem IOU for something of value; the writer wants to redeem it rather than suffer a penalty at the hands of the court. In this example, the terms specified on the IOU are inessential; more generally, if the penalty imposed by the court were on a sliding scale according to the terms on the IOU, it could be used in various states to sustain a variety of trades.

having C deliver gold directly to A .

Enforcing a credit chain becomes more difficult in the general case where A delivers raw material to B only part of the time. Suppose C is uncertain about whether A has delivered a good to B , i.e., whether B owes a debt to A . If C has to rely, for example, on information from B about whether payment should go to A or to B , then there is a strong temptation for B to lie about his own indebtedness, unless the court can again apply heavy penalties to B .

An alternative arrangement for sustaining trade involves the use of transferable debt. In the case that A had delivered a good to B in period 1, suppose that B issues an IOU to A , which is payable in period 3 by transfer of a third party's (C 's) debt to A . C 's debt is then ultimately redeemed by the transfer of gold from A to C in period 6, upon presentation of C 's IOU. In the case where A has not delivered a good to B , C 's debt stays with B and is redeemed by B in period 4.

The court can enforce such an arrangement as follows: If B issues a debt to A and fails to discharge his debt in the manner described above, A reports this to the court and shows B 's debt as evidence. The court announces that B is in default, indicating that any payments by C should be to the court and not to B . The court would also apply penalties to B , sufficient to induce performance, but these could be smaller than before, since by defaulting on his period 3 payment, B removes any chance that he could receive a payment from C in period 5.

Payment with transferable debt thus allows for debt owed to a certain agent (i.e., by C to B) to “cancel out” another debt owed by the same agent (by B to A). So long as such cancellation is understood by all agents to be a feature of the trading environment, it allows for trouble-free enforcement of creditors' claims, even in cases where other enforcement options are limited. This feature of transferable debt closely corresponds to the idea of net settlement (cf. netting the arrangements analyzed in Kahn, McAndrews, and Roberds 2001).

1.2 Institutional and historical context

The foregoing example, while stylized, captures the basic idea of all inside-money payment systems: the settling of a debt between two parties by tender of a third party's debt. Such transfers are risky by nature, and associated with every inside-money payment system is a set of rules that govern the allocation of risks that may arise in the course of exchange.

In the legal and historical literatures, the rules are often described by

terms such as *transferability* and *finality*. Roughly speaking, a debt is “transferable” (“assignable”) if a third party who receives the debt retains the same creditor’s rights against the debtor as original debt holder. Under contemporary U.S. law, for example, a check or similar instrument may be freely transferred to a third party via endorsement, whereas a credit or debit card payment can only be cleared through certain prespecified channels.⁵

A debt transfer is final when it extinguishes an obligation between two parties. Otherwise put, if final discharge has not occurred then one or both parties have recourse, i.e., the right to compel the other side to undertake additional actions in fulfillment of the contract. In practice, finality may hold in some circumstances but not others. A modern check payment, for example, typically does not extinguish an obligation but only suspends the obligation pending settlement of the check. The higher degree of finality, the more money-like the character of a debt transfer.

Modern notions of transferability and finality are the result of a very long evolution. Medieval debt contracts were generally not transferable (Kohn 1999), but as trade expanded, debts of individuals began to be used as a means of payment. Circulating debt became widespread in the Low Countries in 16th and 17th centuries, with the establishment of the legal concept of *negotiability*.⁶ Bills of exchange and similar debt “instruments” became “negotiable,” or generally acceptable in exchange, because they were freely transferable and their transfer was subject to certain widely accepted finality rules.⁷ The use of negotiable instruments quickly spread to other countries. Rogers (1995) argues that the adoption of negotiable instruments in 17th and 18th century England resulted in a gradual reorganization of trade, essentially from sequences of spot transactions to something more like the credit chains that prevail today.⁸

The concept of negotiability survives to the present day in U.S. law,

⁵Strictly speaking, a check is not debt since it represents an order to pay rather than a promise to pay. However, a check or similar instrument may effectively become a debt obligation if it is “accepted,” as in a certified check (a check accepted by a bank).

⁶See van der Wee (1997) on the origins of negotiable instruments.

⁷Like checks, negotiable bills of exchange were “order instruments” that became debt only after they had been accepted by the party instructed to pay (see Rogers 1995). It is worth emphasizing that bills of exchange and related types of instruments were in use for centuries (in non-circulating form) before the development of negotiability.

⁸A parallel development took place in Edo-era Japan. There, according to Tamaki (1995), a number of merchant bankers (*ryogae*) issued notes payable either at a fixed term, or on demand. Like their European counterparts, these notes circulated as means of payment. We are grateful to Masato Shizume for making us aware of the existence of these arrangements.

where it forms the basis for much of the law governing check payments. As the debt of individuals and non-bank firms no longer circulates as money, negotiability per se is less important than in earlier times (Winn 1998, Mann 1999). The use of bills of exchange and similar instruments persists in less developed economies, however. Ickes (1998) describes the widespread use of “veksels,” essentially bills of exchange, by Russian industrial firms during the 1990s.⁹ More modern forms of payment, such as credit cards and wire transfers, incorporate different rules for transferability and finality. Nonetheless, all of these arrangements share the fundamental feature in the course of normal trade, of pulling the middle party out of the credit chain.

The formal model below gives some precise economic meaning to the concepts of transferability and finality. Specifically, the model illustrates how transferable debt can implement optimal allocations in cases where a credit chain cannot.

2 The General Framework

We begin by establishing a general framework for modeling trading economies with debt. The framework is too general to be of use in itself, but all of the examples with which we deal will be special cases.¹⁰

There are a finite number of periods t and a finite number of agents i . Each period consists of a trading stage followed by an announcement stage. In the trading stage, a subset of the agents are paired for trading.

An agent enters the trading stage with a set X_{it} of feasible offers to make to his trading partner. Each agent in the pair simultaneously makes a *trade proposal*, which consists of a feasible offer in X_{it} plus a demand from the other individual (not necessarily feasible, since the agent may not know what is feasible for the partner). If the proposals match then the trade takes place, otherwise it does not.¹¹ Only the trading pair observe their own activity.

In the announcement sub-period, an agent may learn about his cost of making an immediate announcement to the center. Each player indepen-

⁹Unlike early negotiable instruments, the veksels could not be freely transferred but instead were only transferable to specific parties, usually the next participant in a given supply chain.

¹⁰Indeed, except for the assumptions of finite agents and finite periods, the framework encompasses *all* existent models of inside money.

¹¹It would also be natural to assume that the round of trade proposals is preceded by a round of cheap talk between the trading partners; however, for the examples we deal with in this paper, this extra flexibility is not needed.

dently decides whether to send a message to the center, and if so what message to send. Let \mathbb{M} denote the set of possible messages. (The center cannot observe the trades themselves; we use ‘0’ to indicate that no message was sent). The cost does not vary with the announcement an agent decides to make; however, if he decides not to make an announcement at all, he pays no announcement cost. After receiving any individual announcements, the center may make its own public announcement. If it does, then the cost of so doing is borne by all players equally. We will only consider the two extreme possibilities: prohibitive costs for public announcements in a particular stage or zero costs for public announcements in that stage.

The center can also assess rewards and penalties as a function of the history of announcements; for simplicity we will assume that these are all assessed after the final period of play. We let F denote the maximum fine that the center has the ability to charge an individual, and we will use the fraction $\alpha \in (0, 1)$ to index the efficiency of the transfer mechanism. Rewards must be paid out of the other players’ fines, where a fraction $(1 - \alpha)$ of any fine is wasted. Thus a mechanism is dependent on the power of the enforcement system to penalize and to reward, which ability we take as parametric.

In all the examples we consider, the trading will follow a subset of the six-period pattern noted above. Thus at most one side will have goods to trade. The other side may, however, exchange goods for “evidence” (see below for more details of the physical characteristics of such evidence).

In subsequent sections we use this general framework to analyze a number of trading economies. We begin by examining a simplified, two-player environment in Section 3. By analyzing the two-player case, we are able to more precisely describe the effects of introducing evidence of trades into the environment, in the form of receipts and IOUs. The use of IOUs, in particular, is shown to allow trading to occur in situations where enforcement of trading obligations would otherwise be prohibitively costly. Section 4 then extends the analysis to the three-player, “Wicksell triangle” environment discussed above, again considering the effects of introducing different forms of evidence. Trading with transferable IOUs is shown to require a lesser degree of enforcement than other forms of evidence. Section 4.2 then considers an extension of the environment to the case where there is uncertainty over how many agents will trade. This final section shows an additional benefit from transferable IOUs in providing information about activities of other agents.

3 Two-Player Environment

3.1 The Basic Game

We begin our analysis by considering a simplified environment involving only the last two agents in the Wicksell triangle, B and C . In other words, only period 2 (“the earlier period”) and period 4 (“the later period”) are relevant. In each stage, the trading proposals consist of the pair $\{Y, N\}$. In the earlier period B either gives (Y) or does not give (N) his endowment (bread) to C . In the later period C either gives (Y) or does not give (N) his endowment (gold) to B .¹² Each player values his own endowment at v and the other player’s endowment at u where $u > v > 0$.¹³

At the end of the later period each player i observes his own draw of a cost of reporting to the center t_i . The draws are independent; each is made from a non atomic distribution $G_i(\cdot)$, for $i \in \{B, C\}$, where $G_i(0) = 0$. Let E_i denote the expectation with respect to distribution G_i . For simplicity, after the earlier period costs of reporting to the center are prohibitive, as are the costs of messages from the center.¹⁴

Let P_B and P_C denote the center’s payouts to the two players. Thus a mechanism is a pair of payout functions

$$(P_B, P_C) : (\mathbb{M} \cup \{0\})^2 \rightarrow \mathbb{R}^2 \quad (1)$$

which satisfy the following *feasibility conditions*:

$$P_B \geq -F \quad (2)$$

$$P_C \geq -F \quad (3)$$

$$P_B + \alpha P_C \leq 0 \quad (4)$$

$$P_B + \alpha P_A \leq 0. \quad (5)$$

In the payout function $P_i(\cdot, \cdot)$, the first argument will denote B ’s message, and the second argument will denote C ’s message.

An *efficient mechanism* is one such that the game has a subgame perfect Nash equilibrium¹⁵ in which

¹²In each trading stage, without loss of generality, we simply assume that the recipient’s trading proposal is to receive the other’s good. Thus only the donor’s decisions are relevant.

¹³In the proofs in the appendix, we generalize the two-player model slightly by allowing player-specific payoffs.

¹⁴Allowing reporting after the earlier period would not affect the results. See below.

¹⁵Extensions to the case of correlated equilibrium, allowing for a publicly observed signal before messages are sent, would probably increase the generality of the results significantly, at the cost of considerable additional overhead.

1. Along the equilibrium path, B plays Y , C plays Y and both agents send no message.
2. Along the equilibrium path, the center provides neither rewards nor punishments.

The space of mechanisms is enormous, so difficulty might be expected in the determination of whether an efficient mechanism exists for a particular set of parameter values. Matters are greatly simplified by the following lemma:

Lemma 1 *If an efficient mechanism exists, then there is an efficient mechanism with $|\mathbb{M}| = 1$.*

Proof (outline). Consider the strategies of the individuals in the reporting subgames, which can be denoted by the history of play up to then $h = \{YY, YN, NY, NN\}$. Each such strategy is a function of the individual's cost draw. Because of single crossing there is a critical value t_{hi} such that for t higher the strategy is to make no report. (By condition 1 of an efficient mechanism, $t_{YYi} = 0$.) Given the strategies, define $p_{hi;jk}$ as the expected payoff to i after history h conditional, if $j = 1$ (respectively $k = 1$) on B (respectively C) not reporting and if $j = 0$ (respectively $k = 0$) on B (respectively C) making some report. In other words,

$$p_{hi;10} = E\{P_i|h, t_B \leq t_{hB}, t_C > t_{hC}\} \quad (6)$$

$$p_{hi;01} = E\{P_i|h, t_B > t_{hB}, t_C \leq t_{hC}\} \quad (7)$$

etc. (Also given condition 2 of an efficient mechanism, $p_{hi;00} = 0$). Since in every realization, the pair of payoffs is drawn from the same convex set, the pair $(p_{hA;jk}, p_{hB;jk})$ is also in this set of feasible payments. Given h, i , for all t the expected payout from the center to/from the individual is the same, otherwise the strategy would not be payoff maximizing. In other words, in the equilibrium,

$$E\{P_B|h, t_B\} = G_C(t_{hC})p_{hB;11} + (1 - G_C(t_{hC}))p_{hB;10} = G_C(t_{hC})p_{hB;01} \quad (8)$$

independent of the realization of t_A , and similarly for C . Thus we may replace the initial game by an equivalent one in which each player announces a value of h or not, and in which payoffs are zero if neither player makes an announcement, are $p_{hi;jk}$ if h is the announcement of both players or only one player makes an announcement and the announcement is h , and

– F if the two players make announcements and the two announcements do not match. There is an equilibrium of this game which corresponds to the initial equilibrium: a player’s expected payoffs are identical as a function of history and of his own announcement cost, announcements occur in the same states that they did in the initial game, and payoffs are zero along the equilibrium path, YY . In other words, we have reduced the message space to three signals $\{YN, NY, NN\}$. Finally, suppose that the messages NY and NN are dropped. It is then clearly an equilibrium to send no message in these events, and to continue to play as before in the event of YY or of YN . These subgame equilibria support the path YY . To see this, note that given that he will face no difference in payoffs, player C prefers to play N following N , and as before he prefers to play Y following Y . Given this behavior, player B prefers the outcome from YY to that from NN . ■

In other words, when players make a report, only one message is needed (We will use ‘1’ to denote the message; the intuitive interpretation of the message will be “ C misbehaved”). Clearly this equilibrium behavior must be supported by threats off the equilibrium path. There is no efficient mechanism in which the Nash equilibrium is unique (that is, the efficient outcome is not strongly implementable). Instead, implementation requires exploiting the multiplicity of Nash equilibria in the reporting subgame. In the implementing equilibrium, the selection among these subgame equilibria provides incentives for efficient behavior in trading sub-periods.

Theorem 2 *An efficient mechanism exists if and only if there are a pair of feasible payoff functions P_B and P_C , and values t_B and t_C satisfying the following conditions:*

$$P_B(0, 0) = 0; P_C(0, 0) = 0 \quad (9)$$

$$P_B(1, 0) \leq 0; P_C(0, 1) \leq 0 \quad (10)$$

$$G_B(t_B) = G_B([P_B(1, 0)(1 - G_C(t_C)) + P_B(1, 1)G_C(t_C)] - [P_B(0, 0)(1 - G_C(t_C)) + P_B(0, 1)G_C(t_C)]) \quad (11)$$

$$G_C(t_C) = G_C([P_C(0, 1)(1 - G_B(t_B)) + P_C(1, 1)G_B(t_B)] - [P_C(0, 0)(1 - G_B(t_B)) + P_C(1, 0)G_B(t_B)]) \quad (12)$$

$$P_C(0, 0) \geq v + E_C[\max\{P_C(0, 0)(1 - G_B(t_B)) + P_C(1, 0)G_B(t_B), P_C(0, 1)(1 - G_B(t_B)) + P_C(1, 1)G_B(t_B) - t_C\}]. \quad (13)$$

Proof. Following Lemma 1, we assume that there are two equilibria in the reporting subgame, one in which with certainty each player makes no announcement, one in which player i reports ‘1’ if his cost is at most t_i and reports nothing otherwise. Condition (10) states that each player prefers to keep silent if he knows the other player is keeping silent. In the other equilibrium, the probability that a player makes an announcement is the probability that the cost of making an announcement is less than the difference in the expected payoffs from making an announcement or not, given the other player’s probability of making an announcement; this is conditions (11-12). For the mechanism to be efficient, we then make the second equilibrium the continuation when B plays Y , but C plays N and the first equilibrium the continuation otherwise. Given a particular mechanism, for this set of strategies to be an equilibrium, condition (13) is necessary and sufficient. Finally, an efficient mechanism entails no fines along the equilibrium path; this is condition (9). ■

Note that B ’s play in the early period is self-enforcing: the threat of not receiving goods from C in return is sufficient to induce good behavior. The difficulty is in enforcing C ’s good behavior once he receives the goods from B . The reports must entail sufficient punishment for C to prefer good behavior. Working with the above conditions, the appendix establishes the following necessary and sufficient conditions for the existence of an efficient equilibrium:

Theorem 3 *An efficient mechanism exists if and only if $v \leq -\Omega$, where Ω is the minimand of the following problem:*

$$\min_{\varphi_A, \varphi_C, z, m, n} -\varphi_B F + \int_0^{G_C^{-1}(\varphi_C)} G_C(t) dt \quad (14)$$

subject to the following conditions:

$$0 \leq \varphi_i \leq 1, \text{ for } i = B, C \quad (15)$$

$$-F \leq z \leq 0 \quad (16)$$

$$\alpha m + n \leq 0; \alpha n + m \leq 0, \text{ with complementary slackness} \quad (17)$$

$$\varphi_C = G_C[(1 - \varphi_B)z + \varphi_B(n + F)] \quad (18)$$

$$\varphi_B = G_B[\varphi_C(m + F)]. \quad (19)$$

It also establishes somewhat simpler necessary conditions:

Corollary 4 *The following condition is necessary for the existence of an efficient mechanism*

$$\max_t G'_B(t) \max_t G'_C(t) F^2 \geq \Lambda \quad (20)$$

where

$$\begin{aligned} \Lambda &= \frac{1}{2} \text{ if } \alpha \in (0, \frac{1}{2}] \\ &= \frac{4\alpha}{(1+2\alpha)^2} \text{ if } \alpha \in [\frac{1}{2}, 1) \end{aligned} \quad (21)$$

The appendix also establishes an implementation result:

Theorem 5 *If an efficient mechanism exists, the efficient outcome can be implemented with the payoff function shown in the following matrix (B is row player; C is column player):*

	‘0’	‘1’
‘0’	0	$-F$
‘1’	$-F$	n

with

$$\varphi_i = G_i(t_i). \quad (22)$$

Since there is no difficulty in inducing good behavior on the part of B , nothing would be gained by providing a round of messages after the earlier period. However, allowing the center to make announcements immediately before the reporting subgame would allow additional possibilities (see footnote 15).

3.1.1 A particular case

(All proofs for this subsection are in the appendix.) If the distributions of reporting costs are uniform, the necessary condition (20) can be strengthened:

Lemma 6 *When the distributions of reporting costs are uniform on $[0, T_i]$, for $i = B, C$, an efficient mechanism exists only if*

$$F^2 \geq T_B T_C. \quad (23)$$

In this case the implementing payoff matrix can be further simplified:

Theorem 7 *If any efficient mechanism exists, then the efficient outcome can be implemented with the following payoff function:*

	‘0’	‘1’
‘0’	0	-F
‘1’	-F	$-\alpha^{-1}m$

where m is the unique non negative number such that

$$(F + m)(F - \alpha^{-1}m) = T_B T_C. \quad (24)$$

The mechanism has an intuitive form: A player is neither rewarded nor fined as long as the other player says no deviation occurred. Since reporting of deviations is inherently costly to a player, the payoff structure must encourage reporting. Therefore a player receives the maximum fine if he does not report a deviation but the other player does report a deviation. Moreover, if a deviation occurs (and both players report it) then Player C (the misbehaving player) is punished but not as much as he is punished for failure to report. Player B is rewarded to the maximum extent possible given the magnitude of C 's fine.

Lemma 8 *If any efficient mechanism exists, then there is an efficient mechanism in which one of the two players reports deviations with probability one. If*

$$2T_B \geq (1 + \alpha)F \quad (25)$$

and

$$T_B + \alpha T_C \geq F(1 + \alpha) \quad (26)$$

then player C reports with probability one. Otherwise player B reports with probability 1.

Finally, in the case of uniform distributions, the necessary and sufficient conditions for existence of an efficient mechanism can be stated explicitly in terms of the parameters of the model (α, F, T_B, T_C) .

Theorem 9 *Suppose that*

$$F^2 \geq T_B T_C. \quad (27)$$

If (25-26) hold then an efficient mechanism exists if and only if

$$v + \frac{T_B T_C - (1 + \alpha)F^2 - F\sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C}}{2T_B} \leq 0. \quad (28)$$

Otherwise, an efficient mechanism exists if and only if

$$v - F + \frac{1}{T_C \alpha^2} \left(F^2 (1 + \alpha)^2 - 2\alpha T_B T_C - 2F(1 + \alpha)\sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C} \right) \leq 0. \quad (29)$$

Note that provided that $u > v$ the existence of an efficient mechanism in general does not depend on u and is less likely as v increases. Similarly, in general the existence of an efficient mechanism becomes more likely as F increases and as α increases—that is as the enforcement mechanism becomes more efficient. All of these claims follow from Theorem 3. The above results show furthermore, that efficiency becomes more likely as T_B and T_C decrease—that is, as reporting costs decrease.

3.1.2 Constrained Optimality when only B can report

Corollary 4 demonstrates that for any particular distribution G_B , there is a distribution with C 's costs sufficiently high that no efficient mechanism is possible. In this section we assume that costs for C are sufficiently great that the probability of his making a report is negligible. We then consider the constrained optimal mechanisms—that is, mechanisms which maximize the expected sum of the two players' payoffs. In describing mechanisms we now drop the argument which denotes C 's report.

Theorem 10 *When C 's reporting costs are prohibitive, a second best mechanism dominating autarchy exists if and only if*

$$u - v \geq \Omega \quad (30)$$

where Ω is the maximand of the following problem

$$\max_{m,n,s,\varphi} (1 - \alpha + \varphi\alpha)s + \varphi n + \int^m G_B(t_B) dt_B \quad (31)$$

subject to

$$s \leq 0 \quad (32)$$

$$\varphi n \geq v - \varphi F \quad (33)$$

$$s \geq -F \tag{34}$$

$$\varphi = G_B(m). \tag{35}$$

and to

$$\alpha n + s + m \leq 0 \tag{36}$$

$$\alpha n + s + m \leq 0 \tag{37}$$

with complementary slackness.

In the constrained optimal mechanism, C gives his endowment to B , but not vice versa.

Theorem 11 *If the constrained optimal mechanism dominates autarchy, then it takes the following form:*

$$\mathbb{M} = \{ '1', '2' \} \tag{38}$$

$$P_B(1) = P_B(2) = m + s \tag{39}$$

$$P_B(0) = s \tag{40}$$

$$P_C(0) = -\alpha s \tag{41}$$

$$P_C(1) = -F \tag{42}$$

$$P_C(2) = \min\{-\alpha(m + s), -\alpha^{-1}(m + s)\} \tag{43}$$

for some $s \in [-F, 0], m \in [0, \alpha F - s]$.

The values of s and m in the mechanism are identical to the maximizing parameters of theorem 10.

The mechanism works as follows: If B makes no report, he pays an amount $|s|$ (possibly zero), and C receives $\alpha|s|$. If B makes a report, he receives m more than he would have if he makes no report, regardless of the contents of the report. We interpret report '1' as " C misbehaved" and report '2' as " C behaved correctly." If B claims C misbehaved, C is fined the maximal amount. If B says that C behaved correctly, then C receives the maximum feasible amount given what B is set to receive.

Next we examine this mechanism more closely in the case of a uniform distribution.

Theorem 12 *When C 's reporting costs are prohibitive and B 's costs are uniformly distributed on $[0, T]$, then the following conditions are necessary for the existence of an arrangement which dominates autarchy:*

$$(1 + \alpha)F \geq \alpha T + v \quad (44)$$

$$F^2 \geq vT \quad (45)$$

Corollary 13 *In the case of a uniform distribution the maximand Ω is equal to*

$$\max_m (1 - \alpha) \left[\frac{F}{\alpha} - m - \frac{vT}{\alpha m} \right] + \frac{m^2}{2T} (1 - 2\alpha). \quad (46)$$

in the range

$$m \in \left[\frac{(1 + \alpha)F - \sqrt{(1 + \alpha)^2 F^2 - 4\alpha vT}}{2\alpha}, \min\left\{T, \frac{vT}{F}\right\} \right] \quad (47)$$

provided that

$$vT \geq 2\alpha F^2 \quad (48)$$

or

$$v \geq F. \quad (49)$$

Otherwise, the maximand is the greater of the this amount and

$$\max_m (1 - \alpha^2)v + (1 - \alpha) \left[\alpha F - m \left(1 + (1 + \alpha) \frac{F}{T} \right) - \frac{\alpha v T}{m} \right] + \frac{m^2}{2T} (1 - 2\alpha) \quad (50)$$

on the interval $m \in \left[\frac{vT}{F}, \min\{T, \sqrt{2\alpha vT}\} \right]$.

The maximizing m in this problem is the m in the optimal contract of the previous theorem. The level n is set low enough that player C is just indifferent between giving up his endowment or not.

In the case where both B 's and C 's distributions are uniform, the phrase "prohibitively costly" can be made more precise. If

$$T_C > (1 + \alpha) \frac{F^2}{v} \quad (51)$$

then it is infeasible to induce player B to give up his endowment. As T_C increases beyond this bound, the optimal mechanism includes occasional reporting by C , but the reporting becomes more and more rare, and the expected payoffs of the optimal mechanism approach those in Theorem 11.

3.2 Introducing Receipts

We incorporate *evidence* into the model. Now, when an individual makes an announcement to the center, he can at the same time present any evidence he possesses; however, it is not required that he do so. The payoff function depends on reports, which now consist of cheap talk *and* evidence. The first kind of evidence we introduce is the *receipt*. A receipt is a differentiated document, costless for the issuer to produce, but impossible for the recipient to forge or transfer.

Clearly, there is no harm in including a receipt in a trade; thus we now assume that all trade proposals include receipts. Again, without loss of generality, the set of trade proposals at any stage is reduced to the set $\{Y, N\}$, where Y indicates a proposed trade; a trade consists of an exchange of endowment for a receipt.¹⁶

The space of B 's reports can without loss of generality be restricted to pairs in which the second dimension denotes the number of C 's receipts that B presents in evidence (0 or 1). It is intuitive, and can be verified, that in any optimal mechanism it will be in the interest of the individual to present any receipt in his possession as part of any announcement he makes, but this will of course impose restrictions on incentive compatible mechanisms.

Theorem 14 *When C 's reporting costs are prohibitive, the constrained optimal mechanism is either as described in Theorem 10 or it takes the following form: Along the equilibrium path both agents trade their endowment. B receives a receipt in return for his endowment. The message space is as follows:*

$$\mathbb{M} \times \mathbb{Z} = \{ '1', '2' \} \times \{0, 1\} \quad (52)$$

And the payoff function takes the following form:

$$P_B(0) = s \quad (53)$$

$$P_B(1, 1) = P_B(2, 1) = m + s \quad (54)$$

$$P_B(1, 0) = P_B(2, 0) = -F \quad (55)$$

$$P_C(0) = -\alpha s \quad (56)$$

$$P_C(2, 0) = P_C(2, 1) = \min\{-\alpha(m + s), -\alpha^{-1}(m + s)\} \quad (57)$$

$$P_C(1, 0) = P_C(1, 1) = -F \quad (58)$$

for some $s \in [-F, 0]$, $m \in [0, \alpha F - s]$.

¹⁶As before, there is no reason for the potential recipient of endowment not to play Y ; the only real strategic choice is on the part of the donor of the endowment.

Again, ‘1’ has the interpretation “ C misbehaved,” and ‘2’ has the interpretation “ C behaved correctly.” Note that the constraints on C are essentially unchanged from Theorem 11: C is punished or not on B ’s say so. The receipt, however, modifies B ’s behavior. B only receives the receipt if he provides goods, and he is only able to collect the reporting reward if he shows the receipt. Nonetheless, the use of receipts does not lead to full efficiency, because the mechanism still requires expensive reporting along the equilibrium path.

The usefulness of evidence is confirmed by the following corollary:

- Corollary 15** *1. In an environment without receipts, when C ’s reporting costs are prohibitive, the choice of second best contract does not depend on the level of u .*
- 2. When C ’s reporting costs are prohibitive, if there exists a second best contract without receipts, there exists a finite u such that for all greater values of u , the second best contract with receipts strictly dominates it.*

The proof shows in fact that for sufficiently high values of u , the court’s payments along the equilibrium path in the two contracts are identical. In the case of the uniform distribution we have a simple bound on u :

Corollary 16 *For the uniform distribution the critical u is no greater than $v + \frac{T}{2}$.*

Under the uniform distribution, the analysis is similar to that for pure messages. For example we can show that

Corollary 17 *When C ’s reporting costs are prohibitive and B ’s costs are uniformly distributed on $[0, T]$, then the following conditions are necessary and sufficient for the existence of a contract which induces both B and C to trade their endowment:*

$$(1 + \alpha)F \geq \alpha T + v \tag{59}$$

$$F^2 \geq vT \tag{60}$$

$$u \geq v\left(1 - \frac{1}{2\alpha}\right) + \frac{(1 + \alpha)F}{4\alpha^2 T} \left[(1 + \alpha)F - \sqrt{(1 + \alpha)^2 F^2 - 4\alpha v T} \right] \tag{61}$$

In this extreme case, although evidence improves B ’s behavior, it has no effect on C ’s behavior, since C cannot make reports. On the other hand, evidence has *no effect* on the set of parameter values where fully efficient mechanisms exist.

Theorem 18 *A fully efficient mechanism exists in the game with receipts, if and only if it exists in the game without receipts.*

Proof (outline). If we take the message space to be $\mathbb{M} \times \{0, 1\}$, then the proof of lemma 1 shows that without loss of generality a fully efficient mechanism is one in which there is only one message, and that message is only delivered in the case where C misbehaves and B does not. Again, without loss of generality we can assume that in this case C shows the receipt from B and B does not show a receipt (having none). The only effect then of the possession of a receipt is to prevent C from claiming that B has cheated when B has not cheated. But this event is not announced in an efficient mechanism in any case, thus C 's possession of a receipt adds nothing to the mechanism. ■

Intuitively, in a fully efficient mechanism, B 's behavior is unproblematic; it is enforced by C 's opportunities for retaliation. The restriction on the equilibrium comes from inducing B to report, not from the particular reports he makes.

3.3 The Model with IOUs

We now consider a new kind of evidence, an *IOU*. An IOU represents a promise to repay at a future date. Like a receipt, it is costlessly produced, and impossible to copy. Unlike a receipt it can be returned to the borrower when repayment is made. Again, we assume, without loss of generality, that each player chooses Y or N in each period, and that trade occurs only if both choose Y in a period. In period 1 a trade consists of player B giving endowment to player C and receiving an IOU from C . Trade in period 2 consists of B receiving endowment from C and, if he holds C 's IOU, returning it.

Like a receipt, an IOU can be offered as evidence when a report is made. The crucial difference between the receipt and the IOU is that the receipt only provides evidence about the behavior of the holder: he must have given endowment in order to have the receipt. The IOU provides evidence about both the players: if B presents an IOU as evidence, it must be the case that C received endowment from B and that B has not received endowment from C .

Theorem 19 *Let*

$$m = G_B^{-1}(v/F). \tag{62}$$

When C 's announcement costs are prohibitive, an efficient mechanism exists with IOU's if and only if

$$\int^m G_B(t_B) dt_B \leq u \quad (63)$$

$$m \leq \alpha F. \quad (64)$$

Under these conditions, the following mechanism is efficient: only B makes announcements, and

$$\mathbb{M} \times \mathbb{Z} = \{ '1' \} \times \{0, 1\} \quad (65)$$

$$P_B(0) = P_C(0) = 0 \quad (66)$$

$$P_B(1, 0) = -F \quad (67)$$

$$P_B(1, 1) = m \quad (68)$$

$$P_C(1, 0) = 0 \quad (69)$$

$$P_C(1, 1) = -F. \quad (70)$$

In other words, B receives the reward m if he presents C 's IOU to the court. This reward is big enough to make B 's reporting sufficiently likely to deter C from refusing to redeem the IOU. It is not so large as to encourage B not to return the IOU for C 's gold. Note therefore that in particular, unlike pure messages or receipts, mechanisms with IOU's can induce efficiency even if it is prohibitively costly for C to make announcements.

An IOU is in effect a hostage. As a hostage it has value to the donor: redeeming it protects the donor from fines by the court. This value at a future date translates into value-in-exchange today, above the value of a mere promise to repay.

4 The Three-Player Environment

Recall that there are six periods in the three-player environment:

1. A can deliver a good (flour) to B
2. B can deliver a good (bread) to C
3. B and A meet again (no goods can be exchanged; this meeting will only become significant when we introduce transferable debt)

4. C can deliver a good (gold) to B
5. B can deliver a good (gold) to A (if he receives it from C)
6. C can deliver a good (gold) to A (to the extent that he did not already deliver it to B).

As before, an agent's utility of consuming his desired consumption good is u , and the opportunity cost of the good to its supplier is given by $v < u$. In other words, we are investigating an intertemporal Wicksell triangle (periods 1, 2, and 6) with additional meetings. As an alternative to delivering the gold directly to A , C can deliver it indirectly, passing it to B for transfer to A . In what follows we will also need to consider the value of gold to B , we will denote the value by w and assume that $v < w < u$.

Agent A can make a report in period 3 or period 6; agent B can make a report in period 4. In each case the agent learns the cost of reporting in that period immediately before he decides whether to make a report. The costs of reporting are independent draws from the distribution G_i for $i = A, B$. We confine attention to the case where reports by C are always prohibitively expensive. We assume the center can make a public announcement at the end of period 3 or period 4. The announcement at the end of period 4 simplifies calculations but is otherwise unimportant. The report and announcement at the end of period 3 is significant when we introduce uncertainty about the number of players, but not before.

Can trade be sustained in this environment purely by announcements? The answer is no, for the same reason as in the two-agent case.

Theorem 20 *When C 's announcement costs are sufficiently high, there is no efficient mechanism in the three-player environment with just announcements, or with receipts.*

Intuitively, we cannot only reward C 's creditor (i.e., B) for announcing non-performance by C , for otherwise B would always announce this. Thus we need to reward B for making an expensive announcement along the equilibrium path, otherwise C will not perform.

Next we consider IOU's (i.e., non-transferable debt).

Theorem 21 *When C 's announcement costs are prohibitive, there exists an efficient mechanism in the three-player environment with IOU's if and only if*

$$G_A(2\alpha F) \geq \frac{w}{F} \tag{71}$$

$$\int^{G_A^{-1}(w/F)} G_A(t) dt \leq u \quad (72)$$

Corollary 22 *The efficient outcome can be enforced by the following mechanism: if A presents B's IOU to the center, he receives the reward $G_A^{-1}(w/F)$ and B and C each receive the maximal punishment $-F$. If B presents his IOU to the center B and C receive the maximal punishment $-F$ and A receives nothing. In all other cases the court provides neither rewards nor punishments.*

The following trade pattern achieves the efficient outcome: B provides A an IOU in return for A's endowment. C provides B an IOU in return for B's endowment. C redeems his IOU from B with gold, and B uses the gold to redeem his IOU from A. In other words, a credit chain of IOUs implements the efficient outcome. If everybody delivers, no IOU is left at the end. Note that in this mechanism guarantees that B never makes an announcement even if C fails to deliver the gold; instead the punishments are entirely dependent on A's announcement. In other words the same mechanism works even if B's announcement costs are prohibitive as well. If A does not receive gold, then both B and C are punished. Collective punishment is adequate for the job at hand. We simply want to thwart misbehavior; when it happens, it is of no consequence which of the two possible miscreants is guilty.¹⁷

¹⁷Collective punishment does however have a limitation. While the mechanism implements with a subgame perfect Nash equilibrium, the equilibrium does not survive some reasonable refinements. In particular, even if C did not receive bread from B, as the mechanism stands he would prefer to pass the gold to B for transmission to A, rather than face the possibility of punishment due to A's announcement. This means that B is in a powerful position in period 2. If he were to refuse to supply the bread to C, it would be reasonable for C to take this as a signal that B will make an offer to receive the gold in period 5 without returning an IOU. C would find it better to comply with this extortion than to suffer the expected penalties from A's announcement. Since this deviation requires implicit coordination by B and C, it does not violate subgame perfection, but it does violate some refinements of perfection.

Since the point of this result is to demonstrate that transferable debt works in some cases where ordinary debt does not work, there is no harm in making the equilibrium concept used in the above theorem relatively weak, provided we use the same concept for both transferable and non-transferable debt.

Nonetheless it is also of interest to know what the necessary and sufficient conditions are for implementing with non transferable IOU's in an equilibrium which does not suffer from this limitation—the proof in the appendix establishes such conditions as well.

4.1 Transferable Debt

In this context, *transferable debt* is an special kind of IOU which can be passed from the creditor to a third party. So far we have not allowed IOU's to be transferable; when debt is extinguished, the IOU is returned to the debtor; in the meanwhile it has remained in the hands of the creditor. In this section we demonstrate the usefulness of transferability, by showing that it relaxes the requirements for mechanisms to deliver the efficient outcome. As before, transferable debt is costless for the issuer to manufacture and impossible for other parties to forge. A party other than the initial debtor can only deliver that debtor's transferable debt if he has previously received it.

Allowing for debt to be transferable increases the options available in designing a mechanism. Thus any mechanism which was feasible with non-transferable IOUs remains feasible. The point is to demonstrate that transferable IOUs lead to efficiency in situations where efficiency could not be attained otherwise. Our next result show that one of the benefits of transferable debt is *finality*. That is, transferable debt allows the timely exit of a less-than-perfectly-reliable party in a credit chain.

Theorem 23 *When B and C 's announcement costs are sufficiently high, there exists an efficient mechanism in the three-player environment with transferable IOU's if*

$$G_A(2\alpha F) \geq \frac{v}{F} \quad (73)$$

$$\int^{G_A^{-1}(v/F)} G_A(t) dt \leq u \quad (74)$$

Since $v < w$, the set of enforcement systems which can support the efficient outcome has increased—that is, it takes less dramatic fines (and/or less efficient levels of transfers) to achieve efficient outcomes.

Corollary 24 *The efficient outcome can be achieved under the following reward structure: If A presents C 's transferable debt or B 's IOU to the court in period six, he receives reward $G_A^{-1}(v/F)$ and B and C receive punishment $-F$. In all other cases, the court makes no rewards or fines.*

Trade takes the following form: In period 1, A trades flour for B 's IOU. In period 2 B trades bread for C 's transferable debt. In period 3, A returns B 's IOU in exchange for C 's transferable debt. In period 4 and 5 no trade takes place, and in period 6 C redeems his transferable debt with gold.¹⁸

¹⁸Under conditions analogous to those presented in the appendix in the proof of the

4.2 An Example with Uncertainty

The previous section shows that an advantage of transferable debt is that it allows payment to bypass relatively unreliable parties, making compliance easier. In the above example, because there was no underlying uncertainty, the correct set of trades in equilibrium was automatically known by all individuals. In the presence of uncertainty transferable debt provides additional benefits, in that it provides evidence to the debtor as to who should receive ultimate payment. In this section we provide an example incorporating uncertainty in order to demonstrate this point. The example combines the two-player and three-player cases.

With probability $\frac{1}{2}$, meetings and preferences are as described previously. With probability $\frac{1}{2}$, player A does not meet player B in period 1 but all other meetings are unaffected. B and C 's preferences are as before. In this event we assume A 's values gold at v . Recall that $v < w$, so that in this case efficient trade is as in the two-player case. As before, A can make reports in periods 3 or 6 and B can make reports in period 4.

Theorem 25 *In the three-person example with uncertainty, transferable debt implements the efficient outcome if*

$$u \geq \int^{G_A^{-1}(w/F)} G_A(t_A) dt_A \quad (75)$$

$$w \geq \int^{G_B^{-1}(v/F)} G_B(t_B) dt_B \quad (76)$$

$$2\alpha F \geq G_A^{-1}(w/F) \quad (77)$$

$$2\alpha F \geq G_B^{-1}(v/F) \quad (78)$$

It does so in the following way: If A presents B 's IOU to the court in period 3, he receives reward $G_A^{-1}(w/F)$, B and C pay F , and the center immediately announces that B is bankrupt. If B is not declared bankrupt, then if B presents C 's transferable debt to the court in period 6, he receives reward $G_B^{-1}(v/F)$ and A and C pay F . If no reports have been made earlier, and if A presents C 's transferable debt to the court in period 6, he receives reward $G_A^{-1}(v/F)$ and B and C pay F .

In the mechanism if B is declared bankrupt, subsequent announcements by him are ignored. (Intuitively, once B is declared bankrupt, C makes

previous theorem, the mechanism can also be made immune to extortion by player B in period 2 (see previous footnote).

payment directly through the court to A , rather than paying B).¹⁹

Trade in this mechanism works as follows: In period 1, if A appears, he trades flour for B 's IOU. In period 2 B trades bread for C 's transferable debt. In period 3, if A holds B 's IOU, he trades it for C 's transferable debt. In period 4, if B has not been declared bankrupt and if B possesses C 's transferable debt, then B trades it for gold. In period 5 no trade takes place. In period 6, if A possesses C 's transferable debt, C redeems it for gold.

Note that the implementation of this arrangement requires that A have the ability to contact the center at an earlier stage, and that the center have the ability to make announcements. These additional powers provided no additional benefit or power to the mechanism without uncertainty. The communication only is of use because it allows the center to check in a timely way on whether B has transferred value to A in the form of transferable debt.²⁰ Without transferable debt earlier announcements would have no bite. Moreover, note that these announcements are only used off the equilibrium path.

The conditions are sufficient, not necessary. By exploiting the fact that A has two opportunities to make a report, and thus two chances to get low costs of reporting on B 's malfeasance, we could expand the set of efficient implementation slightly.

5 Relationship to the literature

In Arrow-Debreu economies, no one makes payments. Instead all agents keep running tabs with a reputable and powerful central authority, secure in the knowledge that budgets will be balanced in the fullness of time. Payments only become necessary with diminution of the central authority and a consequent limitation on agents' reliability: under certain circumstances, an agent is "good" for debt only up to a limit (less than the value of his future income and possibly zero). If there is ample liquidity, i.e., if there are sufficient durable assets and such assets are attachable as collateral, payments can be made in these assets. Techniques for economizing on liquidity become

¹⁹Since bankruptcy never happens on the equilibrium path, we make the payment from C to A maximal for convenience. If we extended the model in such a way that agents occasionally went bankrupt, it would be important to consider the minimal payments necessary from A to make the mechanism effective.

²⁰In effect, a default by B (failure to transfer value to A) renders B unable to enforce debt against another agent (C). This is a standard consequence of default in models with limited commitment (see for example, Azariadis and Lambertini 2003).

important when such assets are scarce, or where the legal structure’s ability to enforce takings of collateral deteriorates. This is the basic rationale for the circulation of debt claims.

Various analyses of circulating debt can be found in the recent literature on inside money (e.g., Freeman 1996, 1999, Cavalcanti and Wallace 1999, Kiyotaki and Moore 2000, Bullard and Smith 2003, Mills 2004). Each of these approaches contains at its core a cycle of trade that begins the issue of debt and ends with its redemption. A common conclusion is that, as long as enforcement is possible at the end of the cycle, that same enforcement can be used to give value to promises traded at earlier stages and thus to allow mere promises to circulate as assets do. Often such enforcement is possible, or economical, only for a certain class of agents. The creditworthiness of these “strong credits” gives rise to the circulation of their debts, which in equilibrium may relax liquidity constraints for all agents. In other papers (Williamson 1992, Williamson 1999, Temzelides and Williamson 2001), fairly broad classes of agents may issue circulating claims, but if there is private information concerning the creditworthiness of the issuers, the potential efficiencies conferred by circulating debt may be undermined by adverse selection.

Despite apparent differences in structure, our approach borrows much from the literature described above. As in many of the above models, trade is restricted to a succession of bilateral encounters, enforcement is limited, and only the debt of the strongest credit circulates. What is different in our approach, however, is the enforceability of agents’ debt is not associated with their inherent creditworthiness (all debtors are subject to the same enforcement technology), but instead with the mixture of assets held. A decision to re-order holdings—to pay a debt—changes the reliability of the agent. Transferable debt becomes complementary with illiquid debt: even an inherently unreliable “middle” agent can temporarily borrow with the assurance that transferable debt will be used to pay off later. The circulation of debt is as much as a cause of creditworthiness as a consequence of it.

The advantages of circulation in our structure can be illustrated using a Wicksell triangle example from Kiyotaki and Moore (2000). Their example, depicted in figure 2, is similar to our economy but is different in a fundamental way. In our model we want A to transfer something to B who then transfers something to C who then transfers something to A . For Kiyotaki and Moore, the first two transfers are reversed in time: the desired arrangement is for B to transfer something to C and then for A to transfer something to B and then for C to transfer something to A . Because the transfers are in this reverse order, a single piece of inside paper money can

pass from hand to hand to accomplish all the trades in the Kiyotaki-Moore example: C passes the paper to B who passes the paper to A who passes it back to C . Such timing is natural in an endowment economy, where the typical individual receives the payment and then uses it to buy goods.²¹

In a production economy, a producer receives raw materials and uses them to make finished product. In other words, there is a need for working capital. If inside claims must be used to accomplish the work, the number of claims needed will be much larger in our economy than in the Kiyotaki-Moore example. Thus in our economy, unlike theirs, there is a role for settling debt with other debts.

Crucial to our story is idea of setoff. Even with limited enforcement, creditors' priority can be maintained as long as debts are allowed to cancel each other (which is effectively what occurs when B transfers C 's debt to A). The role of final debt transfer in this regard is closely related to that of net settlement of payment obligations. What is different here is that the practice of net settlement is typically associated with centralized clearing arrangements, whereas final payment by debt transfer can and does occur in decentralized settings.

Another related paper is Townsend (1987), which considers the role of "tokens" in overcoming trading frictions posed by private information and spatial separation. In Townsend's setup, "patient" agents may lack incentives to provide consumption goods in an early period, because there is a risk that they may be relocated, where such relocation eliminates all records of them having provided goods. Since an agent's patience is private information, all agents claim to be patient once relocated. These frictions may be overcome if patient movers carry with them a token as proof of having provided a consumption good at their original location. Once at a new location, patient agents can present tokens to the planner and be rewarded with higher levels of late consumption.

Townsend's tokens thus play an evidentiary role similar to that of the receipts and IOUs considered above. A key distinction is that Townsend focuses on private information rather than limited enforcement as an impediment to trade. Townsend's tokens can be presented to the planner at zero cost and, for the simplest cases, are able to completely overcome the trading frictions he considers. Absent private information, tokens would not be needed. In our environment, by contrast, enforcement is always costly even in the presence of evidence. Hence the existence of evidence may in of itself not deliver efficiency (cf. Theorems 18 and 20 above).

²¹This timing also underlies cash-in-advance economies.

The other literature with which this account has important links is the literature on trade credit. The advantages and disadvantages of trade credit as an alternative to bank lending have been examined by a variety of authors. Frank and Maksimovic (1998), for example, emphasize the advantage of the supplier of raw materials as a lender to the downstream firm, because of comparative advantage at using repossessed collateral. In the context of emerging markets of Eastern Europe, Hege and Ambrus-Lakatos (2000) emphasize the threats inherent in continuing relationships among chains of upstream and downstream firms as the reason that trade credit can provide an imperfect substitute for bank lending in times of illiquidity.

However, Ickes' (1998) description of veksel in Russia emphasizes several aspects of the arrangement that seem to support our view of these instruments as important for their transferability. In particular, payment in the form of veksel was less subject to opportunistic diversion than payment in relatively anonymous bank funds. By diminishing the scope for successive defaults, Ickes argues, the use of veksel allowed firms to maintain the integrity of their supply chains in spite of difficulties posed by the prevalent legal environment.

6 Conclusion

Inside money is first and foremost, debt, but it is debt with a difference. Associated with every type of inside money is a system of rules that govern the circumstances of monetary transfer. The most critical rules are those that determine when a transfer may occur and when a transfer discharges another debt.

In a model with centralized, but limited enforcement, we have examined the usefulness of a variety of debt arrangements, from simple promises through evidence in the form of receipts to IOUs and finally transferable debt. We have shown that it can be valuable to allow the use of transferable debt to discharge other debt. Transferable debt possesses two advantages: it allows less-than-perfectly-reliable agents to exit a credit chain in a timely fashion, and it provides useful information about the behavior of individuals at a distance from the current transaction.

The laws governing the paper-based transfer have existed for centuries in essentially unaltered form (at least within the Anglo-Saxon legal tradition; see Winn 1999). However, these are becoming less relevant as paper-based payments instruments are being supplanted by electronic, centralized forms of funds transfer. Electronic systems can offer their participants a higher

degree of assurance than paper-based systems, both in terms of payment finality and in terms of protection against fraud. There are downsides to such systems, however, including lack of flexibility (e.g., one must have a merchant account to receive a credit card payment) and a loss of privacy (since there is a centralized record of all transactions). A comparative analysis of traditional and centralized systems for the transfer of inside money, as well as a comparison of the finality aspects of inside and outside money, should provide fertile ground for future research.

7 Appendix: Proofs

7.1 Proofs in the Two-Player Environment

In the two-player case, v_i will denote a player's payoff from his own endowment, and u_i will denote a player's payoff from the other player's endowment, where $i = B, C$. We assume $\min_i u_i > \max_i v_i$, (in other words, that trading endowments is efficient and individually rational).

Denote the payoffs in the payoff table as follows

		'0'	'1'
		0	z
'0'	0		y
		x	n
'1'	w		m

and define

$$\Psi_B(\varphi_C) = (1 - \varphi_C)w + \varphi_C(m - y) \quad (79)$$

$$\Psi_C(\varphi_B) = (1 - \varphi_B)z + \varphi_B(n - x) \quad (80)$$

7.1.1 Proof of Theorems 3 and 5

The proof proceeds through a series of lemmas

Lemma 26 *An efficient mechanism exists if and only if $v_C \leq -\Omega$, where Ω is the minimand of the following minimization problem:*

$$\min_{(w,x,y,z,m,n,\varphi_B,\varphi_C)} \varphi_A x + \int_0^{\Psi_C(\varphi_A)} G_C(t) dt \quad (81)$$

subject to the following restrictions

$$z \leq 0 \quad (82)$$

$$w \leq 0 \quad (83)$$

$$\alpha z + y \leq 0 \quad (84)$$

$$\alpha x + w \leq 0 \quad (85)$$

$$\alpha m + n \leq 0 \quad (86)$$

$$\alpha n + m \leq 0 \quad (87)$$

$$w \geq -F; x \geq -F; y \geq -F; z \geq -F; m \geq -F; n \geq -F \quad (88)$$

$$\varphi_C = G_C(\Psi_C(\varphi_B)) \quad (89)$$

$$\varphi_B = G_B(\Psi_B(\varphi_C)) \quad (90)$$

Proof. Condition (88) is equivalent to feasibility conditions (4-5). Conditions (82-83) are the equivalent of condition (10). Conditions (84-87) come from feasibility conditions (4-5); the other forms of these two feasibility conditions are redundant given (82-83). Conditions (89-90) are equivalent to conditions (11-12) in the current notation. Condition (13) can be restated as

$$\begin{aligned}
0 &\geq v_C + E_C[\max\{\varphi_A x, (1 - \varphi_B)z + \varphi_B n - t_C\}] & (91) \\
&= v_C + \varphi_B x + E_C[\max\{0, \Psi_C(\varphi_B) - t_C\}] \\
&= v_C + \varphi_B x + \int^{\Psi_C(\varphi_B)} \Psi_C(\varphi_B) - t_C dG_C(t_C) \\
&= v_C + \varphi_B x + \int^{\Psi_C(\varphi_B)} G_C(t_C) dt_C
\end{aligned}$$

Thus if we choose variables to minimize (81) we can then determine whether v_C is small enough to make the total expression negative. ■

Lemma 27 *If there is a non-zero solution to (81), there is one with $x = y = -F$.*

Proof. If $(w, x, y, z, m, n, \varphi_B, \varphi_C)$ is a non-zero set of feasible parameters, then so is $(w, -F, -F, z, m - y - F, n - x - F, \varphi_B, \varphi_C)$. To see this, note that $y \geq -F$ implies $m \geq m - y - F$, and $m - y \geq 0$ implies $m - y - F \geq -F$. Together with the corresponding results for n these facts imply that the new values for m and n satisfy the required restrictions; the new values of x and y obviously do. Moreover, since the transformation leaves $m - y$ and $n - x$ unchanged, none of the other parameters are affected by the transformation. Since φ_B and φ_C are unchanged and x is weakly decreasing, the objective is weakly improved by the transformation. ■

Note that when $\varphi_B = 0$, the expression (81) is 0, so that no positive v yields an efficient mechanism. Moreover when $\varphi_B = 0$, $\varphi_C = G_C(\Psi_C(\varphi_B)) = 0$.

Without loss of generality we can assume $\Psi_C(\varphi_B)$ is no greater than the upper bound of the support of the distribution of G_C . (Otherwise, we could reduce z or n without changing φ_B unless $z = n = -F$. But then $\Psi_C(\varphi_B) < 0$, which is below the support. Thus we can rewrite the objective (81) as

$$\min_{(w, y, z, m, n, \varphi_B, \varphi_C)} -\varphi_A F + \int^{G_C^{-1}(\varphi_C)} G_C(t_C) dt_C \quad (92)$$

Since, in any solution φ_B must be strictly positive and w is at most zero, φ_C must be strictly positive as well, thus $(m - y)$ and $(n - x)$ are positive (otherwise Ψ_B and Ψ_C are negative on the unit interval).

Lemma 28 *In any non zero solution to (81), $w = 0$ and either (86) or (87) is binding.*

Proof. Any change which simultaneously increases φ_B and reduces φ_C improves the objective. If n and m are interior, then there exists a small increase in m and a small decrease in n which will improve the situation. It is not possible for $n = -F$, otherwise $n - x$ is not positive. This leaves (86) or (87) to be binding. Similarly, if $w < 0$, we can simultaneously increase w and decrease n so as to improve the situation. ■

7.1.2 Proof of Corollary 4

We know that

$$0 = G_C(\Psi_C(G_B(\Psi_B(0)))) \quad (93)$$

Suppose there is a solution with φ_A and φ_C greater than zero. Then we have

$$\varphi_C = G_C(\Psi_C(G_B(\Psi_B(\varphi_C)))) \quad (94)$$

Thus

$$\begin{aligned} 1 &\leq (\max G'_C)\Psi'_C(\max G'_B)\Psi'_B & (95) \\ &= (\max G'_C)(\max G'_B)(F + n - z)(F + m) \\ &\leq (\max G'_C)(\max G'_B)(2F + n)(F + m) \end{aligned}$$

Given the restrictions (86-87) the last expression is maximized at $m = n = 0$ when $\alpha \leq \frac{1}{2}$ and at $m = -\alpha n = \frac{1}{2}F(2\alpha - 1)$ when $\alpha \geq \frac{1}{2}$.

7.1.3 Proof of Lemma 6

Define the “truncation function” (figure A1)

$$\tau(p) \doteq \min\{p, 1\} - \min\{0, p\}. \quad (96)$$

For the uniform distribution, Ω simplifies to

$$\varphi_B x + \frac{T_B}{2} \varphi_C^2 \quad (97)$$

and (89) and (90) become

$$\varphi_C = \tau(T_C^{-1}\Psi_C(\varphi_B)) \quad (98)$$

$$\varphi_B = \tau(T_B^{-1}\Psi_B(\varphi_C)) \quad (99)$$

We divide solutions into three (not necessarily mutually exclusive) types: The two equations always have the *zero solution*, $\varphi_B = \varphi_C = 0$, but at these values $\Omega = 0$. This does not generate an efficient mechanism since v is positive. This may be the only simultaneous solution to the equations (figure A2) or there may also be an *interior solution*, one in which

$$\tau(T_C^{-1}\Psi_B(\varphi_B)) = T_C^{-1}\Psi_C(\varphi_B) \quad (100)$$

and vice versa (figure A3). Suppose either w or z is not zero. If there is an interior solution to the pair of equations, it is unique and non zero. If and only if there is an interior solution there will also be a unique *upper boundary* solution (one in which $\varphi_B = 1$ or $\varphi_C = 1$).²² Thus a necessary condition for an efficient mechanism is that there be an interior solution to the two equations. If both w and z are zero, then there is generally unique interior solution at the origin. The exception is the “degenerate case” (figure A4) in which both $T_B^{-1}\Psi_B$ and $T_C^{-1}\Psi_C$ go through the origin and have slopes whose product is 1.

If either w or z is not zero the simultaneous solution is

$$\varphi_A = \frac{-z(m-y-w) - wT_C}{(m-y-w)(n-x-z) - T_B T_C} \quad (101)$$

$$\varphi_C = \frac{-w(n-x-z) - zT_B}{(m-y-w)(n-x-z) - T_B T_C} \quad (102)$$

In this case, for an interior solution to the equations the following conditions are necessary:

$$(m-y-w)(n-x-z) - T_B T_C > -w(n-x-z) - zT_B \quad (103)$$

$$(m-y-w)(n-x-z) - T_B T_C > -z(m-y-w) - wT_C. \quad (104)$$

These simplify to

$$(m-y)(n-x) - T_B T_C > -z(T_B - (m-y)) \quad (105)$$

$$(m-y)(n-x) - T_B T_C > -w(T_C - (n-x)) \quad (106)$$

²² A solution could be both an interior solution and an upper boundary solution if one of the equations intersects the other at a “kink.”

for which,

$$(m - y)(n - x) > T_B T_C \quad (107)$$

is necessary. The same condition is necessary for the existence of an upper boundary solution in the non degenerate case when $w = z = 0$.

In the degenerate case, $w = z = 0$ and the product of the slopes equals 1, so that

$$(m - y)(n - x) = T_B T_C. \quad (108)$$

In any case,

$$(m - y)(n - x) \leq (m + F)(n + F) \leq F^2 \quad (109)$$

given restrictions (86-88). Combining the last inequality with either (107) or (108) proves the lemma.

Henceforward we will assume that condition (23) holds.

7.1.4 Proof of theorem 7

Lemma 29 *If there is a non-zero solution to (81), there is a solution where $w = z = 0$ and $(m - y)(n - x) = T_B T_C$ (in other words, a “degenerate” solution).*

Proof. If $(w, z, m, n, \varphi_B, \varphi_C)$ is a non-zero set of feasible parameters, then so is $(0, 0, m', n', \varphi_B, \varphi_C)$ where

$$\frac{n' + F}{T_C} = \frac{T_B}{m' + F} = \frac{\varphi_C}{\varphi_B}. \quad (110)$$

That is,

$$\varphi_B(n' + F) = \varphi_C T_C = T_C \tau(T_C^{-1}[(1 - \varphi_B)z + \varphi_B(n + F)]) \quad (111)$$

$$\leq (1 - \varphi_B)z + \varphi_B(n + F) \quad (112)$$

or

$$n' = \frac{1 - \varphi_B}{\varphi_B} z + n \leq n \quad (113)$$

and furthermore since $\varphi_B(n' + F) = \varphi_C T_C > 0$, we conclude that $n' > -F$. Similar conditions hold for m . These conditions imply that m' and n' satisfy the restrictions on m and n . The product of the slopes is one, and both of the new functions $T_B^{-1}\Psi_B$ and $T_C^{-1}\Psi_C$ go through the points $(0, 0)$ and (φ_B, φ_C) . ■

Henceforward we will assume, w.o.l.o.g. $w = z = 0$. For a given feasible pair (m', n') any pair $(\varphi_B, \varphi_C) \in (0, 1]^2$ which satisfies condition (110) satisfies the restrictions to the minimization problem (81).

Lemma 30 *If there is a non-zero solution to (81), there is a solution where $m \geq 0$ and $n = \alpha^{-1}m$.*

Proof. The objective Ω is decreasing in φ_B and increasing in φ_C . Thus the Ω -minimizing combination (φ_B, φ_C) on the locus of points where

$$\frac{\varphi_C}{\varphi_B} = r \quad (114)$$

for some given r is dominated by the Ω -minimizing combination (φ_B, φ_C) on any locus with lower value of r . In other words, if the pair (m', n') are interior we can simultaneously increase m' and decrease n' so as to maintain the left equality in (110), while reducing the value of the common ratio. This process will only stop when either (86) or (87) binds. It only remains to show that $m \geq 0$ (so that it is (86) that binds). If there is a feasible upper boundary solution with $m \geq 0$, it dominates one with $m < 0$. But we know that

$$(m + F)(-\alpha^{-1}m + F) \quad (115)$$

is decreasing in m for m positive, and by (23) it is non-negative when $m = 0$. Moreover it reaches zero when $m = \alpha F$ (and so $n = -F$). By continuity there is a non-negative value of m satisfying (24). ■

By the quadratic formula, we have

$$m = \frac{-(1 - \alpha)F + \sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C}}{2} \quad (116)$$

7.1.5 Proof of Lemma 8

The minimization problem thus reduces to the following convex problem:

$$\min_{\varphi_B, \varphi_C} -F\varphi_B + \frac{T_B}{2}\varphi_C^2 \quad (117)$$

subject to

$$0 < \varphi_B \leq 1; 0 < \varphi_C \leq 1 \quad (118)$$

$$\frac{T_B}{F + m} = \frac{\varphi_C}{\varphi_B}. \quad (119)$$

along the ray defined by condition (119) the objective function is minimized at $\varphi_C = T_B/(F + m) = F/(F - \alpha^{-1}m) > 1$. Thus the solution to the problem lies on a boundary. If

$$\frac{T_B}{F + m} \geq 1 \quad (120)$$

then φ_C can be increased to 1. If the inequality is reversed then φ_B hits 1 before φ_C does. The condition (120) is equivalent to

$$T_B \geq \frac{(1 + \alpha)F + \sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C}}{2} \quad (121)$$

which in turn, given (23) is equivalent to the following pair of conditions:

$$2T_B \geq (1 + \alpha)F \quad (122)$$

and

$$(2T_B - (1 + \alpha)F)^2 \geq (1 + \alpha)^2 F^2 - 4\alpha T_B T_C \quad (123)$$

the latter of which simplifies to

$$T_B + \alpha T_C \geq F(1 + \alpha) \quad (124)$$

(see figure A5).

7.1.6 Proof of Theorem 9.

If $\varphi_C = 1$, then $\varphi_A = (F + m)/T_B$ and

$$\begin{aligned} \Omega &= -F \frac{F + m}{T_B} + \frac{T_C}{2} \\ &= \frac{T_B T_C - (1 + \alpha)F^2 - F \sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C}}{2T_B} \end{aligned} \quad (125)$$

If $\varphi_B = 1$, then $\varphi_C = (F - \alpha^{-1}m)/T_C$ and

$$\begin{aligned} \Omega &= -F + \frac{1}{2T_C} \left(F + \frac{(1 - \alpha)F - \sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C}}{2\alpha} \right)^2 \\ &= -F + \frac{1}{8T_C \alpha^2} (F^2 (1 + \alpha)^2 \\ &\quad - 2F(1 + \alpha) \sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C} + (1 + \alpha)^2 F^2 - 4\alpha T_B T_C) \\ &= -F + \frac{1}{4T_C \alpha^2} \left(F^2 (1 + \alpha)^2 - 2\alpha T_B T_C - F(1 + \alpha) \sqrt{(1 + \alpha)^2 F^2 - 4\alpha T_B T_C} \right) \end{aligned} \quad (126)$$

7.1.7 Proof of Theorems 10 and 11

Clearly it is impossible to get B to give away his endowment when only he can make a report. Suppose the optimum gets C to give away his endowment. The contract should maximize the net payments to the two parties

less the costs of announcement, while inducing trades. Let s be B 's receipt when he makes no announcement, and let $s + m$ be his receipt when he makes an announcement. Clearly the receipt in the case of an announcement cannot depend on the announcement made. Let r be C 's receipt if no announcement is made, and let n be his receipt if an announcement is made that C was in compliance. Clearly if C is not in compliance he should suffer the maximum possible fine. Let φ denote the probability that B makes an announcement. That is we wish to solve

$$\max_{m,n,r,s,\varphi} (1 - \varphi)(r + s) + \varphi(s + m + n) - \int^m t_A dG_B(t_B) \quad (127)$$

subject to

$$\alpha r + s \leq 0 \quad (128)$$

$$r + \alpha s \leq 0 \quad (129)$$

$$\alpha n + s + m \leq 0 \quad (130)$$

$$n + \alpha s + \alpha m \leq 0 \quad (131)$$

$$\varphi n \geq v_C - \varphi F \quad (132)$$

$$r \geq -F \quad (133)$$

$$s \geq -F \quad (134)$$

$$\varphi = G_B(m) \quad (135)$$

We know the result will be less than zero. If the result of the problem is less than $-(u_B - v_C)$ then the costs exceed the benefit and the constrained optimum is to have no trade. Note that by (132), φ must be positive. Therefore by (135) m is positive.

Lemma 31 *In any optimal solution, either (130) or (131) binds.*

Proof. Otherwise, n can increase, improving the objective. ■

Lemma 32 *In any optimal solution $s \leq 0$.*

Proof. The problem is linear in r, n and s . Without loss of generality assume (134) does not bind. Let $\kappa, \lambda, \mu, \nu$ represent the (non-negative) multipliers for constraints (128-131). Then by differentiating with respect to

$r, n,$ and $s,$ we see that

$$1 - \varphi - \alpha\kappa - \lambda \leq 0 \quad (136)$$

$$\varphi - \alpha\mu - \nu \leq 0 \quad (137)$$

$$1 - \kappa - \alpha\lambda - \mu - \alpha\nu = 0 \quad (138)$$

Since φ cannot be zero, the second restriction demonstrates that either (130) or (131) must bind. If φ is not one then the first restriction demonstrates either (128) or (129) must bind (if $\varphi = 1$ then the variable r is irrelevant to the objective and we can without loss of generality continue to assume either (128) or (129) must bind). Adding the first two restrictions and comparing with the third, we conclude that it cannot be that both

$$\kappa + \alpha\lambda > \alpha\kappa + \lambda \quad (139)$$

and

$$\mu + \alpha\nu > \alpha\mu + \nu, \quad (140)$$

which implies that it cannot be that both κ and μ are positive, that is it cannot be that both (128) and (130) bind. Given (129) and (131) this means that it cannot be that both

$$s > 0 \quad (141)$$

$$s + m > 0 \quad (142)$$

and since m must be positive s cannot be. ■

We can therefore simplify the problem by using (129) to eliminate r from the problem, and rewriting the objective function as in the theorem (expression 31).

7.1.8 Case of uniform density.

If $G_B(t_B) = t_B/T$ for $0 \leq t_B \leq T$, then problem (31) can be rewritten as follows:

$$\max_{m,n,s} (1 - \alpha)s + \frac{\alpha ms}{T} + \frac{mn}{T} + \frac{m^2}{2T} \quad (143)$$

subject to

$$s \leq 0 \quad (144)$$

$$\alpha n + s + m \leq 0 \quad (145)$$

$$n + \alpha s + \alpha m \leq 0 \quad (146)$$

with complementary slackness,

$$m(n + F) \geq v_C T \quad (147)$$

$$s \geq -F \quad (148)$$

$$m \leq T. \quad (149)$$

Lemma 33 *There exist (m, s, n) satisfying these restrictions if and only if*

$$(1 + \alpha)F \geq \alpha T + v \quad (150)$$

$$F^2 \geq v_C T. \quad (151)$$

Proof. Without loss of generality we can assume $s = -F$. Then the largest possible value $m(n + F)$ with m and n satisfying (145) and (146) is F^2 (occurring at $m = F, n = 0$). This demonstrates (151) is necessary and sufficient for a triplet satisfying all the conditions except (149). Given these conditions, the lowest possible value for m occurs when (147) and (146) are binding. (Point *A* in figure A6). At this point, m is

$$\frac{(1 + \alpha)F - \sqrt{(1 + \alpha)^2 F^2 - 4\alpha v_C T}}{2\alpha} \quad (152)$$

(note that (151) is sufficient for the discriminant to be positive). For the last inequality to be consistent, it is necessary and sufficient for this expression to be less than T . That requirement simplifies to (150). ■

Theorem 12 is an immediate consequence of this lemma.

Lemma 34 *Constraint (147) is binding.*

Proof. We first consider the case with (146) binding. The problem simplifies to

$$\max_{m,s} (1 - \alpha)s + \frac{m^2}{2T}(1 - 2\alpha) \quad (153)$$

subject to

$$s + m \leq 0 \quad (154)$$

$$m(F - \alpha(m + s)) \geq v_C T \quad (155)$$

$$s \geq -F \quad (156)$$

$$m \leq T. \quad (157)$$

Since the objective is increasing in s , either the first or the second constraint will be binding at the optimum. But if the first constraint is binding, then along the constraint the objective reduces to

$$\max_m -(1 - \alpha)m + \frac{m^2}{2T}(1 - 2\alpha) \quad (158)$$

which is decreasing in m for $m \leq T$. As we decrease m along the constraint eventually the second constraint must be binding.

Now we consider the case with (145) binding. Replacing s using $z = m + s$, the problem simplifies to

$$\max_{m,z} (1 - \alpha)\left[z - m - \frac{(1 + \alpha)mz}{T\alpha}\right] + (1 - 2\alpha)\frac{m^2}{2T} \quad (159)$$

subject to

$$z \leq m \quad (160)$$

$$z \geq 0 \quad (161)$$

$$m(F\alpha - z) \geq \alpha v_C T \quad (162)$$

$$z \geq m - F \quad (163)$$

$$m \leq T. \quad (164)$$

Now the partial derivative of the objective with respect to m is negative for $m \leq T$. Thus either (160) or (162) binds. If it is the former, then the objective reduces to

$$\max_m \left[-\frac{(1 - \alpha^2)}{T\alpha} + \frac{(1 - 2\alpha)}{2T}\right]m^2 \quad (165)$$

which is decreasing in m along the constraint (see figure A7). In short (162) binds. ■

Lemma 35 *There are values satisfying the restrictions with (145) binding if and only if*

$$(1 + \alpha)F \geq \alpha T + v_C \quad (166)$$

$$F \geq v_C \quad (167)$$

Proof. Again, (160)-(163) are consistent if and only if (151) holds (point B is above the m axis in figure A7) In addition we need to verify that (164) is consistent with the rest—that is that T is at least as great as the values of m at points C and D . That is the two restrictions, together they imply (151). ■

Once we know that constraint (147) is binding, we can further simplify the problem:

$$\max_{m,s} v_C + (1 - \alpha)s + \frac{\alpha ms}{T} - \frac{mF}{T} + \frac{m^2}{2T} \quad (168)$$

subject to

$$\begin{aligned} s &\leq 0 \\ s &\geq -F \\ m &\leq T \end{aligned} \quad (169)$$

and to

$$\begin{aligned} \alpha\left(\frac{v_C T}{m} - F\right) + m + s &\leq 0 \\ \frac{v_C T}{m} - F + \alpha m + \alpha s &\leq 0 \end{aligned} \quad (170)$$

with complementary slackness.

Now the partial of the objective with respect to m is

$$\frac{1}{T}(\alpha s - F + m). \quad (171)$$

By (170) this is less than

$$-(1 - \alpha)(F + s) \quad (172)$$

which is negative by (169). This leads to the conclusion that we look for the maximal value of m as follows: in the range

$$m \in \left[\frac{(1 + \alpha)F - \sqrt{(1 + \alpha)^2 F^2 - 4\alpha v_C T}}{2\alpha}, \min\left\{T, \frac{v_C T}{F}\right\} \right] \quad (173)$$

we search for the maximum of

$$\max_m (1 - \alpha) \left[\frac{F}{\alpha} - m - \frac{v_C T}{\alpha m} \right] + \frac{m^2}{2T} (1 - 2\alpha). \quad (174)$$

If

$$v_C T \geq 2\alpha F^2 \quad (175)$$

(only possible if $\alpha \leq \frac{1}{2}$) or if $F < v$ then this is the answer. Otherwise we compare with the maximal value of

$$(1 - \alpha^2)v_C + (1 - \alpha)\left[\alpha F - m\left(1 + (1 + \alpha)\frac{F}{T}\right) - \frac{\alpha v_C T}{m}\right] + \frac{m^2}{2T}(1 - 2\alpha) \quad (176)$$

on the interval

$$m \in \left[\frac{v_C T}{F}, \min\{T, \sqrt{2\alpha v_C T}\}\right]. \quad (177)$$

(see figure A8), proving Corollary 13. (If $\alpha \geq 1/2$, we also know the left end of the first case is not optimal).

7.1.9 Proof of Theorem 14

If the contract does not take advantage of evidence, clearly it is identical to that described in Theorem 10. If it does take advantage of evidence, the maximization problem is identical to that of Theorem 10, with the addition of the following constraint:

$$\varphi(s + m) + (1 - \varphi)s - \int^m t_A dG_B(t_B) + u_B \geq s + v_B. \quad (178)$$

In other words, player B prefers the equilibrium play to a defection in which he does not deliver his endowment, does not receive a receipt and never reports. (It is clear that he never reports, since a report without evidence incurs the maximal punishment). This condition can be simplified:

$$u_B - v_B \geq \int^m G_B(t_B) dt_A. \quad (179)$$

Then any solution to the problem in Theorem 10 provides payments which are also optimal for this problem, as long as the above constraint does not bind. On the other hand if this constraint binds, it limits the size of m , but has no other effect on the problem. The rest of the conclusions in Theorem 10 therefore remain valid.

7.1.10 Proof of Corollaries 16 and 17

In the case of a uniform distribution the restriction (149) is augmented by (179), becoming

$$m \leq \{T, \sqrt{2T(u_B - v_B)}\}. \quad (180)$$

Now the calculations follow those of lemma (33) with the additional requirement that expression (152) also be less than $\sqrt{2T(u_B - v_B)}$. This provides the third requirement in lemma 17.

7.2 Proofs in the Three-Player Environment

7.2.1 Proof of Theorem 19

From Theorem 18 we know that there is no efficient mechanism without IOUs. Any mechanism with IOUs and one sided reporting must work as follows: if player B makes a report without possessing an IOU from C , then B is assessed the maximum fine. If B makes a report with C 's IOU, B is rewarded with the amount m and C is assessed the maximum fine. For such a mechanism to induce the correct behavior, it must be that

$$v_C \leq G_B(m)F \quad (181)$$

so that C prefers to give up the gold rather than face the chance of a fine, and

$$\int^m (m - t_B) dG_B(t_B) \leq u_B \quad (182)$$

so that B prefers to receive the gold rather than the fine. The range of permitted m is $[0, \alpha F]$. Integrating by parts we rewrite (182) as

$$\int^m G_B(t_B) dt_B \leq u_B \quad (183)$$

and a value of m in the required range satisfies the the conditions exists if and only if conditions (63-64) of the text are satisfied.

7.2.2 Proof of Theorem 21

As before, in a world without transferable debt, when C cannot report, IOU's are necessary to enforce C 's payment efficiently. That is, along the credit chain, A will give B flour in return for an IOU; B will give C bread in return for an IOU. (If C is expected to give the goods directly to A in period 6 then there will be no evidence as to whether the goods were actually given; only costly reports could be used to enforce C 's behavior).

Therefore, an efficient mechanism without transferable debt entails no transfers in period 6. Suppose everybody has behaved correctly up to period 4, so that both A and B hold IOUs. The remaining decisions are as follows: At period 4, C must choose whether to offer to redeem his IOU and B must

choose whether to accept the offer. If B retains the IOU he must choose whether to report or not. Without loss of generality we assume that if B reports, the center announces the fact at the end of period 4, and in this case, any report by A does not affect payoffs. (Thus we know A makes no report if B reports). In period 5, if B has the gold he must choose whether to redeem his IOU, and A must choose whether to accept the gold. Finally, if A retains the IOU he must choose whether to report or not. Thus a mechanism specifies $P_i(x)$ where $x \in \{0, A, B\}$ indicates who (if anyone) made an announcement. The payoff functions are subject to the following feasibility restrictions:

$$P_i(0) = 0 \text{ for all } i \quad (184)$$

$$P_i(x) \geq -F \text{ for all } i, \text{ for } x = A, B \quad (185)$$

$$\sum_i \min\{P_i(x), \alpha P_i(x)\} \leq 0 \text{ for } x = A, B \quad (186)$$

A will choose to report when he holds an IOU, if $t_A < P_A(A)$. B will choose to report, when he holds an IOU if $t_B < s$, where

$$s = P_B(B) - G_A(P_A(A))P_B(A) \quad (187)$$

The following restrictions are necessary conditions for efficient behavior by the players:

$$w + G_A(P_A(A))P_B(A) \leq 0 \quad (188)$$

—that is, B must prefer to give up gold if he holds it.

$$\int^{P_A(A)} G_A(t) dt \leq u \quad (189)$$

—that is, A must prefer to accept the gold than to hold the IOU.

$$v + G_B(s)P_C(B) + (1 - G_B(s))G_A(P_A(A))P_C(A) \leq 0 \quad (190)$$

—that is, C must prefer to give up the gold.

$$G_A(P_A(A))P_B(A) + \int^s G_B(t) dt \leq 0 \quad (191)$$

—that is, B must prefer to accept the gold than to hold the IOU.

Inspection of the conditions demonstrates that provided that

$$\int^{G_A^{-1}(w/F)} G_B(t_B) dt_B \leq u \quad (192)$$

$$\frac{w}{F} \leq G_A(2\alpha F). \quad (193)$$

then there exists a value m such that the conditions are satisfied when $P_C(A) = P_C(B) = P_B(A) = P_B(B) = -F$; $P_A(A) = G_A^{-1}(w/F)$. Under these values the following behavior forms the backbone of a subgame perfect Nash equilibrium: In period 1, A offers and B accepts flour in return for an IOU. In period 2, B offers and C accepts bread in return for an IOU. In period 3, no offers are made. In period 4, C offers and B accepts gold in order to redeem the IOU. In period 4 B never makes a report. In period 5, if he has the gold B offers and A accepts gold in order to redeem the IOU. In period 6, A reports if he still has the IOU and the costs of reporting are less than $G_A^{-1}(w/F)$. This proves the theorem.

Mechanism Refinement As noted in the footnote 17, page 23 of the text, the collective punishment inherent in this mechanism leaves C vulnerable to extortion by B . For the mechanism make C invulnerable to extortion ex post, it must be the case that if B does not possess an IOU from C , then C prefers for A to make an announcement rather than to give up his gold. For this to be the case, the following condition is necessary:

$$v + G_A(P_A(A))P_C(A) \geq 0 \quad (194)$$

That is, the expected payoff to C in the case where B deviates is nonnegative. Combining this condition with the other necessary conditions leads to the following conclusion: there exists a mechanism immune to extortion if and only if the parameters satisfy the following more stringent conditions:

$$\int^{G_A^{-1}(w/F)} G_B(t_B) dt_B \leq u \quad (195)$$

$$\frac{w}{F} \leq G_A(\alpha F(1 + \frac{v}{w})). \quad (196)$$

7.2.3 Proof of Theorem 23

Any mechanism which does not depend on transferable debt can continue to be used. But in addition, we can consider mechanisms in which in period 3 agent A receives C 's transferable debt from B , as payment for B 's IOU. No trade occurs in periods 4 or 5; instead we consider the incentives for C to redeem his transferable debt from A in period 6. Let $P_i(AC)$ denote the

payoffs if A presents C 's transferable debt to the court in period 6. Then it is necessary that

$$v + G_A(P_A(AC))P_C(AC) \leq 0 \quad (197)$$

—that is, C must prefer to give up the gold and

$$\int^{P_A(AC)} G_A(t) dt \leq u \quad (198)$$

—that is, A must prefer to accept the gold. These two conditions can be satisfied if and only if the conditions stated in the theorem hold.

In period 4 A and B must be willing to exchange debt instruments. Let $P_i(AB)$ denote the payoff if A presents B 's IOU to the court. (Without loss of generality, we can assume that there is no benefit to B from presenting C 's transferable debt to the court). Then we need that

$$G_A(P_A(AB))P_C(AB) \leq 0 \quad (199)$$

and that

$$\int^{P_A(AB)} G_A(t) dt \leq u \quad (200)$$

—in other words, any small penalty and reward can always enforce this trade.

The final step is to verify that in period 1 each agent has an incentive to trade flour for an IOU, and in period 2 each agent has incentive to trade bread for transferable debt.

7.2.4 Proof of Theorem 25

There are three possible announcements: an announcement by A at period 6 in which C 's transferable debt is presented, an announcement at period 4 by B in which C 's transferable debt is presented, and an announcement by A at period 3 in which B 's IOU is presented. We will denote these announcements as AC , BC , and AB , respectively. As before, provided

$$v + G_A(P_A(AC))P_C(AC) \leq 0 \quad (201)$$

and

$$\int^{P_A(AC)} G_A(t) dt \leq u \quad (202)$$

then in period 6 A and C prefer to make the trade (provided C has the gold and A has the transferable debt). Provided

$$v + G_B(P_B(BC))P_C(BC) \leq 0 \quad (203)$$

and

$$\int^{P_B(BC)} G_B(t) dt \leq u \quad (204)$$

then in period 4 B and C prefer to make the trade (provided B has the transferable debt and has not been declared bankrupt). And provided

$$w + G_A(P_A(AB))P_B(AB) \leq 0 \quad (205)$$

and

$$\int^{P_A(AB)} G_A(t) dt \leq u \quad (206)$$

then in period 3 A and B prefer to trade financial claims. Given the conditions specified in the theorem, the payoffs specified for each announcement satisfy these requirements.

For completeness, we specify strategies that form the subgame perfect equilibrium yielding efficient outcomes under this mechanism:

Player C offers in period 2 to trade his transferable debt for bread. In period 4, if B has not been declared bankrupt, he offers to trade gold for transferable debt. Otherwise he makes no offer. In period 6, if C still has gold, he offers to trade gold for transferable debt.

Player B offers in period 1 to trade his IOU for flour. In period 2 he offers to trade bread for transferable debt. In period 3, if he has transferable debt, he offers to trade it for his IOU (otherwise he makes no offer). In period 4, if he has transferable debt, he offers to trade it for gold (otherwise he makes no offer). If he still has C 's transferable debt at the end of period 4 he announces it to the center, if the draw of costs is sufficiently low.

If A meets B in period 1 he offers to trade flour for an IOU. In period 3 if he has an IOU, he offers to trade it for transferable debt. If he has B 's IOU at the end of the period he announces it to the center if the draw of costs is sufficiently low. He makes no offers in period 5. If he has C 's transferable debt in period 6, he offers to trade it for gold. If he still has C 's transferable debt at the end of period 6 he announces it to the center if the draw of costs is sufficiently low.

References

- [1] Azariadis, C. and L. Lambertini, 2003, Endogenous debt constraints in lifecycle economies, *Review of Economic Studies* 70, 461-487.
- [2] Bullard, J. D. and B. D. Smith, 2003, Intermediaries and payments instruments, *Journal of Economic Theory* 109(2), 172-197.
- [3] Cavalcanti, R. and N. Wallace, 1999, A model of private banknote issue, *Review of Economic Dynamics* 2, 104-136.
- [4] Frank, M. and V. Maksimovic, 1998, Trade credit, collateral, and adverse selection, working paper, University of British Columbia.
- [5] Freeman, S., 1996, The payments system, liquidity, and rediscounting, *American Economic Review* 86: 1126-38.
- [6] Freeman, S., 1999, Rediscounting under aggregate risk, *Journal of Monetary Economics* 43: 197-216.
- [7] Hege, U. and L. Ambrus-Lakatos, 2000, Trade credit chains and liquidity supply, working paper, Tilburg University.
- [8] Ickes, B. W., 1998, Notes on veksels, unpublished manuscript, Pennsylvania State University.
- [9] Kahn, C. M., J. McAndrews, and W. Roberds, 2001, Settlement risk under gross and net settlement, *Journal of Money, Credit, and Banking* 47(2): 299-319.
- [10] Kiyotaki, N. and J. Moore, 2000, Inside money and liquidity, unpublished manuscript, London School of Economics.
- [11] Kohn, M., 1999, Bills of exchange and the money market to 1600, working paper 99-04, Department of Economics, Dartmouth College.
- [12] Mann, R. J., 1999. *Payment Systems and Other Financial Transactions*. (New York, Aspen Law and Business).
- [13] Mills, D. C., 2004, Mechanism design and the role of enforcement in Freeman's model of payments, *Review of Economic Dynamics* 7(1): 219-236.

- [14] Rogers, J. S., 1995, *The Early History of Bills and Notes: A Study of the Origins of Anglo-American Commercial Law* (Cambridge: Cambridge University Press).
- [15] Tamaki, N., 1995, *Japanese Banking: A History 1859-1959* (Cambridge: Cambridge University Press).
- [16] Temzelides, T. and S. D. Williamson, 2001, Private money, settlement, and discounts, *Carnegie-Rochester Conference Series on Public Policy* 54: 85-108.
- [17] Townsend, R.M., 1987, Economic organization with limited communication, *American Economic Review* 77: 954-971.
- [18] van der Wee, H., 1997, The influence of banking on the rise of capitalism in northwest europe, fourteenth to nineteenth century, in: A. Teichova, G. Kurgan-van Hentenryk, and D. Ziegler eds., *Banking, Trade, and Industry* (Cambridge University Press: New York) 173-190.
- [19] Wicksell, K., 1935, *Lectures on Political Economy, Volume II: Money* (New York: Macmillan).
- [20] Williamson, S. D., 1992. Laissez faire banking and circulating media of exchange, *Journal of Financial Intermediation* 2: 134-167.
- [21] Williamson, S. D., 1999, Private money, *Journal of Money, Credit and Banking* 31: 469-491.
- [22] Winn, J. K., 1998, Couriers without luggage: negotiable instruments and digital signatures, *South Carolina Law Review* 29: 739-785.
- [23] Winn, J. K., 1999, Clash of the titans: regulating the competition between established and emerging electronic payment systems, *Berkeley Technology Law Journal* 14: 675.