The Optimal Capital Structure and Endogenous Bankruptcy for a Fixed Term Debt Issued at Par

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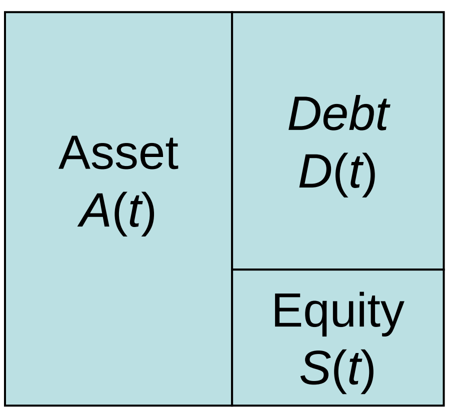
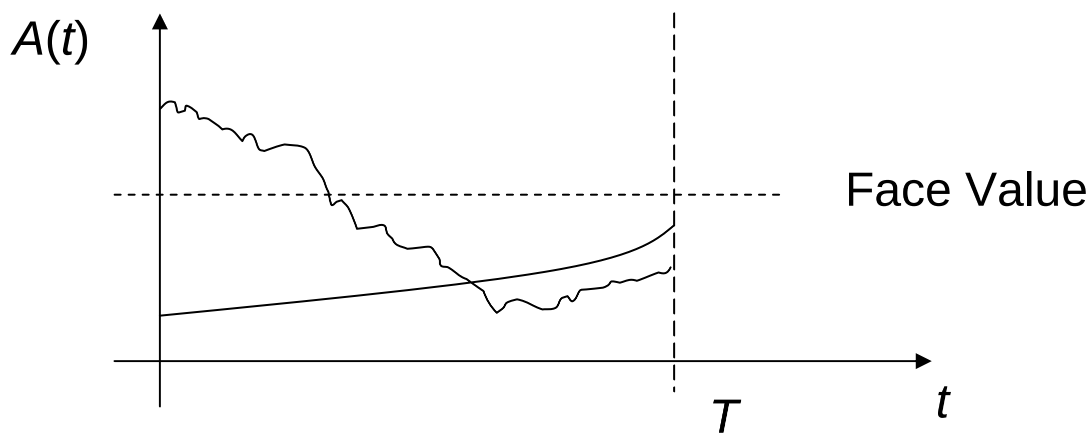
Contents

Literature (1)

- Structural Model
  - Merton (1974), Black and Cox (1976)
  - Exogenous bankruptcy, discount bond
  - No corporate tax, Finite Maturity

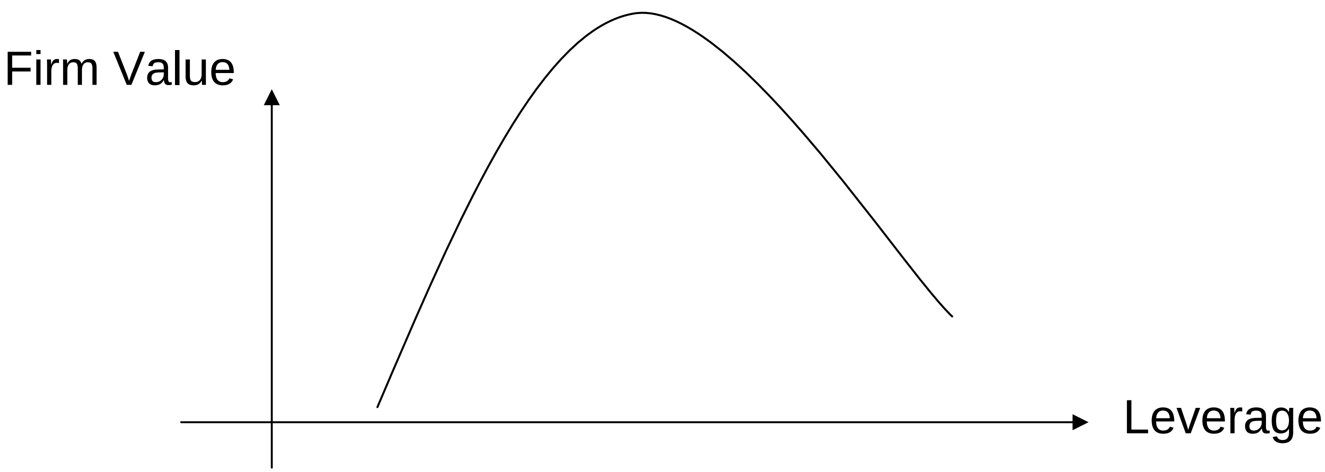




Literature (2)

- Brennan and Schwartz (1978)
  - First to examine optimal capital structure
  - Protected coupon debt, Finite maturity
  - Balancing (trade off) Theory

Firm Value = Asset Value + Tax benefit – Bankruptcy cost



Literature (3)

- Leland (1994)
  - Perpetual debt: coupon determines leverage
  - Endogenous and exogenous bankruptcy
- Leland and Toft (1996)
  - Continuous issuing of fixed term debt
  - Stationary debt structure
- Dynamic Capital choice:
  - Goldstein, Ju and Leland (2001)
  - Hennessy and Whited (2006)
  - Ju and Ou-Yang (2006)….

Motivation and Objective

1. Motivation
   - ✓ No infinitely lived corporate debt.
   - ✓ In primary market, firms issue debts at par.
2. Objective
   - ✓ Examine optimal leverage and optimal coupon rate for protected or unprotected debts.
3. Questions
   - ✓ Can high levered firms fully get tax benefit?
   - ✓ Are there any tax benefit to discount debt?

Our Model

Firm's total asset value

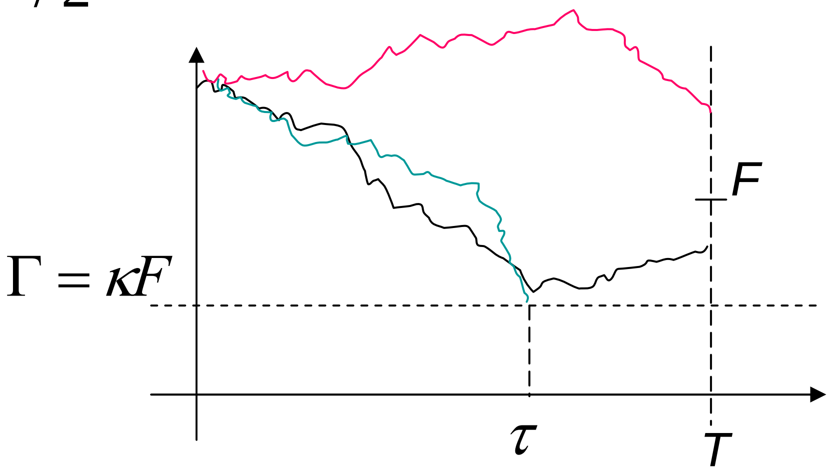
$$A(t) = \exp X(t), \ \ X(t) = x + \mu t + \sigma W(t)$$

$$\mu = r - \delta - \sigma^2 / 2$$

Default time epoch

$$\tau = \inf\{t : A(t) \le \Gamma\}$$

(Exogenous Case):



Notations:

F : Face value, T : Maturity, δ : Dividend

r : Risk-free rate, $\Gamma = \kappa F$: Exogenous threshold

Payoff functions (1)

Debt value:

$$D(A,T,C,F) = E\left[\int_0^{\tau\wedge T} CFe^{-rt}dt + 1_{\{\tau<T\}}(1-\alpha)\Gamma e^{-r\tau} + 1_{\{\tau\geq T, A(T)>F\}}Fe^{-rT} + 1_{\{\tau\geq T, A(T)<F\}}(1-\alpha)A(T)e^{-rT}\right]$$

Bankruptcy cost:

$$BC(A,T,C,F) = E\left[1_{\{\tau<T\}}\alpha\Gamma e^{-r\tau} + 1_{\{\tau\geq T, A(T)<F\}}\alpha A(T)e^{-rT}\right]$$

Tax benefit:

$$TB(A,T,C,F) = E\left[\int_0^{\tau\wedge T} \lambda CFe^{-rt}dt\right]$$

where α : Default loss rate, λ : tax rate

Payoff functions (2)

Equity value:

$$S(A,T,C,F) = E\left[\int_0^{\tau\wedge T} \delta A(t)e^{-rt}dt - \int_0^{\tau\wedge T}(1-\lambda)CFe^{-rt}dt + \max\{A(T)-F,\ 0\}1_{\{\tau\geq T\}}\right]$$

By definition, the firm value is defined by

$$v(A,T,C,F) = D(A,T,C,F) + S(A,T,C,F)$$

It can be shown that

$$\boxed{v(A,T,C,F) = A + TB(A,T,C,F) - BC(A,T,C,F)}$$

Also, these formulas are obtained in closed form.

Optimal Leverage with Par Issuing Constraint

$$\max_{F} \left\{ v(A, T, \hat{C}, F) \right\}$$

$$\text{s.t. } D(A, T, \hat{C}, F) = F$$

Note: Maximizing the firm value is equivalent to maximizing the equity value

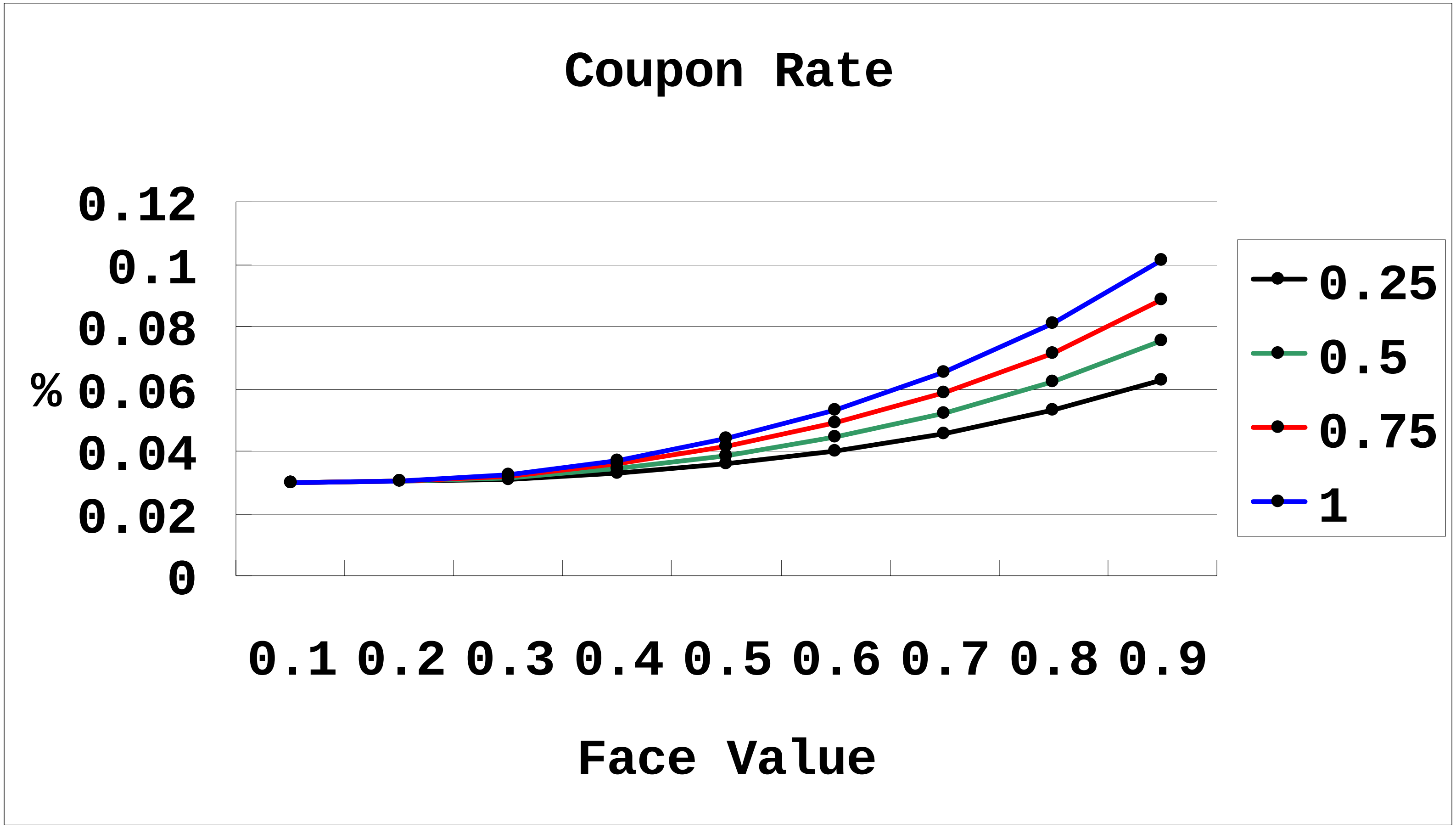
Standard Parameter Settings:

$$r = 0.03, \delta = 0.01, T = 10,$$

$$\sigma = 0.2, \lambda = 0.4, A(0) = 1.0, \kappa = 0.7$$

Constraint for par issuing debt: $D(A, T, \hat{C}, F) = F$

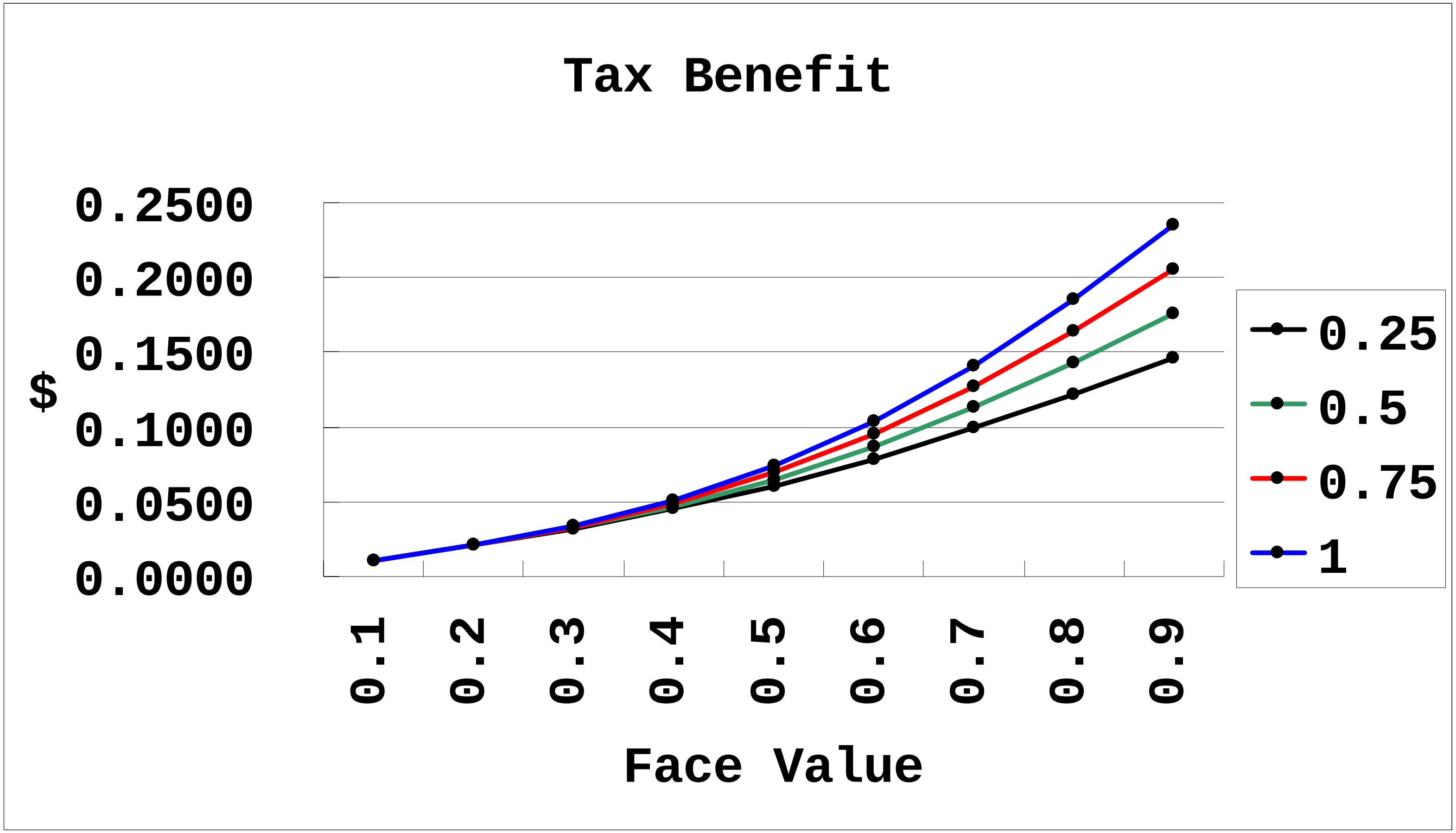
Given F , the par coupon rate is obtained



Note: Yield Spread = $\hat{C} - r$

Firm value:

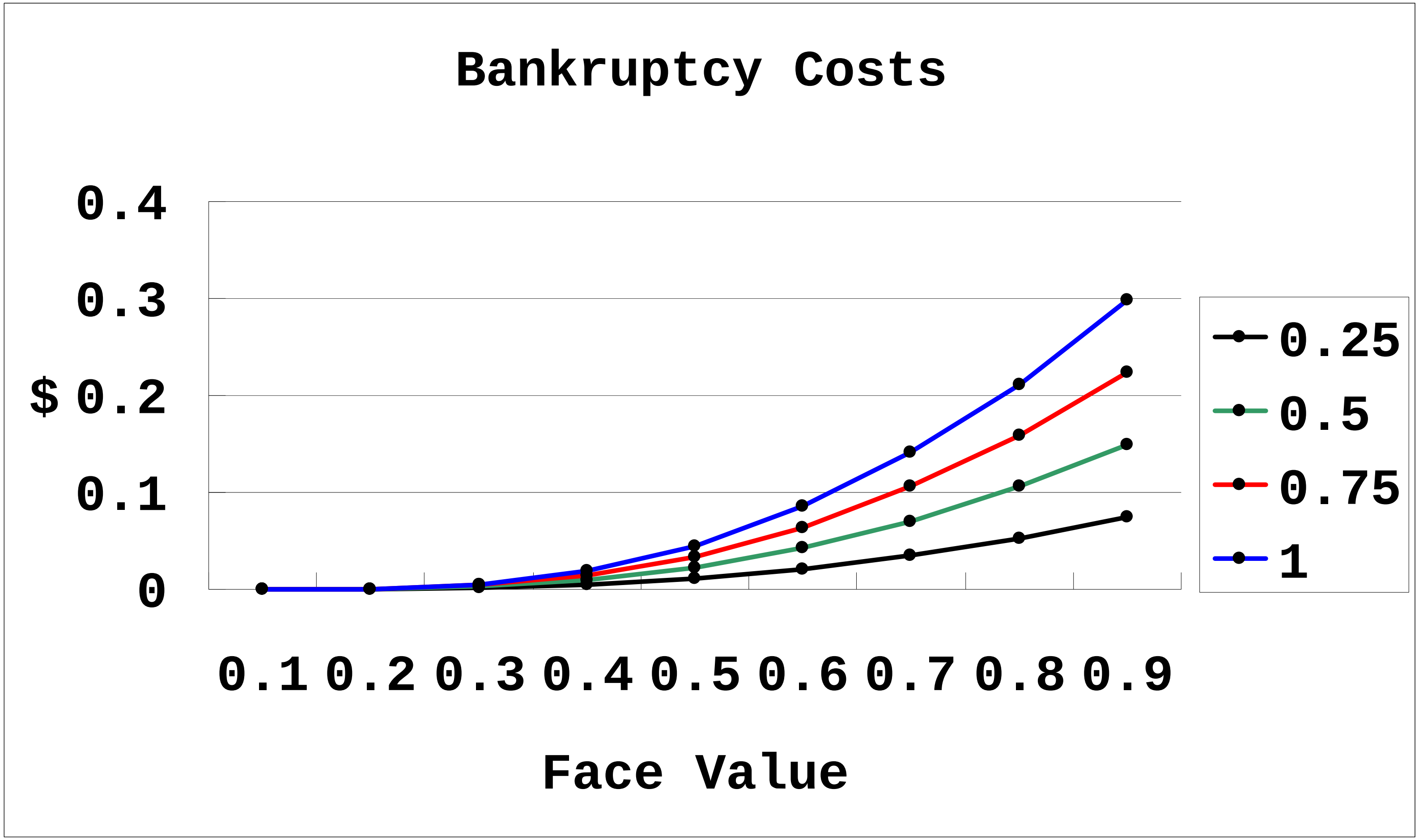
$$v(A,T,\hat{C},F) = A + \underline{TB(A,T,\hat{C},F)} - BC(A,T,\hat{C},F)$$



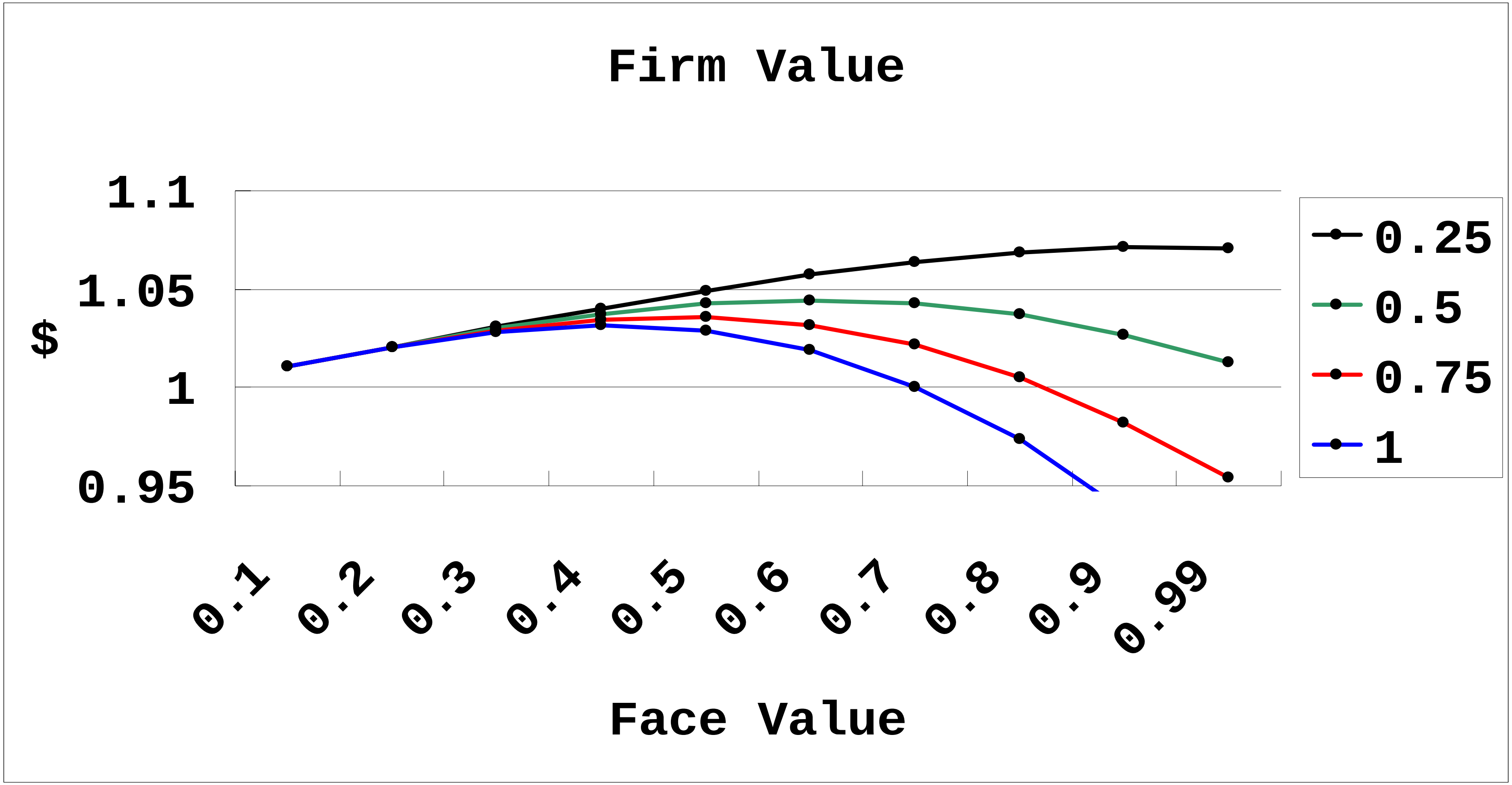
Note: $\alpha \uparrow \Rightarrow \hat{C} \uparrow$ (due to par issuing constraint) $\Rightarrow TB \uparrow$

Firm value:

$$v(A,T,\hat{C},F) = A + TB(A,T,\hat{C},F) - \underline{BC(A,T,\hat{C},F)}$$



Optimal Leverage (1)
(Exogenous Bankruptcy)



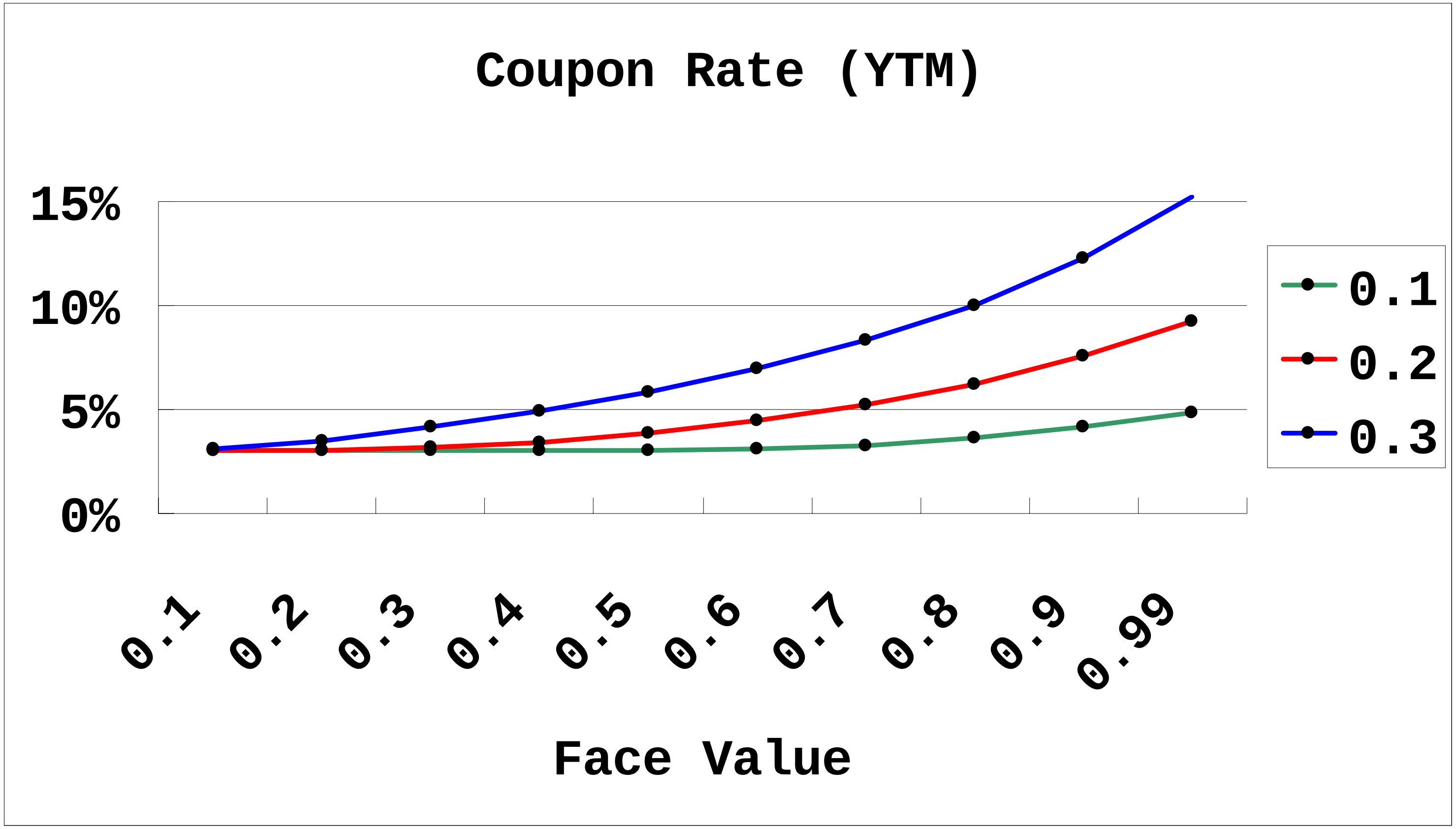
Optimal Leverage (2)
(Exogenous Bankruptcy)

Face Value alpha	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
0.25	1.0104	1.0207	1.0307	1.0403	1.0492	1.0572	1.0639	1.0687	1.0712	1.0709
0.5	1.0104	1.0206	1.0299	1.0375	1.0425	1.0444	1.0427	1.037	1.0265	1.0127
0.75	1.0104	1.0205	1.0291	1.0346	1.0357	1.0316	1.0216	1.0052	0.9818	0.9544
1	1.0104	1.0204	1.0283	1.0318	1.029	1.0188	1.0005	0.9734	0.9371	0.8961

Note: Red Zone indicates $v < A(0) \Leftrightarrow BC > TB$

Comparative Statics (1)

$$\sigma = 0.1, 0.2, 0.3 \quad [\alpha = 0.5,\ T = 10]$$



Note: Yield Spread = $\hat{C} - r$

Comparative Statics (1)

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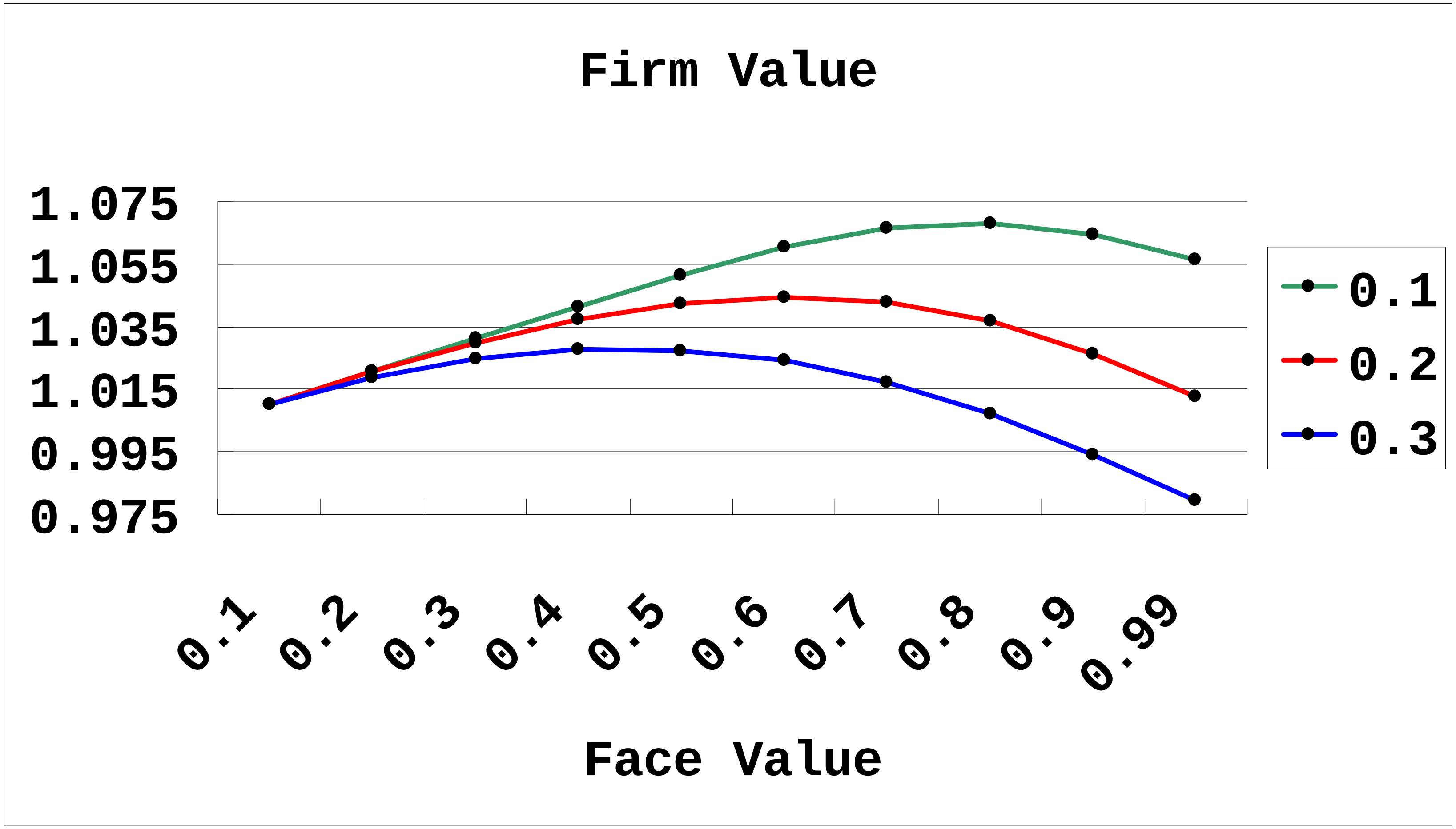
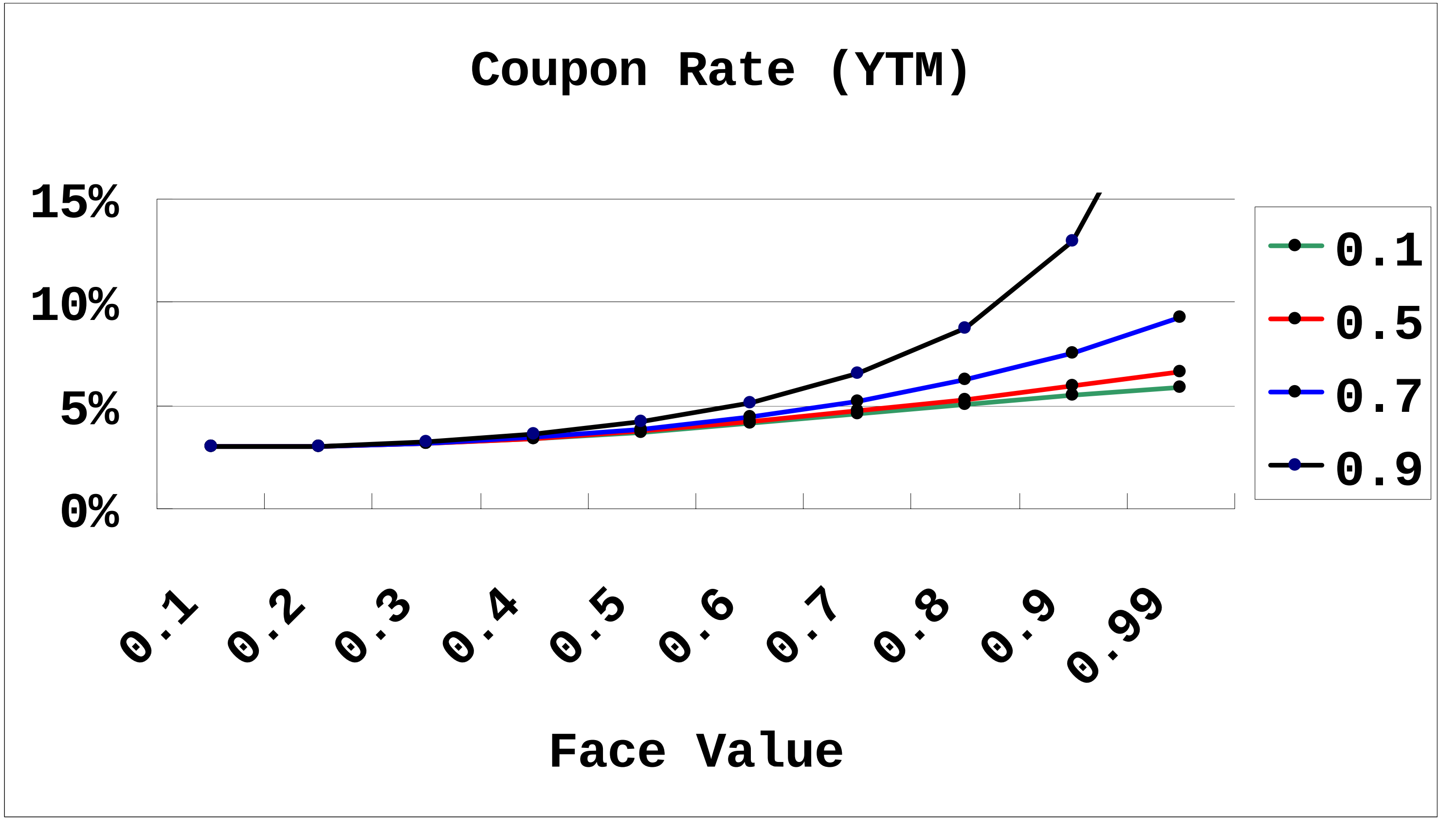


Comparative Statics (2)

$$\kappa = 0.1, 0.5, 0.7, 0.9 \quad [\alpha = 0.5,\ T = 10,\ \Gamma = \kappa F]$$



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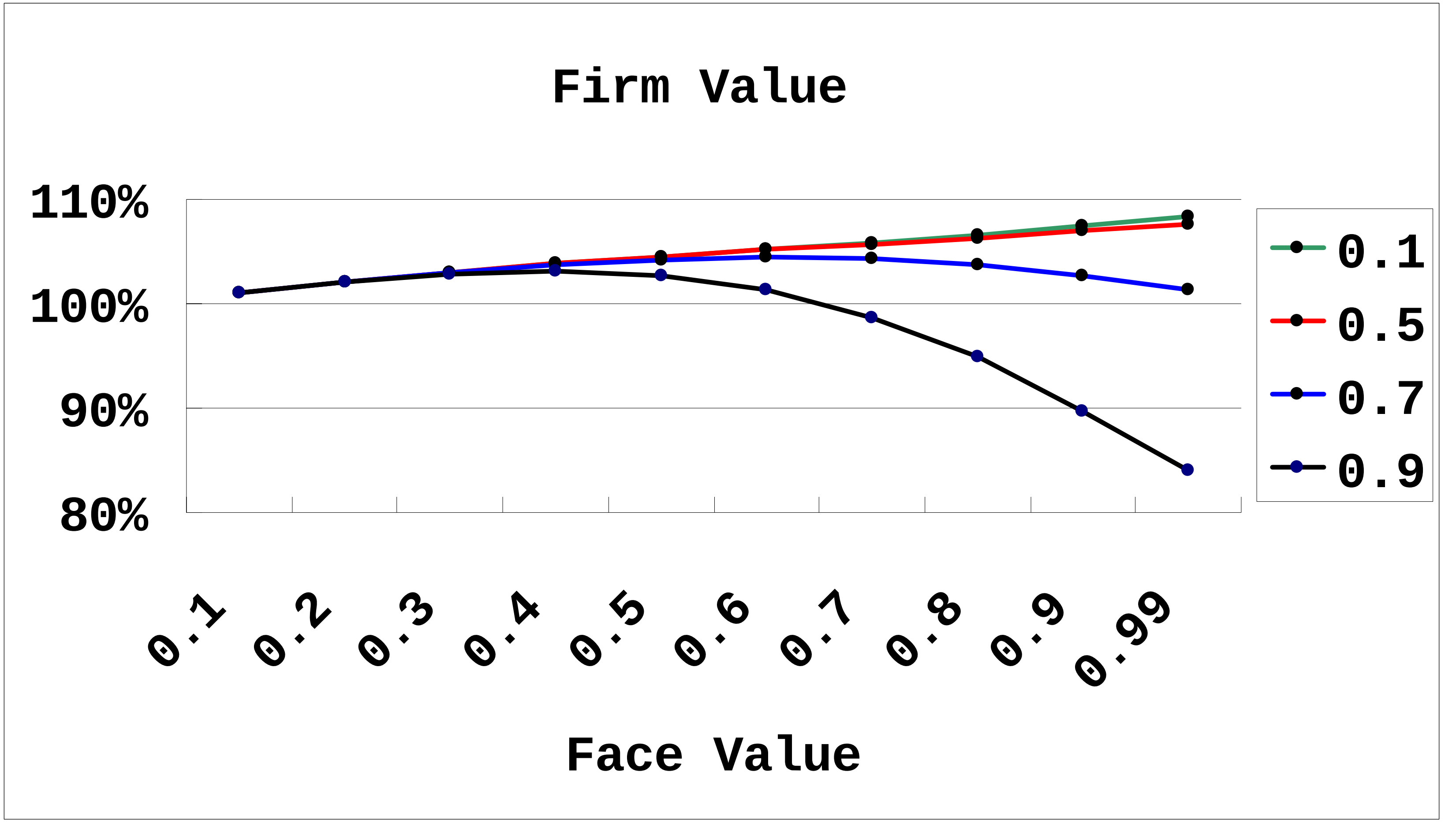
Comparative Statics (2)

$$\kappa = 0.1, 0.5, 0.7, 0.9 \quad [\alpha = 0.5,\ T = 10,\ \Gamma = \kappa F]$$



Endogenous Bankruptcy

Stock holders can choose the default time to maximize equity value:

$$\tau^* = \arg\max_{\tau} \left\{ S(A, T, \hat{C}, F, \tau) \right\}$$

Free-boundary problem

The firm value is maximized under the condition of equity holders' strategy and the par issuing constraint.

$$\max_{F} \left\{ v(A, T, \hat{C}, F, \tau^*) \right\}$$

$$\text{s.t. } D(A, T, \hat{C}, F, \tau^*) = F$$

Payoff functions (3)

Debt value:

$$D(A,T,C,F,\tau)=E\left[\int_0^{\tau\wedge T} CFe^{-rt}dt+1_{\{\tau<T\}}(1-\alpha)b(\tau)e^{-r\tau}+1_{\{\tau\geq T,A(T)>F\}}Fe^{-rT}+1_{\{\tau\geq T,A(T)<F\}}(1-\alpha)A(T)e^{-rT}\right]$$

This term disappears!

Bankruptcy cost:

$$BC(A,T,C,F,\tau)=E\left[1_{\{\tau<T\}}\alpha b(\tau)e^{-r\tau}+1_{\{\tau\geq T,A(T)<F\}}\alpha A(T)e^{-rT}\right]$$

Tax benefit:

$$TB(A,T,C,F,\tau)=E\left[\int_0^{\tau\wedge T}\lambda CFe^{-rt}dt\right]$$

where $b(t)$: Default boundary

Payoff functions (4)

Equity value:

$$S(A,T,C,F,\tau) = E\left[\begin{array}{c}\int_0^{\tau \wedge T} \delta A(t) e^{-rt} dt - \int_0^{\tau \wedge T} (1-\lambda) CF e^{-rt} dt \\ + \max\{A(T) - F,\ 0\} 1_{\{\tau \geq T\}}\end{array}\right]$$

By definition, the firm value is defined by

$$v(A,T,C,F,\tau) = D(A,T,C,F,\tau) + S(A,T,C,F,\tau)$$

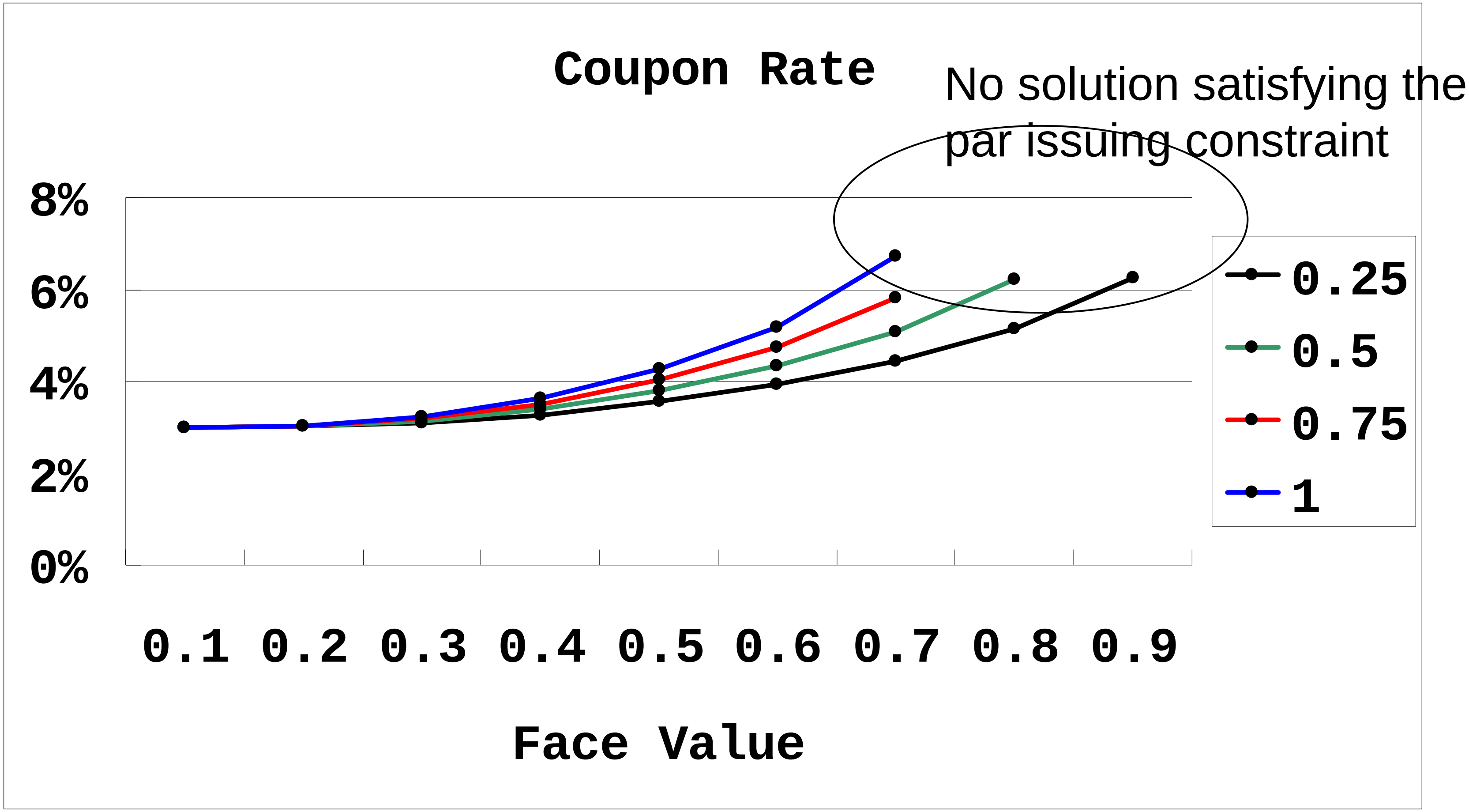
It can be shown, even for this case, that

$$v(A,T,C,F,\tau) = A + TB(A,T,C,F,\tau) - BC(A,T,C,F,\tau)$$

Note: No closed formulas ⇨ We use the *binomial model*

Constraint for par issuing debt: $D(A, T, \hat{C}, F, \tau^*) = F$

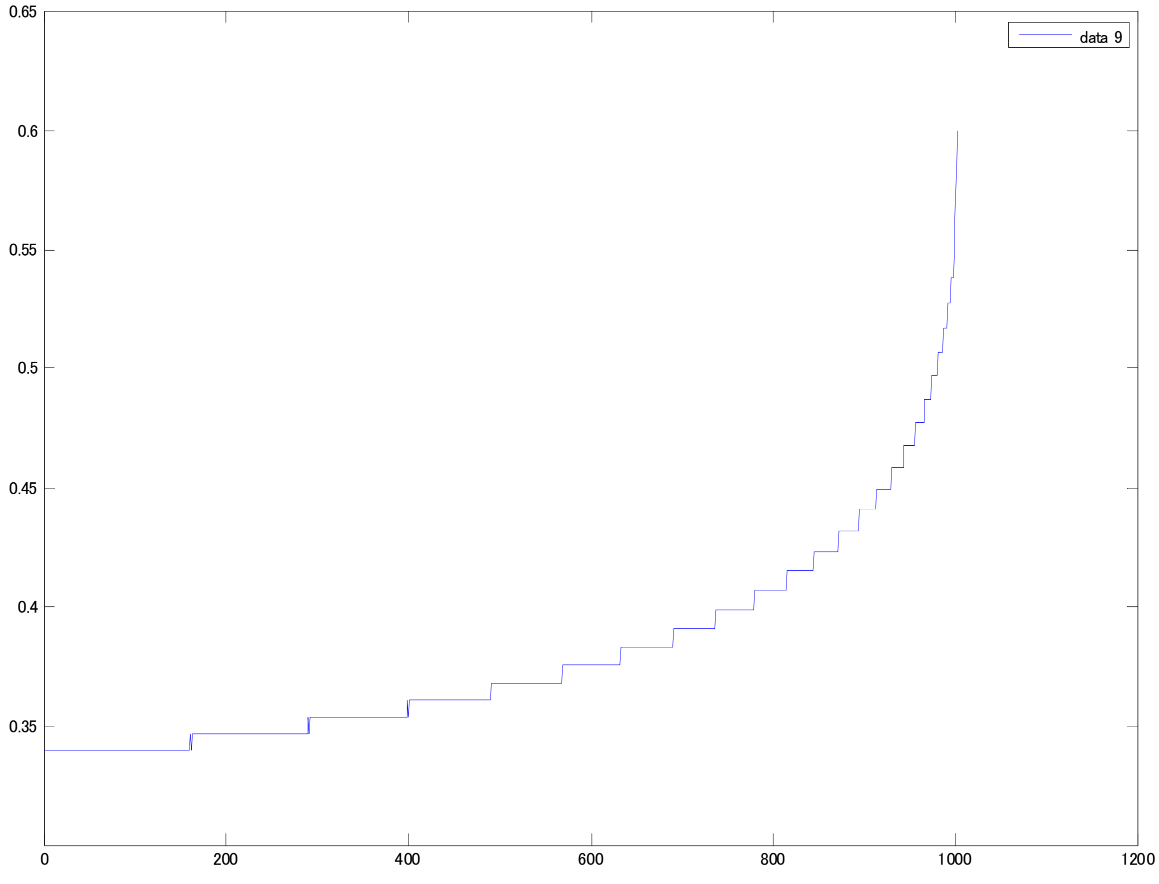
⇨ Given *F* and τ^*, the par coupon rate is obtained



Note: Yield Spread = $\hat{C} - r$

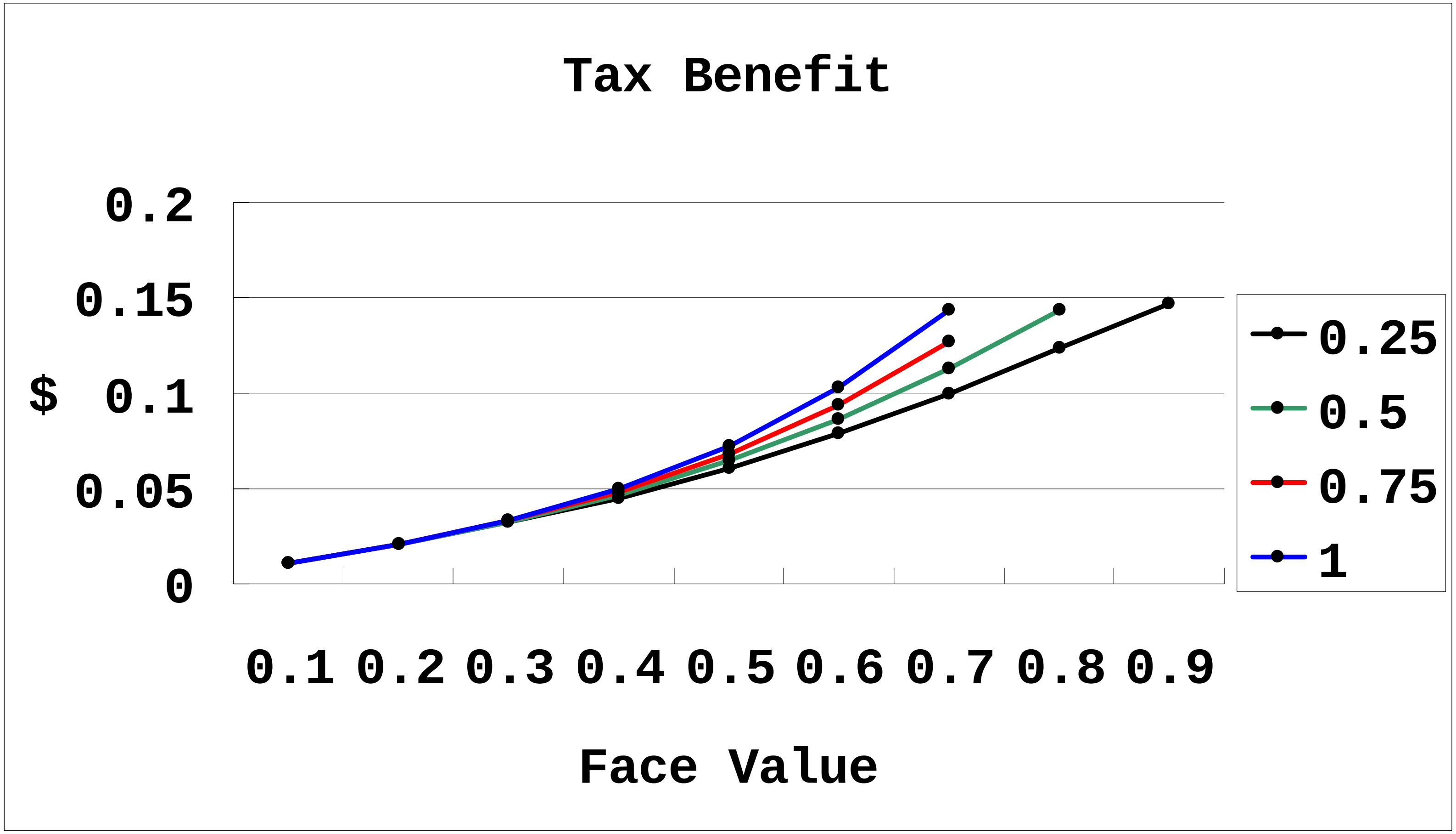
Optimal Default Boundary

$$[F = 0.6,\ \alpha = 0.5,\ T = 10]$$

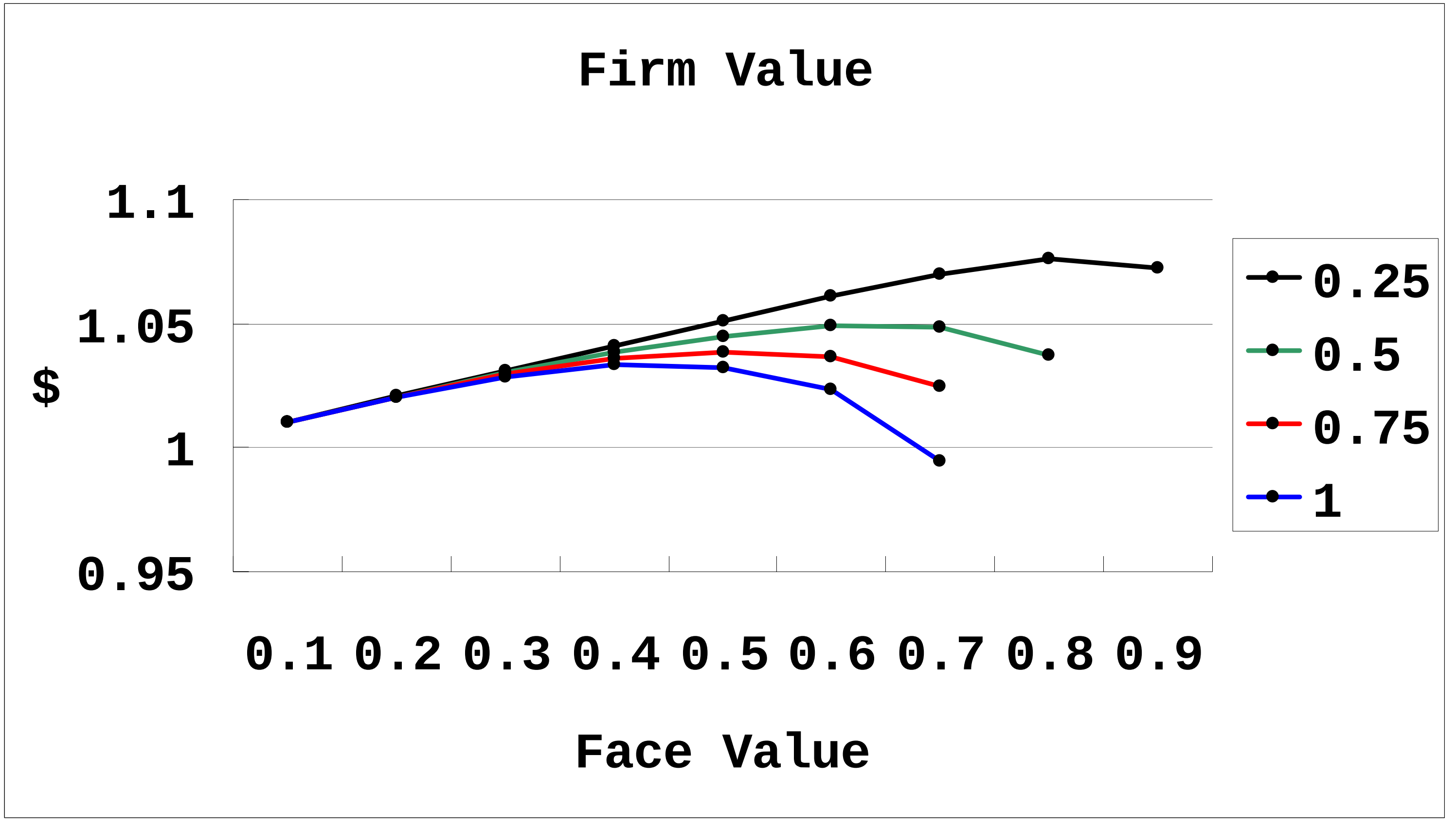


Firm value:

$$v(A,T,\hat{C},F) = A + \underline{TB(A,T,\hat{C},F)} - BC(A,T,\hat{C},F)$$



Optimal Leverage (1) (Endogenous Bankruptcy)



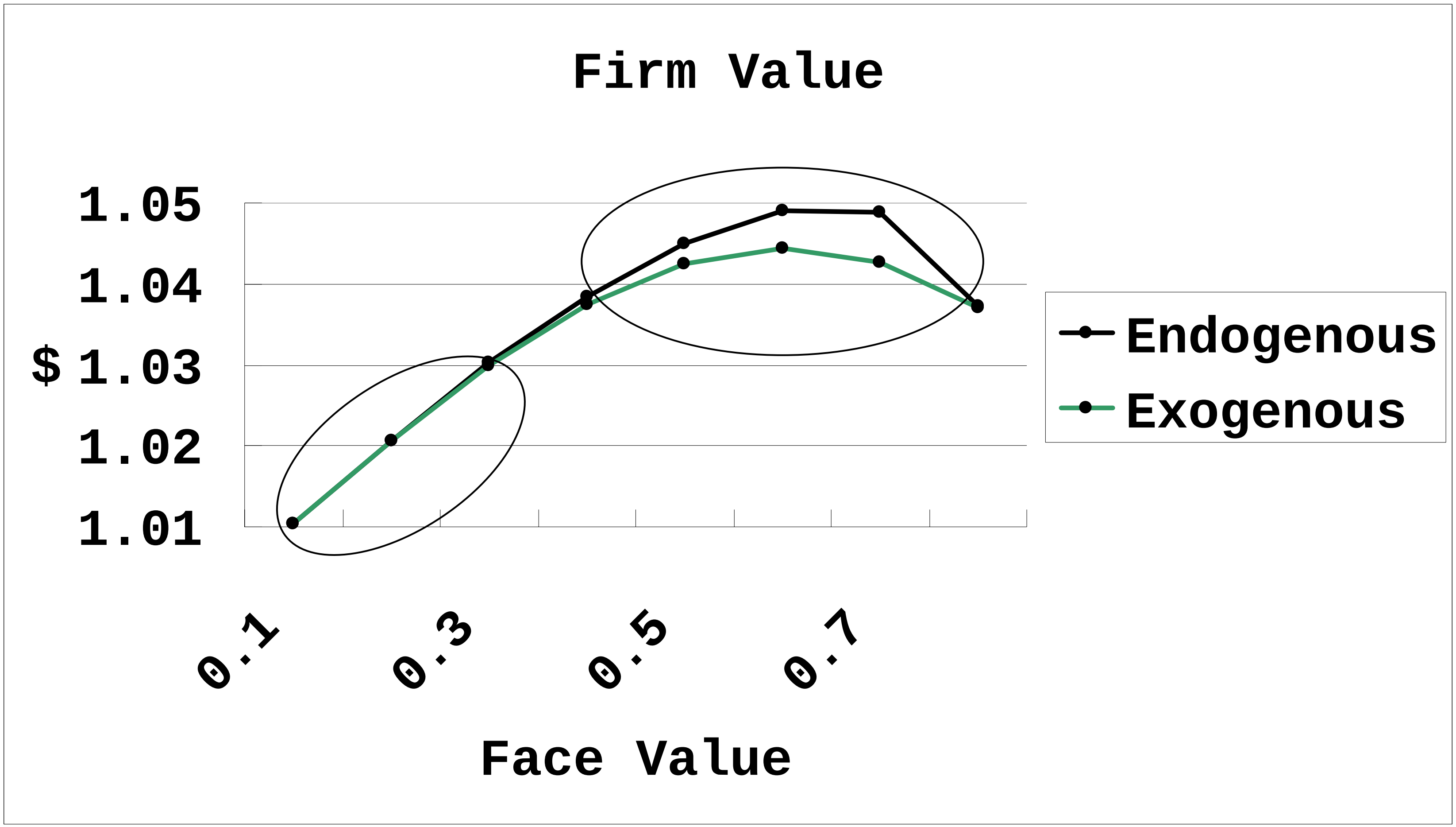
Optimal Leverage (2)
(Endogenous Bankruptcy)

Face Value alpha	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.25	1.0104	1.0207	1.0309	1.041	1.0508	1.0608	1.0696	1.0759	1.0722
0.5	1.0104	1.0206	1.0302	1.0385	1.0449	1.049	1.0488	1.0372	NaN
0.75	1.0104	1.0205	1.0295	1.036	1.0388	1.0366	1.0246	NaN	NaN
1	1.0104	1.0204	1.0287	1.0334	1.0325	1.0233	0.9948	NaN	NaN

Debt Capacity, Par Constraint

Comparison

$[\alpha = 0.5,\ T = 10]$



Shortcomings

Facts:

- Some firms issue pure discount bonds.
- If the revenue is not enough to repay coupons, then coupons may be reduced.

Our model cannot answer what if these happen.

⇨ Need to modify the model!

Assumption:

- *Firms can't issue debts over par.*

Extended Model (1)

Partial tax benefit:

$$\text{If } (r-\delta)A(t) > CF \quad \text{then} \quad TB(t) = \lambda CF$$

$$\text{else} \quad TB(t) = \lambda(r-\delta)A(t)$$

Tax benefit for discount debt:

$$\text{Discount Premium } \Delta = F - D(A, T, C, F)$$

$$TB(t) = \lambda\left\{CF + \Delta\left(\frac{1-e^{-rT}}{r}\right)\right\}$$

We take the issue of discount bonds as deferred charge which provides tax benefit in the future.

Extended Model (2)

New Problem to be solved:

$$\max_{F,C} v(A,T,C,F)$$

$$= A + TB(A,T,C,F,\Delta,\tau^*) - BC(A,T,C,F,\tau^*)$$

$$\text{s.t.} \begin{cases} \Delta = F - D(A,T,C,F,\tau^*) \\ D(A,T,C,F,\tau^*) \le F \end{cases}$$

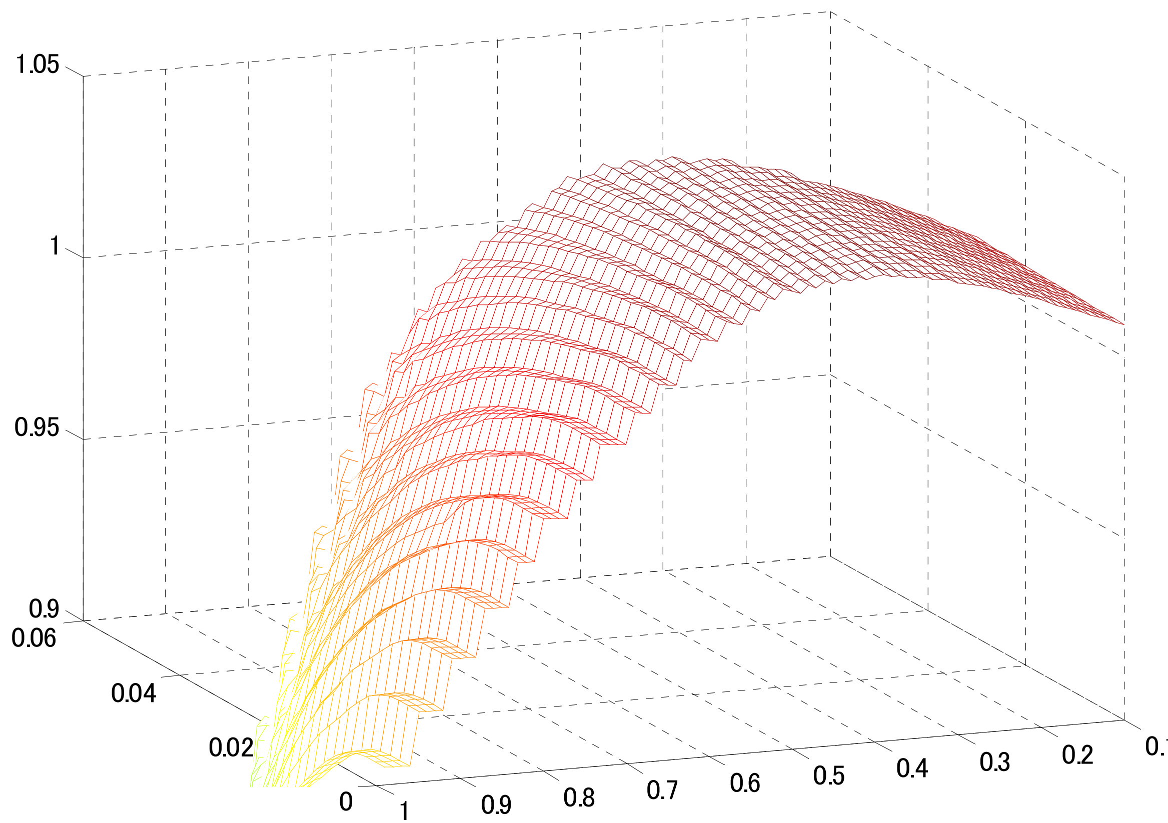
$$TB(A,T,C,F,\Delta,\tau) = \lambda \left\{ CF + \Delta \frac{1-e^{-rT}}{r} \right\} E\left[\int_0^{\tau \wedge T} e^{-rt} dt \right]$$

alpha=0.75

optimal

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	Par Coupon	Firm Value
0.1	1.009	1.0094	1.0099	1.0104	O.P.	O.P.	O.P.	O.P.	0.03	1.0104
0.2	1.0176	1.0186	1.0195	1.0204	O.P.	O.P.	O.P.	O.P.	0.0304	1.0205
0.3	1.0248	1.0261	1.0274	1.0287	O.P.	O.P.	O.P.	O.P.	0.0321	1.0289
0.4	1.028	1.0296	1.031	1.0321	O.P.	O.P.	O.P.	O.P.	0.0358	1.0327
0.5	1.0258	1.0272	1.0277	1.0275	1.0264	O.P.	O.P.	O.P.	0.0417	1.0262
0.6	1.014	1.0146	1.0129	1.0097	1.0043	0.9964	0.9856	O.P.	0.051	0.9955
0.7	0.9907	0.9904	0.986	0.9786	0.9672	0.9508	0.9296	0.8981	–	
0.8	0.965	0.9639	0.9555	0.9417	0.9211	0.8923	0.8521	0.7999	–	
0.9	0.9322	0.9301	0.917	0.8955	0.8632	0.8177	0.7542	0.6729	–	
0.99	0.9001	0.8968	0.8788	0.8488	0.8035	0.7375	0.6486	0.5341	–	

Alpha = 0.75

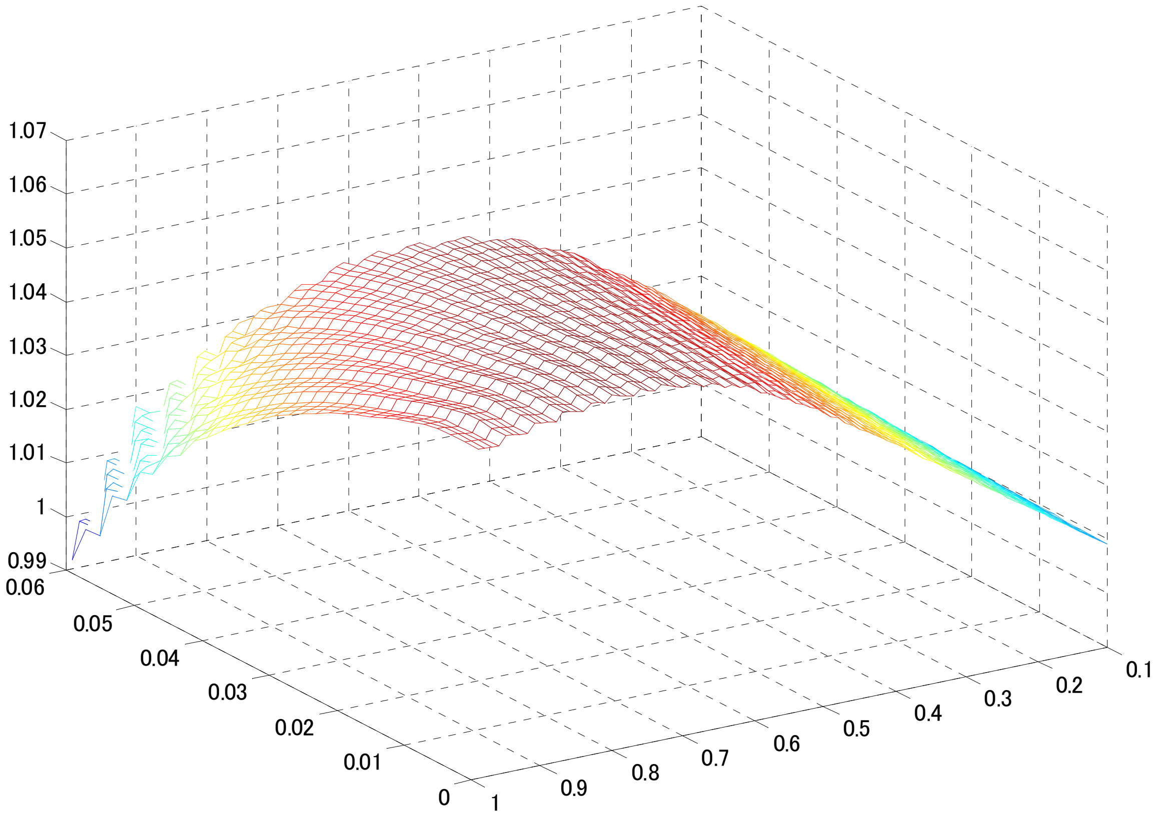


Alpha=0.1

optimal

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	Par Coupon	Firm Value
0.1	1.009	1.0094	1.0099	1.0104	O.P.	O.P.	O.P.	O.P.	0.03	1.0104
0.2	1.0179	1.0189	1.0198	1.0207	O.P.	O.P.	O.P.	O.P.	0.0301	1.0207
0.3	1.0269	1.0283	1.0296	1.031	O.P.	O.P.	O.P.	O.P.	0.0307	1.0311
0.4	1.0358	1.0375	1.0391	1.0407	O.P.	O.P.	O.P.	O.P.	0.0321	1.0411
0.5	1.0443	1.0461	1.0477	1.0492	O.P.	O.P.	O.P.	O.P.	0.0344	1.0499
0.6	1.0514	1.0528	1.0537	1.0544	O.P.	O.P.	O.P.	O.P.	0.0376	1.0546
0.7	1.0548	1.0555	1.0552	1.0545	1.053	O.P.	O.P.	O.P.	0.0419	1.0526
0.8	1.0554	1.0554	1.0537	1.0513	1.0475	O.P.	O.P.	O.P.	0.0479	1.0432
0.9	1.0534	1.0528	1.0496	1.0451	1.0383	1.0287	O.P.	O.P.	0.0586	1.0184
0.99	1.0505	1.0492	1.0444	1.0376	1.0276	1.0128	0.9929	0.9667	-	

Alpha=0.1



1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
0.99
0.06
0.05
0.04
0.03
0.02
0.01
0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

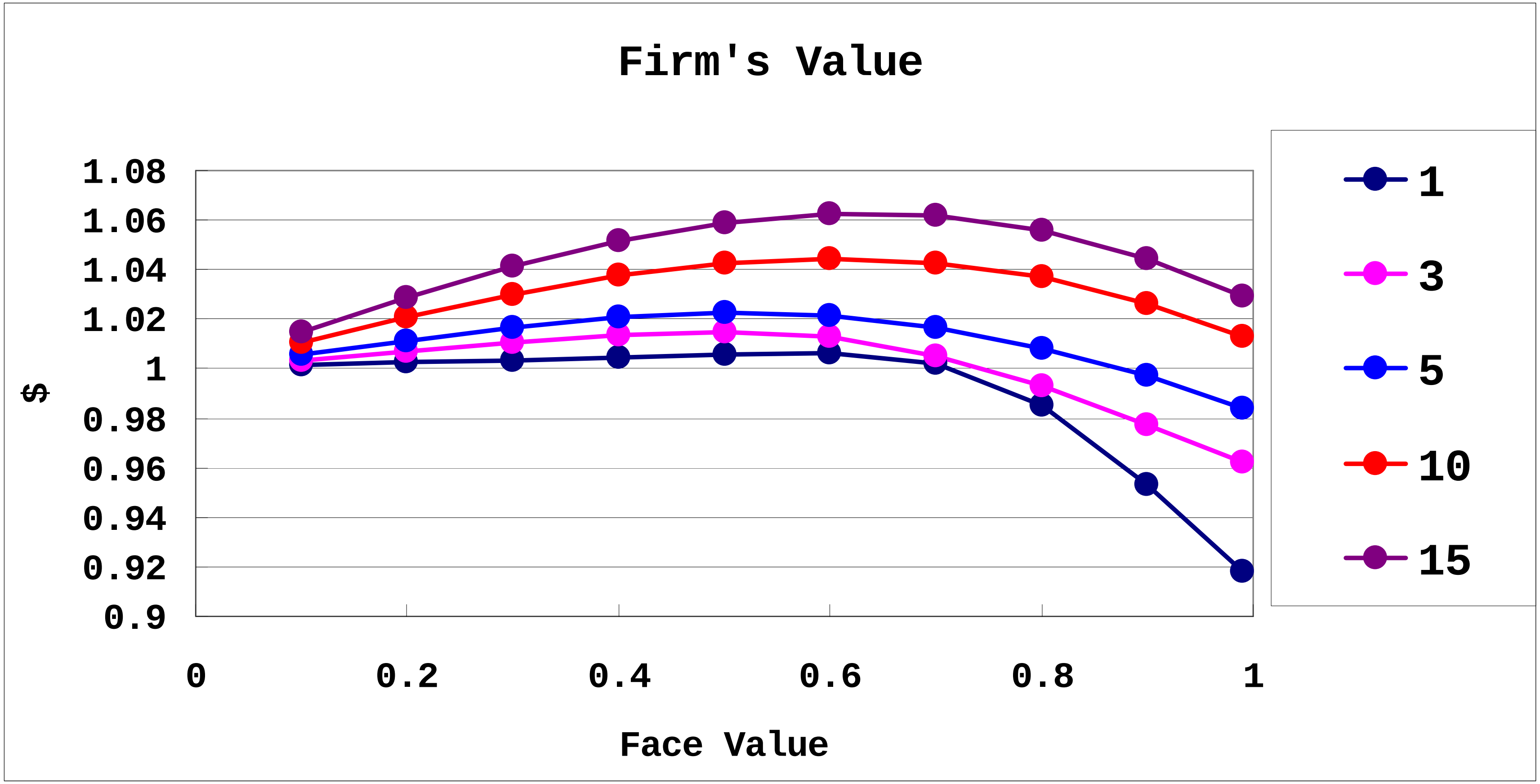
Conclusions

- We extend Brennan-Schwartz (1978) model to include
  - Par issuing constraint
  - Endogenous default
  - Tax benefit for discount bonds
- Our findings are
  - Issuing discount bonds can be optimal
  - Firm value is robust around the optimal

Future Works

- Longer maturity increases the firm value. This is counterintuitive. Need to introduce a mechanism of credit risk so that there is an optimal length of maturity.
- Need to introduce the term structure of interest rates.
- Stock has no maturity. Maybe, an extension of Leland-Toft model in our framework would be of interest.
- And, of course, many othres.

Effect on Firm's Value from Debt Maturity



Thank You for Patience