Random Coefficient Models

$$\underset{n_i \times 1}{\boldsymbol{y}_i} = \underset{n_i \times p}{\boldsymbol{X}_i} \underset{p \times 1}{\boldsymbol{\beta}} + \underset{n_i \times r}{\boldsymbol{Z}_i} \underset{r \times 1}{\boldsymbol{\upsilon}_i} + \underset{n_i \times 1}{\boldsymbol{\varepsilon}_i}$$

$i = 1 \ldots N$ subjects
$j = 1 \ldots n_i$ observations within subject i

$$\boldsymbol{\varepsilon}_i \sim \mathcal{NID}(0, \sigma^2 \boldsymbol{I}_{n_i}) \text{ and } \boldsymbol{\upsilon}_i \sim \mathcal{NID}(0, \boldsymbol{\Sigma}_\upsilon)$$

- typically an order to n_i observations (time)
- account for and model effects of repeated obs.
- $\boldsymbol{X}_i$ includes trends over time
- first columns of $\boldsymbol{X}_i = \boldsymbol{Z}_i$ for subject-varying trends
- first $\boldsymbol{\beta}$ = average trend over time and $\boldsymbol{\upsilon}_i$ are deviations of each individual from average time trend

Can also consider other covariates to be random

- level 1 covariates random at level 2
- moving more stuff from $\boldsymbol{X}_i$ (fixed) to $\boldsymbol{Z}_i$ (random)
- reveals degree of consensus of overall effect
- reveals degree of heterogeneity across subjects

For example,

- time-level effects vary by individuals
- plasma drug-level effects on response vary by patients
- (yearly) tutoring effects on (yearly) grades vary by students
- (repeatedly assessed) mood effects (repeatedly assessed) on behavior vary by individuals

- variable must be at level 1 to be considered random at level 2
  - otherwise it is completely correlated with the first individual random effect
- key is replication within level 2
  - issue of # of random effects vs. # of observations within level 2
  - can't be "constant" replication - must examine whether particular variable does in fact vary at level 1
    e.g., variable that is repeatedly measured that does not vary over time is a level 2 variable *not* a level 1 variable

Replication is vital to 2 important scientific models

1. Mixed-effects regression
2. Past-life regression

Random Coefficient Model of Fishbein & Ajzen's Theory of Reasoned Action (TRA)

Hedeker, Flay, & Petraitis, JCCP, 1996

Condensed Cliff-Note Version: Behavior is only influenced by Behavioral Intentions (BI) which is a function of two components

ATB = attitudes toward the behavior

SN = subjective norms about the behavior

Take a hint from TV advertisers:
⇒ Why use words when a beautiful model can say so much!

$$BI_i = \beta_0 + \beta_1 ATB_i + \beta_2 SN_i + \varepsilon_i$$

where $i = 1, 2, \ldots N$ individuals

TRA also postulates individuals can have different weights for ATB and SN

$$BI_i = \beta_0 + \beta_{1i} ATB_i + \beta_{2i} SN_i + \varepsilon_i$$

- One observation per individual
  - can't estimate individual weights
  - time for theories

- Multiple (or repeated) observations per individual
  - use mixed-model to estimate weights
  - time for action

With repeated observations of BI, ATB, and SN:

$$
\begin{aligned}
BI_{ij} = \; & \beta_0 + \beta_1 ATB_{ij} + \beta_2 SN_{ij} + \\
& \upsilon_{0i} + \upsilon_{1i} ATB_{ij} + \upsilon_{2i} SN_{ij} + \varepsilon_{ij}
\end{aligned}
$$

$i = 1 \ldots N$ individuals
$j = 1 \ldots n_i$ observations within individual i

$\Rightarrow$ Moral: What a difference a j makes!

Data for examining TRA - The Television School and Family Smoking Prevention and Cessation Project (Flay, *et al.*, 1988); a subsample of this project:

- *sample* - 1002 7th-graders (at first measurement)
- *timing* - students assessed at pre and post-intervention (2 months later) and yearly for two years (months 0, 2, 14, 26)
  - for this example, all students were measured at all four timepoints
- *BI* - students intention to smoke cigarettes (range: 2 to 12)
- *ATB* - students attitudes towards smoking (z-score)
- *SN* - perception of normative views on smoking, based on close friends and close adults (z-score)

Questions of interest include:

- What is the relative effects of ATB and SN on BI
- Is there significant individual variation in these effects and if so, what is the relationship between the individual weights
- What student-level variables might be related to individual ATB and SN weights

Behavioral Intentions

BI = sum of two items

1. Do you think you'll ever smoke cigarettes in the future?
2. Do you think you might ever ask anyone to let you try a cigarette?

scored as:

6 Yes, definitely

5 Yes, I'm pretty sure

4 Yes, I guess so

3 I don't know

2 Probably not

1 Definitely not

Subjective Norms

Subjective norms =
sum of [(normative beliefs) $\times$ motivation to comply)]

Normative Belief (NB)	Motivation to Comply (MC)
How many of your 10 closest friends would approve if you smoked a cigarette? (6 = 8 to 10)	How much do you think close friends influence what you do? (4 = a great deal)

SN = z-score of NB $\times$ MC

Attitudes towards the behavior

ATB = z-score of sum of Values $\times$ Expectancies

Values	Expectancies
How worried are you about the possibility that people get lung cancer or heart disease from smoking cigarettes?	Out of 100 smokers, how many do you think will get lung cancer or heart disease from smoking?
How worried are you about the possibilty that people die from smoking?	Out of 100 smokers, how many do you think will die from smoking?
If you smoked, how worried would you be about getting lung cancer or heart disease?	If you smoked, how likely is it that you would get lung cancer or heart disease?
If you smoked, how worried would you be about the possibility of dying?	If you smoked, how likely is it that you would die from smoking?

Coding of Values and Expectancies

Values	Expectancies
1 = extremely worried	1 = extremely likely (81 to 100%)
2 = very worried	2 = very likely (61 to 80%)
3 = moderately worried	3 = somewhat likely (41 to 60%)
4 = a little worried	4 = a little likely (21 to 40%)
5 = not at all worried	5 = not likely (0 to 20%)

Descriptive statistics for BI (n = 1002)

	month 0	month 2	month 14	month 26
mean	3.61	3.91	3.98	4.12
sd	1.96	2.22	2.35	2.67

- intentions to smoke increase slightly over time
- variability increases over time

Correlations of ATB & SN with BI over time (n = 1002)

	month 0	month 2	month 14	month 26
ATB	0.29	0.26	0.32	0.31
SN	0.28	0.34	0.40	0.45

- ATB with BI generally constant over time
- SN with BI generally increasing over time

Model I - no heterogeneity in effect of ATB and SN

$$\underset{4\times 1}{\underset{\boldsymbol{y}_i}{\begin{bmatrix} BI_{i1} \\ BI_{i2} \\ BI_{i3} \\ BI_{i4} \end{bmatrix}}} = \underset{4\times 4}{\underset{\boldsymbol{X}_i}{\begin{bmatrix} 1 & 0 & ATB_{i1} & SN_{i1} \\ 1 & 2 & ATB_{i2} & SN_{i2} \\ 1 & 14 & ATB_{i3} & SN_{i3} \\ 1 & 26 & ATB_{i4} & SN_{i4} \end{bmatrix}}} \underset{4\times 1}{\underset{\boldsymbol{\beta}}{\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}}}$$

$$+ \underset{4\times 2}{\underset{\boldsymbol{Z}_i}{\begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 14 \\ 1 & 26 \end{bmatrix}}} \underset{2\times 1}{\underset{\boldsymbol{\upsilon}_i}{\begin{bmatrix} \upsilon_{0i} \\ \upsilon_{1i} \end{bmatrix}}} + \underset{4\times 1}{\underset{\boldsymbol{\varepsilon}_i}{\begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \varepsilon_{i2} \\ \varepsilon_{i4} \end{bmatrix}}}$$

Orthogonal polynomials (unequal time intervals)

1. compute $\boldsymbol{Z}'\boldsymbol{Z}$
2. obtain the Cholesky $\boldsymbol{S}$ (square-root) of $\boldsymbol{Z}'\boldsymbol{Z}$, where for a real symmetric positive-definite matrix $\boldsymbol{A}$, $\quad \boldsymbol{A} = \boldsymbol{S}\boldsymbol{S}'$
3. obtain the inverse $(\boldsymbol{S}')^{-1}$
4. multiply $\boldsymbol{Z}$ by this inverse $(\boldsymbol{S}')^{-1}$

$$\boldsymbol{Z}' = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 14 & 26 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{Z}'\boldsymbol{Z} = \begin{bmatrix} 4 & 42 \\ 42 & 876 \end{bmatrix}$$

$$\boldsymbol{S}' = \begin{bmatrix} 2 & 21 \\ 0 & 20.86 \end{bmatrix} \quad \Rightarrow \quad (\boldsymbol{S}')^{-1} = \begin{bmatrix} 0.5 & -0.5034 \\ 0 & 0.0479 \end{bmatrix}$$

$$\boldsymbol{Z}(\boldsymbol{S'})^{-1} = \begin{bmatrix} 0.5 & -0.5034 \\ 0.5 & -0.4075 \\ 0.5 & 0.1678 \\ 0.5 & 0.7432 \end{bmatrix}$$

Orthogonal Polynomials via SAS (unequal timepoints)

```
TITLE 'producing orthogonal polynomial matrix';
PROC IML;
 time = { 1  0   ,
          1  2   ,
          1 14   ,
          1 26 } ;
orthpoly = time*INV(ROOT(T(time)*time));
PRINT 'time matrix', time [FORMAT=8.4];
PRINT 'orthogonalized time matrix', orthpoly
[FORMAT=8.4];
```

Within-subjects model

$$BI_{ij} = b_{0i}Cons_j + b_{1i}Lin_j + b_{2i}ATB_{ij} + b_{3i}SN_{ij} + RES_{ij}$$

$$\begin{aligned} i &= 1 \ldots 1002 \text{ students} \\ j &= 1 \ldots 4 \text{ observations per student} \end{aligned}$$

$$\begin{aligned} b_{0i} &= \text{scaled average (across time) BI level for student } i \\ b_{1i} &= \text{linear change in BI for student } i \text{ across months} \\ b_{2i} &= \text{change in BI per unit change in } z\text{-score ATB for student } i \\ b_{3i} &= \text{change in BI per unit change in } z\text{-score SN for student } i \end{aligned}$$

Merits to using z-scores for ATB and SN and orthogonal polynomials for month effect

- intercept is scaled grand mean (when month = 10.5 and ATB and SN values are at their means)
- can more readily compare size of effects
- unit change in ATB and SN corresponds to change of *sd*-units

Between-subjects models

$$
\begin{aligned}
b_{0i} &= \beta_0 + v_{0i} \\
b_{1i} &= \beta_1 + v_{1i} \\
b_{2i} &= \beta_2 \\
b_{3i} &= \beta_3
\end{aligned}
$$

where

$$
\begin{aligned}
\beta_0 &= \text{average (scaled) average } BI \text{ level} \\
\beta_1 &= \text{average } BI \text{ linear monthly change} \\
\beta_2 &= \text{average effect of } ATB \\
\beta_3 &= \text{average effect of } SN \\
v_{0i} &= \text{individual deviation from average } BI \\
v_{1i} &= \text{individual deviation from average linear monthly change}
\end{aligned}
$$

Total Model

$$BI_{ij} = \beta_0 Cons_j + \beta_1 Lin_j + \beta_2 ATB_{ij} + \beta_3 SN_{ij} + \upsilon_{0i} Cons_j + \upsilon_{1i} Lin_j + RESID_{ij}$$

SAS syntax

```
PROC MIXED COVTEST METHOD=ML;
    CLASS subjid;
    MODEL behint = cons linear zatb zsn / SOLUTION NOINT;
    RANDOM cons linear / SUBJECT=subjid TYPE=UN G GCORR;
```

parameter	ML estimate	se	z	$p <$
constant β_0	7.784	0.100	78.01	.0001
linear β_1	0.246	0.060	4.09	.0001
ATB β_2	0.490	0.036	13.60	.0001
SN β_3	0.577	0.033	17.60	.0001
$\sigma^2_{\upsilon_0}$	7.986	0.449		
$\sigma_{\upsilon_0\upsilon_1}$	0.916	0.192		
$\sigma^2_{\upsilon_1}$	1.637	0.173		
σ^2	1.975	0.062		

$\log L = -8151.91$

$\Rightarrow$ *Overall effect of ATB and SN similar*

$$\hat{\boldsymbol{\beta}}_{raw} = \begin{bmatrix} 0.5 & -0.5034 \\ 0 & 0.0479 \end{bmatrix} \begin{bmatrix} 7.784 \\ 0.246 \end{bmatrix} = \begin{bmatrix} 3.768 \\ 0.012 \end{bmatrix}$$

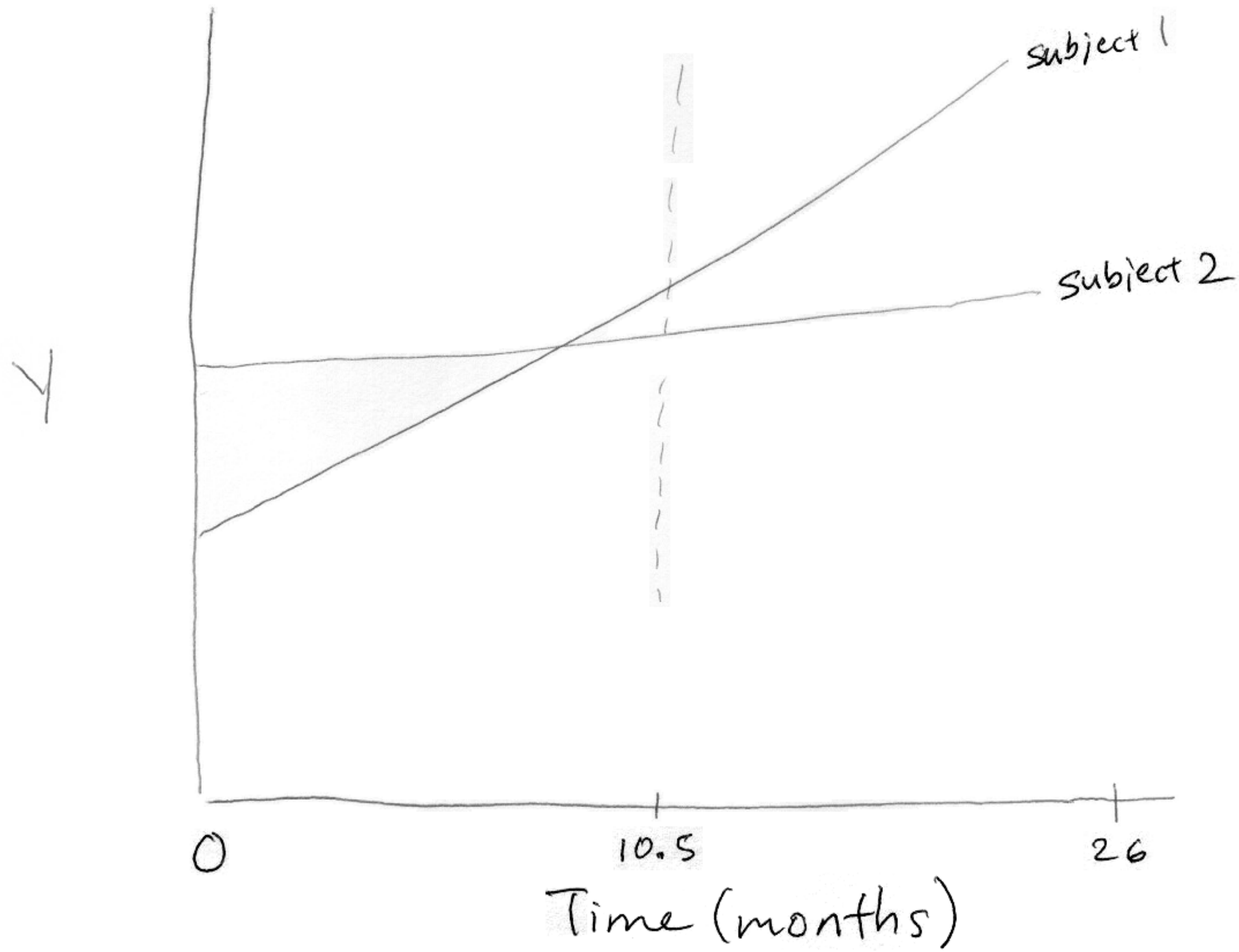
grand mean $= 0.5 \times 7.784 = 3.892 = \left(\Sigma_{i=1}^{N=1002} \Sigma_{j=1}^{4} BI_{ij} \right) / 4N$

Comparing results in orthogonal polynomial versus raw metric:

$\sigma_{\upsilon_0 \upsilon_1}$ as corr between cons and lin terms = 0.25
(higher grand mean associated with greater slope)

However, $\sigma_{\upsilon_0 \upsilon_1}$ as corr between int and month terms = -0.21
(higher intercept associated with less slope)

Explanation?


subject 1
subject 2
Y
0
10.5
26
Time (months)

Fit of model to means and sds

	month 0	month 2	month 14	month 26
obs mean	3.61	3.91	3.98	4.12
est mean	3.77	3.79	3.93	4.08
obs sd	1.96	2.22	2.35	2.67
est sd	1.98	1.97	2.04	2.36

$$est(\bar{y}) = \boldsymbol{X}\hat{\boldsymbol{\beta}}$$

$$est(var(y \mid X)) = \boldsymbol{Z}\hat{\boldsymbol{\Sigma}}_{\upsilon}\boldsymbol{Z}' + \hat{\sigma}^2\boldsymbol{I}_n$$

The estimated means are for when ATB and SN equal zero, which happens to be the average ATB and SN values in the sample since z-scores were used for these variables

Is there variation in ATB and SN effects?

Within-subjects model

$$BI_{ij} = b_{0i}Cons_j + b_{1i}Lin_j + b_{2i}ATB_{ij} + b_{3i}SN_{ij} + RES_{ij}$$

$i = 1 \ldots 1002$ students; $j = 1 \ldots 4$ obs per student

b_{0i} = scaled average (across time) BI level for student i
b_{1i} = linear change in BI for student i
b_{2i} = change in BI per unit change in z-score ATB for student i
b_{3i} = change in BI per unit change in z-score SN for student i

Between-subjects models

$$\begin{aligned} b_{0i} &= \beta_0 + \upsilon_{0i} \\ b_{1i} &= \beta_1 + \upsilon_{1i} \\ b_{2i} &= \beta_2 + \upsilon_{2i} \\ b_{3i} &= \beta_3 + \upsilon_{3i} \end{aligned}$$

where

$$\begin{aligned}
\beta_0 &= \text{average (scaled) average } BI \text{ level} \\
\beta_1 &= \text{average } BI \text{ linear change} \\
\beta_2 &= \text{average effect of } ATB \\
\beta_3 &= \text{average effect of } SN \\
\upsilon_{0i} &= \text{individual deviation from average average} \\
\upsilon_{1i} &= \text{individual deviation from average linear change} \\
\upsilon_{2i} &= \text{individual deviation from average ATB effect} \\
\upsilon_{3i} &= \text{individual deviation from average SN effect}
\end{aligned}$$

SAS syntax

```
PROC MIXED COVTEST METHOD=ML;
    CLASS subjid;
    MODEL behint = cons linear zatb zsn / SOLUTION NOINT;
    RANDOM cons linear zatb zsn / SUBJECT=subjid TYPE=UN G GCORR;
```

parameter	ML estimate	se	z	$p <$
constant β_0	7.700	0.099	78.18	.0001
linear β_1	0.197	0.057	3.47	.0005
ATB β_2	0.467	0.040	11.78	.0001
SN β_3	0.550	0.041	13.27	.0001
constant $\sigma^2_{\upsilon_0}$	7.196	0.451		
$\sigma_{\upsilon_0 \upsilon_1}$	0.605	0.179		
linear $\sigma^2_{\upsilon_1}$	1.301	0.157		
$\sigma_{\upsilon_0 \upsilon_2}$	0.419	0.115		
$\sigma_{\upsilon_1 \upsilon_2}$	0.083	0.066		
ATB $\sigma^2_{\upsilon_2}$	0.152	0.058		
$\sigma_{\upsilon_0 \upsilon_3}$	0.599	0.125		
$\sigma_{\upsilon_1 \upsilon_3}$	0.343	0.071		
$\sigma_{\upsilon_2 \upsilon_3}$	0.111	0.046		
SN $\sigma^2_{\upsilon_3}$	0.318	0.057		
error σ^2	1.765	0.062		

SN and ATB as random effects: $\chi^2_7 = 195, p < .0001$

YES! people do weigh SN and ATB differently; SN var larger than ATB var

$\boldsymbol{\Sigma}_v$ expressed as a correlation matrix:

$$\begin{bmatrix} 1.000 & & & \\ 0.198 & 1.000 & & \\ 0.401 & 0.186 & 1.000 & \\ 0.396 & 0.534 & 0.510 & 1.000 \end{bmatrix}$$

- lots of positive association
- all terms positively associated with constant
- greater association between linear BI trend and SN weight, than linear BI trend and ATB weight
- SN weight positively related to ATB weight

Influences & Interactions of ATB and SN weights

Within-subjects model

$$\begin{aligned} BI_{ij} = & \ b_{0i}Cons_j + b_{1i}Lin_j + b_{2i}ATB_{ij} + b_{3i}SN_{ij} + \\ & b_{4i}\left(ATB_{ij} * Lin_j\right) + b_{5i}\left(SN_{ij} * Lin_j\right) + RES_{ij} \end{aligned}$$

$i = 1 \ldots 1002$ students; $j = 1 \ldots 4$ observations per student

b_{0i} = scaled average (across time) BI level for student i

b_{1i} = linear change in BI for student i with ATB = SN = 0

b_{2i} = change in BI associated with unit change in z-score ATB for student i at $linear = 0$

b_{3i} = change in BI associated with unit change in z-score SN for student i at $linear = 0$

b_{4i} = change in ATB effect across time

b_{5i} = change in SN effect across time

Between-subjects models

$$
\begin{aligned}
b_{0i} &= \beta_0 + \beta_6 Sex_i + v_{0i} \\
b_{1i} &= \beta_1 + v_{1i} \\
b_{2i} &= \beta_2 + \beta_7 Sex_i + v_{2i} \\
b_{3i} &= \beta_3 + \beta_8 Sex_i + v_{3i} \\
b_{4i} &= \beta_4 \\
b_{5i} &= \beta_5
\end{aligned}
$$

where

$$
\begin{aligned}
\beta_0 &= \text{average (scaled) average } BI \text{ level} \\
\beta_1 &= \text{average } BI \text{ linear change for ATB} = \text{SN} = 0 \\
\beta_2 &= \text{average effect of } ATB \text{ at } linear = 0 \\
&\quad\ \text{for females (sex 0=f and 1=m)} \\
\beta_3 &= \text{average effect of } SN \text{ at } linear = 0 \\
&\quad\ \text{for females (sex 0=f and 1=m)} \\
\beta_4 &= \text{average change in } ATB \text{ effect across time} \\
\beta_5 &= \text{average change in } SN \text{ effect across time} \\
\beta_6 &= \text{sex effect for average } ATB \text{ and } SN \text{ values} \\
\beta_7 &= \text{sex influence on } ATB \text{ effect} \\
\beta_8 &= \text{sex influence on } SN \text{ effect} \\
\upsilon_{0i} &= \text{individual deviation from average average} \\
\upsilon_{1i} &= \text{individual deviation from average linear change} \\
\upsilon_{2i} &= \text{individual deviation from average ATB effect} \\
\upsilon_{3i} &= \text{individual deviation from average SN effect}
\end{aligned}
$$

SAS syntax

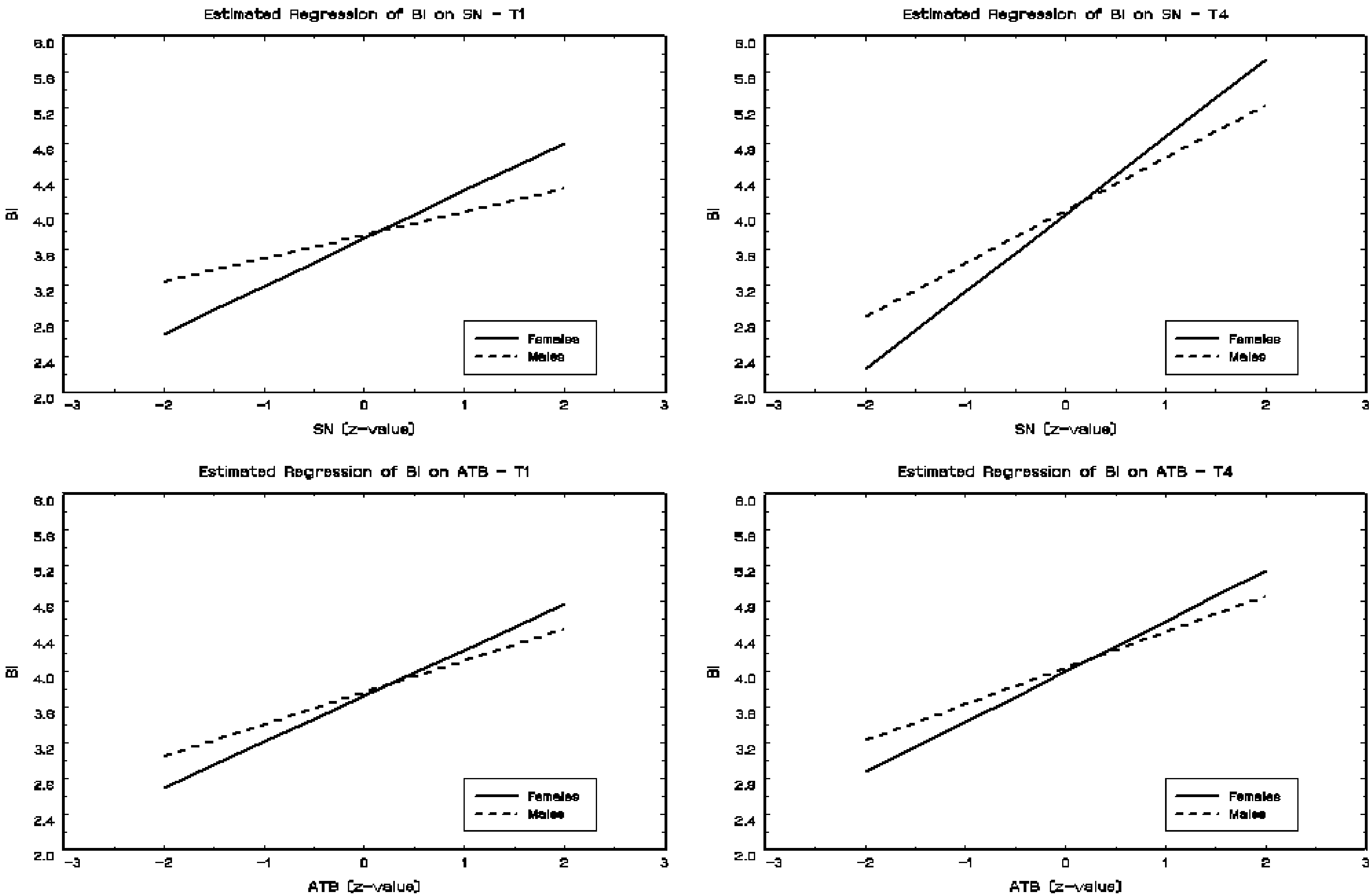
```
PROC MIXED COVTEST METHOD=ML;
    CLASS subjid;
    MODEL behint = cons linear zatb zsn sexm
    zatb*sexm zsn*sexm zatb*linear zsn*linear / SOLUTION NOINT;
    RANDOM cons linear zatb zsn / SUBJECT=subjid TYPE=UN G GCORR;
```

parameter	ML estimate	se	z	$p <$
Constant	7.671	0.130	59.13	.0001
Linear	0.219	0.057	3.87	.0001
ATB	0.537	0.055	9.69	.0001
SN	0.674	0.054	12.44	.0001
ATB*Linear	0.040	0.060	0.67	.50
SN*Linear	0.269	0.065	4.16	.0001
Sex (0=F 1=M)	0.039	0.099	0.39	.69
ATB*Sex	-0.159	0.079	-2.01	.045
SN*Sex	-0.277	0.082	-3.39	.0007

parameter	ML estimate	se	z	$p <$
constant $\sigma^2_{\upsilon_0}$	7.106	0.445		
$\sigma_{\upsilon_0\upsilon_1}$	0.457	0.175		
linear $\sigma^2_{\upsilon_1}$	1.236	0.154		
$\sigma_{\upsilon_0\upsilon_2}$	0.439	0.113		
$\sigma_{\upsilon_1\upsilon_2}$	0.050	0.065		
ATB $\sigma^2_{\upsilon_2}$	0.148	0.057		
$\sigma_{\upsilon_0\upsilon_3}$	0.597	0.123		
$\sigma_{\upsilon_1\upsilon_3}$	0.322	0.069		
$\sigma_{\upsilon_2\upsilon_3}$	0.100	0.045		
SN $\sigma^2_{\upsilon_3}$	0.289	0.055		
σ^2	1.772	0.062		

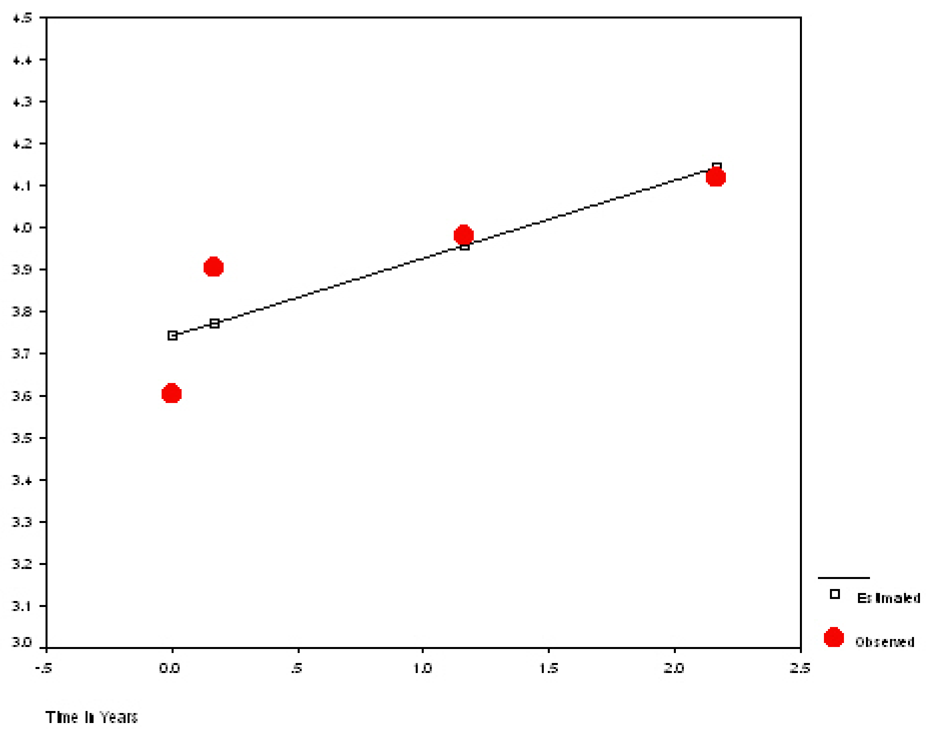
$\log L = -8035.81$

- Variance in SN and ATB effects is less
- $\chi^2_2 = 19.5, p < .0001$ for addition of ATB and SN interactions with (linear) time
  - influence of SN on BI increases over time
  - no significant change in ATB influence on BI over time
- $\chi^2_3 = 17.8, p < .005$ for addition of Sex and ATB and SN interactions with sex
  - average SN weight greater for females than males
  - average ATB weight somewhat greater for females than males


Estimated Regression of BI on SN - T1
Estimated Regression of BI on SN - T4
Estimated Regression of BI on ATB - T1
Estimated Regression of BI on ATB - T4
BI
SN (z-value)
ATB (z-value)
Females
Males

Linear time effect?

- Observed means indicate pronounced change from pre to post-intervention, followed by less pronounced approximate linear trend from post-intervention (2 months) to last followup (26 months)
- Overall linear time effect over-estimates mean at pre-intervention and under-estimates mean at post-intervention



Alternatives to linear time effect

- higher-order polynomials (*e.g.*, quadratic, cubic)
- $p - 1$ contrasts for p timepts (*e.g.*, t1 vs t0, t2 vs t0, t3 vs t0)
- piecewise linear (*e.g.*, one linear trend between t0 and t1; another between t1 and t3)

$\Rightarrow$ match treatment of time with scientific hypotheses of interest

Here, piecewise linear is interesting choice

- slope between pre and post-intervention indicates change related to intervention
- slope from post-intervention to final timepoint indicates change across follow-up

Matrix for linear effect of time (in units of years)

intercept and linear — model

$$\begin{bmatrix} 1 & 0 \\ 1 & 2/12 \\ 1 & 14/12 \\ 1 & 26/12 \end{bmatrix} \qquad \begin{aligned} y &= \beta_0 \\ y &= \beta_0 + (2/12)\beta_1 \\ y &= \beta_0 + (14/12)\beta_1 \\ y &= \beta_0 + (26/12)\beta_1 \end{aligned}$$

Matrix for proposed piecewise linear effects of time

intercept, post, and follow-up — model

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2/12 & 2/12 - 2/12 = 0 \\ 1 & 2/12 & 14/12 - 2/12 = 1 \\ 1 & 2/12 & 26/12 - 2/12 = 2 \end{bmatrix} \qquad \begin{aligned} y &= \beta_0 \\ y &= \beta_0 + (2/12)\beta_1 \\ y &= \beta_0 + (2/12)\beta_1 + \beta_2 \\ y &= \beta_0 + (2/12)\beta_1 + 2\beta_2 \end{aligned}$$

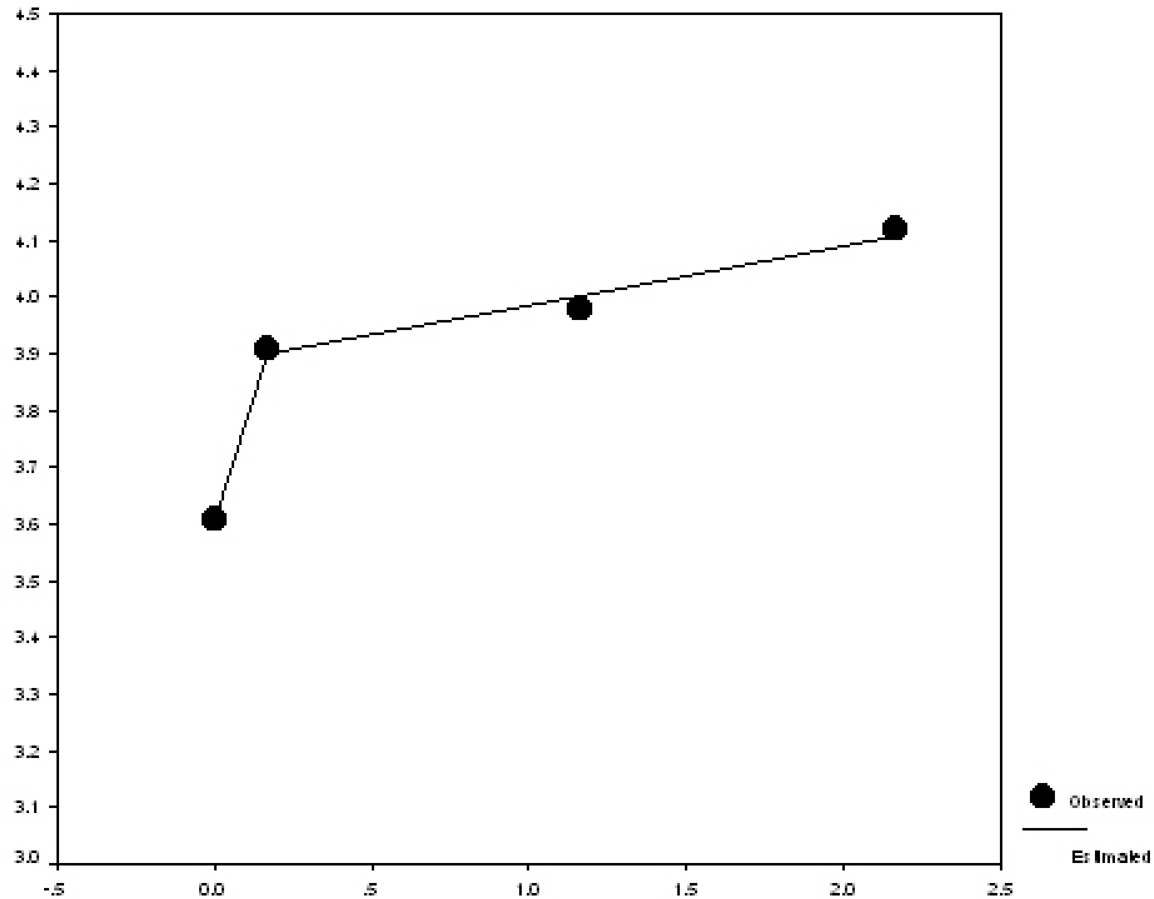
notice, if $\beta_2 = \beta_1$ in latter, then these two approaches are identical
thus, first approach assumes the post and follow-up slopes are the same

parameter	ML estimate	se	z	$p <$
linear trend model				
intercept β_0	3.742	.060	62.44	.0001
year β_1	.185	.038	4.90	.0001
piecewise linear trend model				
intercept β_0	3.607	.068	53.19	.0001
post β_1	1.739	.367	4.74	.0001
follow-up β_2	.106	.042	2.52	.012

comparing models: $\chi_1^2 = 16802.8 - 16784.7 = 18.1$, $p < .0001$

$\Rightarrow$ slope is much greater from pre to post-intervention, than from post-intervention to 2-year follow-up

Observed and estimated means - piecewise linear model



Observed and estimated means $(= \boldsymbol{X}\hat{\boldsymbol{\beta}})$

	year 0	year 2/12	year 14/12	year 26/12
obs	3.607	3.907	3.981	4.119
est	3.608	3.898	4.004	4.109