

Mixed Models for Longitudinal Ordinal Data

Don Hedeker
University of Illinois at Chicago

hedeker@uic.edu
www.uic.edu/~hedeker/long.html

1

Why analyze as ordinal?

- Efficiency: Armstrong & Sloan (1989, Amer Jrn of Epid) report efficiency losses between 89% to 99% comparing an ordinal to continuous outcome, depending on the number of categories and distribution within the ordinal categories.
- Bias: continuous model can yield correlated residuals and regressors when applied to ordinal outcomes, because the continuous model does not take into account the ceiling and floor effects of the ordinal outcome. This can result in biased estimates of regression coefficients and is most critical when the ordinal variables is highly skewed.
- Logic: continuous model can yield predicted values outside of the range of the ordinal variable.

2

Proportional Odds Model - McCullagh (1980)

$$\log \left[\frac{P(\mathbf{y} \leq c)}{1 - P(\mathbf{y} \leq c)} \right] = \gamma_c - \mathbf{x}'\boldsymbol{\beta}$$

$c = 1, \dots, C - 1$ for the C categories of the ordinal outcome

\mathbf{x} = vector of explanatory variables (plus the intercept)

γ_c = thresholds; reflect cumulative odds when $\mathbf{x} = 0$ (for identification: $\gamma_1 = 0$ or $\beta_0 = 0$)

- positive association between x and \mathbf{y} is reflected by $\beta > 0$
- the effect of x is assumed to be the same for each cumulative odds ratio
- odds that the response is less than or equal to c (for any fixed c) is divided by e^β for every unit change in x :

$$\left[\frac{P(\mathbf{y} \leq c)}{1 - P(\mathbf{y} \leq c)} \right] = \exp(\gamma_c - x\beta) = e^{\gamma_c} / (e^\beta)^x$$

- odds that the response is greater than or equal to c (for fixed c) is multiplied by e^β for every unit change in x :

$$\left[\frac{1 - P(\mathbf{y} \leq c)}{P(\mathbf{y} \leq c)} \right] = e^{-\gamma_c} \times (e^\beta)^x$$

3

Ordinal Model for Dichotomous Response: same as it ever was!

$$\log \left[\frac{P(\mathbf{y} = 0)}{1 - P(\mathbf{y} = 0)} \right] = 0 - \mathbf{x}'\boldsymbol{\beta}$$

$$\frac{P(\mathbf{y} = 0)}{1 - P(\mathbf{y} = 0)} = \exp(0 - \mathbf{x}'\boldsymbol{\beta})$$

$$\frac{1 - P(\mathbf{y} = 0)}{P(\mathbf{y} = 0)} = [\exp(0 - \mathbf{x}'\boldsymbol{\beta})]^{-1}$$

$$\frac{1 - P(\mathbf{y} = 0)}{P(\mathbf{y} = 0)} = \exp(\mathbf{x}'\boldsymbol{\beta})$$

$$\log \left[\frac{P(\mathbf{y} = 1)}{1 - P(\mathbf{y} = 1)} \right] = \mathbf{x}'\boldsymbol{\beta}$$

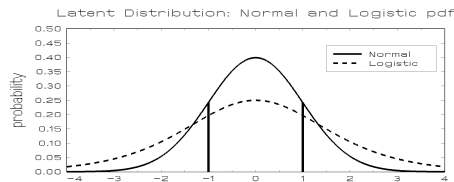
4

Ordinal Response and Threshold Concept

Continuous y_{ij} - unobservable latent variable - related to ordinal response y_{ij} via “threshold concept”

- series of threshold values $\gamma_1, \gamma_2, \dots, \gamma_{C-1}$
- C = number of ordered categories
- $\gamma_0 = -\infty$ and $\gamma_C = \infty$

Response occurs in category c , $y_{ij} = c$ if $\gamma_{c-1} < y_{ij} < \gamma_c$



5

The Threshold Concept in Practice

“How was your day?”

(what is your level of satisfaction today?)

- Satisfaction may be continuous, but we sometimes emit an ordinal response:

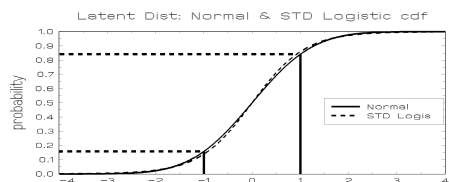
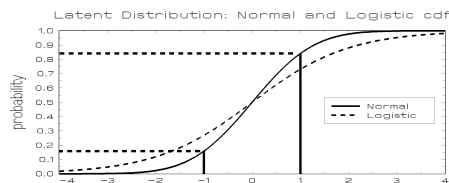
😊 Great Day!

😐 a day ...

😞 *?!*?!? day

6

Latent cumulative distribution function (cdf)



7

Probit Formulation: for a given level 2 unit i , the conditional probability of a response in category 1 for level 1 unit j is:

$$p(y_{ij} = 1 | \mathbf{v}) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\gamma_1} e^{-\frac{1}{2}(y_{ij} - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i))^2/\sigma^2} dy$$

$$= \Phi((\gamma_1 - z_{ij})/\sigma)$$

- $z_{ij} = \mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i$
- $\Phi(\cdot)$ = cumulative std normal dist fn (cdf)

For $C = 3$ the conditional probability equations are

$$p(y_{ij} = 2 | \mathbf{v}) = \Phi((\gamma_2 - z_{ij})/\sigma) - \Phi((\gamma_1 - z_{ij})/\sigma)$$

$$p(y_{ij} = 3 | \mathbf{v}) = 1 - \Phi((\gamma_2 - z_{ij})/\sigma)$$

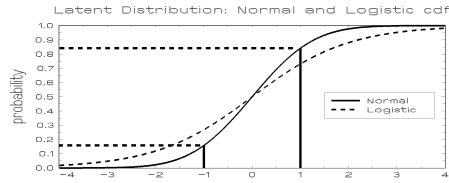
Defining the origin ($\gamma_1 = 0$) and scale ($\sigma = 1$) of latent y ; the conditional probability of a response in category c equals:

$$p(y_{ij} = c | \mathbf{v}) = \Phi(\gamma_c - z_{ij}) - \Phi(\gamma_{c-1} - z_{ij})$$

with $\gamma_0 = -\infty$ and $\gamma_C = \infty$

8

Category probabilities: example



Suppose $z_{ij} = 0$

$$p(y_{ij} = 1 | \mathbf{v}) = \Phi(\gamma_1 - 0) - \Phi(\gamma_0 - 0) = \Phi(\gamma_1) = .159$$

$$\begin{aligned} p(y_{ij} = 2 | \mathbf{v}) &= \Phi(\gamma_2 - 0) - \Phi(\gamma_1 - 0) \\ &= \Phi(\gamma_2) - \Phi(\gamma_1) = .841 - .159 = .682 \end{aligned}$$

$$p(y_{ij} = 3 | \mathbf{v}) = \Phi(\gamma_3 - 0) - \Phi(\gamma_2 - 0) = 1 - \Phi(\gamma_2) = .159$$

9

Logistic representation

$$p(y_{ij} = c | \mathbf{v}) = \Psi(\gamma_c - z_{ij}) - \Psi(\gamma_{c-1} - z_{ij})$$

where the cumulative std logistic dist fn (cdf) is

$$\Psi(\gamma_c - z_{ij}) = \frac{1}{1 + \exp[-(\gamma_c - z_{ij})]}$$

- origin of y : $\gamma_1 = 0$
- unit of measurement: $\sigma = \pi/\sqrt{3}$

Model for cumulative logits

$$\log \left[\frac{P(y_{ij} \leq c)}{1 - P(y_{ij} \leq c)} \right] = \gamma_c - [\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i]$$

\Rightarrow random-effect generalization of proportional odds model with $\mathbf{v}_i \sim \mathcal{NID}(\mathbf{0}, \boldsymbol{\Sigma}_v)$

10

Cumulative Link Models

$$G^{-1}[P(y_{ij} \leq c)] = \gamma_c - [\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i]$$

or, equivalently

$$P(y_{ij} \leq c) = G[\gamma_c - (\mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i)]$$

where

- $G^{-1}(P) = \log[P/(1 - P)]$ gives cumulative logit models (proportional odds models)
- $G^{-1}(P) = \Phi^{-1}(P)$ gives cumulative probit models
- $G^{-1}(P) = \log[-\log(1 - P)]$ (complementary log-log link) gives proportional hazards models

– Hedeker, Siddiqui, & Hu (2000) *Statistical Methods in Medical Research*

11

Multilevel Representation

Within level-2 unit model (*level-1 model*)

observed response

$$\log \left[\frac{Pr(\mathbf{y}_{ij} \leq c)}{1 - Pr(\mathbf{y}_{ij} \leq c)} \right] = \gamma_c - [\mathbf{z}'_{(1)ij}\mathbf{b}_i + \mathbf{x}'_{(1)ij}\boldsymbol{\beta}_{(1)}]$$

latent response

$$y_{ij} = \mathbf{z}'_{(1)ij}\mathbf{b}_i + \mathbf{x}'_{(1)ij}\boldsymbol{\beta}_{(1)} + \varepsilon_{ij} \quad \varepsilon_{ij} \sim \mathcal{LTD}(0, \pi^2/3) \text{ for logistic}$$

Between level-2 unit model: $\mathbf{b}_i = \boldsymbol{\beta}_{(2)} + \mathbf{x}'_{(2)i}\boldsymbol{\beta}_{(2)} + \mathbf{v}_i$

- $\mathbf{x}_{(1)ij}$ and $\boldsymbol{\beta}_{(1)}$ are level-1 covariates and effects
- $\mathbf{x}_{(2)i}$ and $\boldsymbol{\beta}_{(2)}$ are level-2 covariates and effects
- $\mathbf{z}_{(1)ij}$ are level-1 variables that vary at level-2

The level-2 effects \mathbf{b}_i are a function of an overall mean $\boldsymbol{\beta}_{(2)}$, level-2 covariates $\boldsymbol{\beta}_{(2)}$, and a unique random component $\mathbf{v}_i \sim \mathcal{NID}(\mathbf{0}, \boldsymbol{\Sigma}_v)$ (level-2 residuals)

12

Marginal Maximum Likelihood (MML) Estimation

Conditional probability for $n_i \times 1$ response vector \mathbf{y}_i

$$\ell(\mathbf{y}_i | \mathbf{v}) = \prod_{j=1}^{n_i} \prod_{c=1}^C [\Phi(\gamma_c - z_{ij}) - \Phi(\gamma_{c-1} - z_{ij})]^{d_{ijc}}$$

$$\text{where } d_{ijc} = \begin{cases} 1 & \text{if } y_{ij} = c \\ 0 & \text{if } y_{ij} \neq c \end{cases}$$

$$z_{ij} = \mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{v}_i$$

The marginal probability for \mathbf{y}_i in the population is:

$$h(\mathbf{y}_i) = \int_{\mathbf{v}} \ell(\mathbf{y}_i | \mathbf{v}) g(\mathbf{v}) d\mathbf{v}$$

- $g(\mathbf{v})$ represents the population distribution of \mathbf{v}

For integration over \mathbf{v} :

- transform \mathbf{v} space to multivariate unit normal
 - $\mathbf{v} = \mathbf{T}\boldsymbol{\theta}$ (where $\mathbf{T}\mathbf{T}' = \boldsymbol{\Sigma}_v$)
 - $z_{ij} = \mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{T}\boldsymbol{\theta}_i$
- Gauss-Hermite quadrature for numerical integration
 - Q quadrature nodes \mathbf{B}_q and weights $W(\mathbf{B}_q)$
 - integral is replaced with summation over Q nodes

The marginal probability becomes:

$$h(\mathbf{y}_i) = \int_{\boldsymbol{\theta}} \ell(\mathbf{y}_i | \boldsymbol{\theta}) g(\boldsymbol{\theta}) d\boldsymbol{\theta} \approx \sum_{q=1}^Q \ell(\mathbf{y}_i | \mathbf{B}_q) W(\mathbf{B}_q)$$

with the conditional probability as:

$$\ell(\mathbf{y}_i | \boldsymbol{\theta}) \approx \ell(\mathbf{y}_i | \mathbf{B}) \approx \prod_{j=1}^{n_i} \prod_{c=1}^C [\Phi(\gamma_c - z_{iqj}) - \Phi(\gamma_{c-1} - z_{iqj})]^{d_{ijc}}$$

$$z_{iqj} = \mathbf{x}'_{ij}\boldsymbol{\beta} + \mathbf{z}'_{ij}\mathbf{T}\mathbf{B}_q$$

The log likelihood for the response vectors \mathbf{y}_i from the N level 2 units can be written as:

$$\log L = \sum_i^N \log h(\mathbf{y}_i)$$

For MML solution, differentiate $\log L$ with respect to parameters $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, and \mathbf{T}

Empirical Bayes estimates (*univariate case*)

Mean of the posterior distribution

$$\bar{\theta}_i = E(\theta_i | \mathbf{y}_i) = \frac{1}{h_i} \int_{\theta} \theta_i \ell_i g(\theta) d\theta \approx \frac{1}{h_i} \sum_{q=1}^Q \mathbf{B}_q \ell_{iq} W(\mathbf{B}_q)$$

Variance of the posterior distribution

$$V(\bar{\theta}_i | \mathbf{y}_i) = \frac{1}{h_i} \int_{\theta} (\theta_i - \bar{\theta}_i)^2 \ell_i g(\theta) d\theta \approx \frac{1}{h_i} \sum_{q=1}^Q (\mathbf{B}_q - \bar{\theta}_i)^2 \ell_{iq} W(\mathbf{B}_q)$$

At convergence, one more round of quadrature using

- $h_i = h(\mathbf{y}_i)$ which vary by i units
- $\ell_{iq} = \ell(\mathbf{y}_i | \mathbf{B}_q)$ which vary by i units and quadrature points

Treatment-Related Change Across Time

Data from the NIMH Schizophrenia collaborative study on treatment related changes in overall severity. IMPS item 79, *Severity of Illness*, was scored as:

1=normal or borderline mentally ill, 2=mildly or moderately ill, 3=markedly ill, 4=severely or among the most extremely ill

Group	Sample size at Week							completers
	0	1	2	3	4	5	6	
PLC (n=108)	107	105	5	87	2	2	70	65%
DRUG (n=329)	327	321	9	287	9	7	265	81%

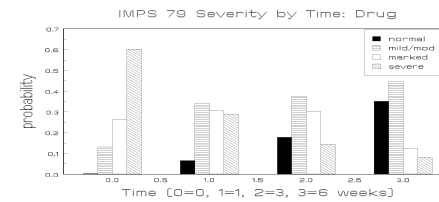
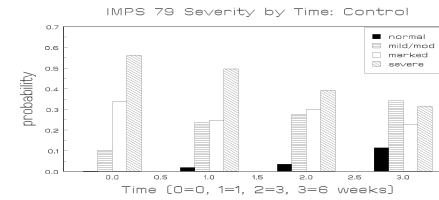
Drug = Chlorpromazine, Fluphenazine, or Thioridazine

Main question of interest:

- Was there differential improvement for the drug groups relative to the control group?

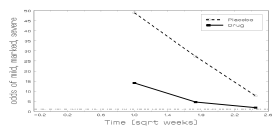
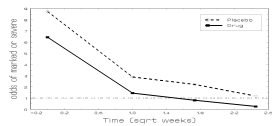
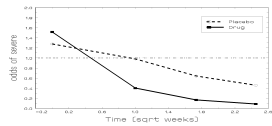
17

Observed Proportions across Time by Condition



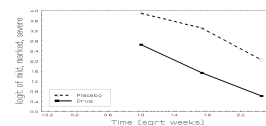
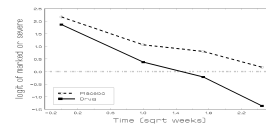
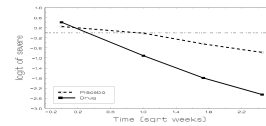
18

Observed Odds across Time by Condition



19

Observed Logits across Time by Condition



20

Within-Subjects / Between-Subjects components

Within-subjects model - level 1 ($j = 1, \dots, n_i$ obs)

$$\text{logit}_{cij} = \gamma_c - [b_{0i} + b_{1i}\sqrt{\text{Week}_j}]$$

Between-subjects model - level 2 ($i = 1, \dots, N$ subjects)

$$b_{0i} = \beta_0 + \beta_2 \text{Grp}_i + v_{0i}$$

$$b_{1i} = \beta_1 + \beta_3 \text{Grp}_i$$

$$v_{0i} \sim \mathcal{NID}(0, \sigma_v^2)$$

21

- $0 - \beta_0$ = week 0 IMPS79 1st logit (1 vs 2-4)
- $\gamma_1 - \beta_0$ = week 0 IMPS79 2nd logit (1-2 vs 3-4)
- $\gamma_2 - \beta_0$ = week 0 IMPS79 3rd logit (1-3 vs 4)

- β_1 = IMPS79 (sqrt) weekly logit change for PLC patients ($\text{Grp} = 0$)
- β_2 = difference in week 0 IMPS79 logit for DRUG patients ($\text{Grp} = 1$)
- β_3 = difference in IMPS79 (sqrt) weekly logit change for DRUG patients ($\text{Grp} = 1$)
- v_{0i} = individual deviation from group trend

22

NIMH Schizophrenia Study: Severity of Illness (N = 437)
Ordinal Logistic ML Estimates (se) - *random intercept model*

	ML estimates	se	z	p <
intercept	5.844	0.331	17.68	.001
threshold ₂	3.028	0.137	22.07	.001
threshold ₃	5.142	0.182	28.28	.001
Drug (0 = plc; 1 = drug)	-0.055	0.313	-0.18	.86
Time (sqrt week)	-0.766	0.131	-5.86	.001
Drug by Time	-1.202	0.153	-7.88	.001
Intercept sd	1.928	0.118		

$$\text{Intra-person correlation} = 1.928^2 / (1.928^2 + \pi^2/3) = .53$$

$$-2 \log L = 3404.2$$

23

Within-Subjects / Between-Subjects components

Within-subjects model - level 1 ($j = 1, \dots, n_i$ obs)

$$\text{logit}_{cij} = \gamma_c - [b_{0i} + b_{1i}\sqrt{\text{Week}_j}]$$

Between-subjects model - level 2 ($i = 1, \dots, N$ subjects)

$$b_{0i} = \beta_0 + \beta_2 \text{Grp}_i + v_{0i}$$

$$b_{1i} = \beta_1 + \beta_3 \text{Grp}_i + v_{1i}$$

$$\mathbf{v}_i \sim \mathcal{NID}(\mathbf{0}, \Sigma_v)$$

24

$0 - \beta_0$ = week 0 IMPS79 1st logit (1 vs 2-4)
 $\gamma_1 - \beta_0$ = week 0 IMPS79 2nd logit (1-2 vs 3-4)
 $\gamma_2 - \beta_0$ = week 0 IMPS79 3rd logit (1-3 vs 4)

β_1 = IMPS79 (sqrt) weekly logit change for PLC patients ($Grp = 0$)
 β_2 = difference in week 0 IMPS79 logit for DRUG patients ($Grp = 1$)
 β_3 = difference in IMPS79 (sqrt) weekly logit change for DRUG patients ($Grp = 1$)
 v_{0i} = individual deviation from group intercept
 v_{1i} = individual deviation from group (sqrt) weekly change

25

Ordinal Logistic Random Intercept and Trend Model				
	ML estimates	se	z	$p <$
intercept	7.283	0.467	15.59	.001
threshold ₂	3.884	0.209	18.57	.001
threshold ₃	6.478	0.290	22.36	.001
Drug (0=plc; 1 = drug)	0.056	0.388	0.15	.88
Time (sqrt week)	-0.879	0.216	-4.07	.001
Drug by Time	-1.684	0.250	-6.74	.001
Intercept var	6.847	1.282		
Int-Time covar	-1.447	0.515	$(r_{v_0v_1} = -.40)$	
Time var	1.949	0.404		

$-2 \log L = 3326.5, \chi^2_2 = 77.7, p < .001$

26

Model Fit of Observed Proportions

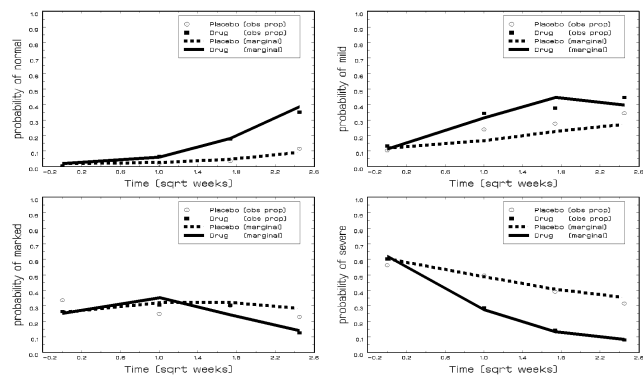
condition	category	model	week 0	week 1	week 3	week 6
Placebo	normal	<i>observed</i>	.00	.02	.03	.11
		random int	.02	.03	.05	.07
		random int & trend	.02	.03	.05	.09
	mild/mod	<i>observed</i>	.10	.24	.28	.34
		random int	.11	.17	.22	.28
		random int & trend	.12	.17	.23	.27
	marked	<i>observed</i>	.34	.25	.30	.23
		random int	.25	.30	.33	.33
		random int & trend	.26	.32	.32	.28
severe	<i>observed</i>	.56	.50	.39	.31	
	random int	.61	.49	.40	.32	
	random int & trend	.60	.49	.41	.35	

27

condition	category	model	week 0	week 1	week 3	week 6
Drug	normal	<i>observed</i>	.00	.07	.18	.35
		random int	.02	.07	.17	.35
		random int & trend	.02	.06	.18	.39
	mild/mod	<i>observed</i>	.13	.34	.38	.45
		random int	.12	.30	.43	.45
		random int & trend	.11	.31	.45	.40
	marked	<i>observed</i>	.26	.31	.30	.12
		random int	.26	.33	.26	.14
		random int & trend	.25	.35	.24	.14
	severe	<i>observed</i>	.60	.29	.14	.08
		random int	.61	.30	.14	.06
		random int & trend	.62	.27	.13	.08

28

Model Fit of Observed Proportions



29

SAS NLMIXED code: random-intercepts logistic regression - ordinal model

```

FILENAME IN1 'C:SCHIZX1.DAT';
DATA ONE; INFILE IN1 ;
INPUT ID IMPS79 IMPS79B IMPS79O INT TX WEEK SWECK TXSWK ;

/* get rid of observations with missing values */
IF IMPS79 > -9;

PROC FORMAT;
VALUE TX 0 = 'Placebo' 1 = 'Drug';

/* ORDINAL LOGISTIC RANDOM-INTERCEPT MODEL - adaptive quadrature */
PROC NLMIXED ;
PARMS B0=0 B1=0 B2=0 B3=0 SD=1 I1=1 I2=1;
Z = B0 + B1*TX + B2*SWECK + B3*TX*SWECK + U;
IF (IMPS79O=1) THEN P = 1 / (1 + EXP(-(0-Z)));
ELSE IF (IMPS79O=2) THEN P = (1/(1 + EXP(-(I1-Z)))) - (1/(1 + EXP(-(0-Z))));
ELSE IF (IMPS79O=3) THEN P = (1/(1 + EXP(-(I1+I2-Z)))) - (1/(1 + EXP(-(I1-Z))));
ELSE IF (IMPS79O=4) THEN P = 1 - (1 / (1 + EXP(-(I1+I2-Z))));
LL = LOG(P);
MODEL IMPS79O ~ GENERAL(LL);
RANDOM U ~ NORMAL(0,SD*SD) SUBJECT=ID;
ESTIMATE 'THRESH2' I1;
ESTIMATE 'THRESH3' I1 + I2;
ESTIMATE 'ICC' SD*SD/(3.289868134+SD*SD);
RUN;

```

30

SAS NLMIXED code: random-trend logistic regression - ordinal model using data from the earlier NLMIXED example

```

/* ORDINAL LOGISTIC RANDOM-TREND MODEL - adaptive quadrature */
PROC NLMIXED ;
PARMS B0=0 B1=0 B2=0 B3=0 V0=1 C01=0 V1=.5; I1=1 I2=1
Z = B0 + B1*TX + B2*SWECK + B3*TX*SWECK + U0 + U1*SWECK;
IF (IMPS79O=1) THEN P = 1 / (1 + EXP(-(0-Z)));
ELSE IF (IMPS79O=2) THEN P = (1/(1 + EXP(-(I1-Z)))) - (1/(1 + EXP(-(0-Z))));
ELSE IF (IMPS79O=3) THEN P = (1/(1 + EXP(-(I1+I2-Z)))) - (1/(1 + EXP(-(I1-Z))));
ELSE IF (IMPS79O=4) THEN P = 1 - (1 / (1 + EXP(-(I1+I2-Z))));
LL = LOG(P);
MODEL IMPS79O ~ GENERAL(LL);
RANDOM U0 U1 ~ NORMAL([0,0], [V0,C01,V1]) SUBJECT=ID;
ESTIMATE 'THRESH2' I1;
ESTIMATE 'THRESH3' I1 + I2;
ESTIMATE 'RE CORR' C01/SQRT(V0*V1);
RUN;

```

31

SAS IML code: computing marginal probabilities - ordinal models

```

TITLE1 'NIMH Schizophrenia Data - Estimated Marginal Probabilities';
PROC IML;
/* Results from NLMIXED analysis: random intercept model */;

x0 = { 1 0 0.00000 0,
       1 0 1.00000 0,
       1 0 1.73205 0,
       1 0 2.44949 0};
x1 = { 1 1 0.00000 0.00000,
       1 1 1.00000 1.00000,
       1 1 1.73205 1.73205,
       1 1 2.44949 2.44949};

sdu = {2.114};
beta = {5.844, -.055, -.766, -1.202};
thresh = {3.028, 5.142};

```

32

```

/* Approximate Marginalization Method */;

pi = 3.141592654;
nt = 4;
ivec = J(nt,1,1);
zvec = J(nt,1,1);
evec = (15/16)**2 * (pi**2)/3 * ivec;

/* nt by nt matrix with evec on the diagonal and zeros elsewhere */;
emat = diag(evec);

/* variance-covariance matrix of underlying latent variable */;
vary = zvec * sdu*sdu * T(zvec) + emat;

sdy = sqrt(vecdiag(vary) / vecdiag(emat));

```

33

```

za0 = (0 - x0*beta) / sdy ;
zb0 = (thresh[1] - x0*beta) / sdy;
zc0 = (thresh[2] - x0*beta) / sdy;
za1 = (0 - x1*beta) / sdy;
zb1 = (thresh[1] - x1*beta) / sdy;
zc1 = (thresh[2] - x1*beta) / sdy;

grp0a = 1 / ( 1 + EXP(0 - za0));
grp0b = 1 / ( 1 + EXP(0 - zb0));
grp0c = 1 / ( 1 + EXP(0 - zc0));
grp1a = 1 / ( 1 + EXP(0 - za1));
grp1b = 1 / ( 1 + EXP(0 - zb1));
grp1c = 1 / ( 1 + EXP(0 - zc1));

print 'Random intercept model';
print 'Approximate Marginalization Method';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (grp0c-grp0b) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 4' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (grp1c-grp1b) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 4' (1-grp1c) [FORMAT=8.4];

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34

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/* Random Intercept and Trend Model */;

varu = {6.847 -1.447,
        -1.447 1.949};
beta = { 7.283, .056, -.879, -1.684};
thresh = {3.884, 6.478};

/* Approximate Marginalization Method */;

pi = 3.141592654;
nt = 4;
ivec = J(nt,1,1);
zmat = {1 0.00000,
        1 1.00000,
        1 1.73205,
        1 2.44949};
evec = (15/16)**2 * (pi**2)/3 * ivec;

/* nt by nt matrix with evec on the diagonal and zeros elsewhere */;
emat = diag(evec);

/* variance-covariance matrix of underlying latent variable */;
vary = zmat * varu * T(zmat) + emat;

sdy = sqrt(vecdiag(vary) / vecdiag(emat));

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35

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za0 = (0 - x0*beta) / sdy ;
zb0 = (thresh[1] - x0*beta) / sdy;
zc0 = (thresh[2] - x0*beta) / sdy;
za1 = (0 - x1*beta) / sdy;
zb1 = (thresh[1] - x1*beta) / sdy;
zc1 = (thresh[2] - x1*beta) / sdy;

grp0a = 1 / ( 1 + EXP(0 - za0));
grp0b = 1 / ( 1 + EXP(0 - zb0));
grp0c = 1 / ( 1 + EXP(0 - zc0));
grp1a = 1 / ( 1 + EXP(0 - za1));
grp1b = 1 / ( 1 + EXP(0 - zb1));
grp1c = 1 / ( 1 + EXP(0 - zc1));

print 'Random intercept and trend model';
print 'Approximate Marginalization Method';
print 'marginal prob for group 0 - catg 1' grp0a [FORMAT=8.4];
print 'marginal prob for group 0 - catg 2' (grp0b-grp0a) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 3' (grp0c-grp0b) [FORMAT=8.4];
print 'marginal prob for group 0 - catg 4' (1-grp0c) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 1' grp1a [FORMAT=8.4];
print 'marginal prob for group 1 - catg 2' (grp1b-grp1a) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 3' (grp1c-grp1b) [FORMAT=8.4];
print 'marginal prob for group 1 - catg 4' (1-grp1c) [FORMAT=8.4];

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36