

# Constitutive Model for MSW Considering Creep and Biodegradation Effects

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**ABSTRACT:** In this paper, a generalized constitutive model for MSW, based on the framework of critical state concepts is proposed to incorporate the effects of mechanical creep and time dependent biodegradation to calculate total compression under loading with time. To illustrate the general applicability of the model, detailed parametric studies considering variations of different parameters are conducted in term of variations of the settlement with time as affected by parameters. The influences of strength and stiffness of MSW, compressibility parameters, and biodegradability parameters in settlement-time response of MSW are highlighted. The model is useful for assessing the deformation and stability of landfills and any post-closure development structures located on landfills.

## INTRODUCTION

Prediction of municipal solid waste (MSW) landfill settlement is required to assess the integrity of cover systems and appurtenant systems (gas and leachate collection pipes), estimate the landfill airspace, and design the end-use facilities (e.g., golf course, industrial/commercial building). MSW settlement is mainly attributed to: (1) physical and mechanical processes that include the reorientation of particles, movement of the fine materials into larger voids, and collapse of void spaces; (2) chemical processes that include corrosion, combustion and oxidation; (3) dissolution processes that consist of dissolving soluble substances by percolating liquids and then forming leachate; and (4) biological decomposition of organics with time depending on humidity and the amount of organics present in the waste.

Reddy and his coworkers (Reddy et al. 2009a, 2009b, 2009c) presented considerable data on the geotechnical characteristics of landfills at different stages of degradation. Babu et al. (2009a) proposed a generalized MSW landfill settlement model which accounts for the stress-strain characteristics through a constitutive model based on critical state concepts. The advantage of this model is that it is based on the stress-strain (constitutive) response of MSW and it is a general model which can be applied to determine spatial variations in settlement depending on the landfill conditions (thickness, age, etc.). The model accounts for mechanical compression and time-dependent mechanical creep and biodegradation. The model is validated with reference to the test data of (a) fresh MSW obtained from working phase of a landfill, (b) landfilled waste retrieved from a landfill after a year of degradation, and (c) synthetic MSW with controlled composition. The model captures the stress-strain and pore water pressure response of these three types of MSW adequately. Babu et al. (2010) illustrated the

applicability of the model for a typical MSW landfill. The predicted settlement results were compared with the predicted settlement results obtained using fourteen different reported models. Two of these reported models were developed by Marques and his collaborators (Marques, 2001; and Marques et al., 2003) which also account for mechanical compression, mechanical creep and biodegradation similar to the constitutive model.

## FOR PARAMETRIC STUDY

The settlement predictive models involve many parameters related to the strength, compressibility and biodegradation which vary widely. Literature review indicates that the influence of the parameters related to (i) shear strength ( $\phi$ ), (ii) compressibility ( $C_c$ ), (iii) total biodegradable strain ( $E_{DG}$ ), and (iv) biodegradation rate constant ( $d$ ) on the settlement response is significant.

It may be stated that while the qualitative influence of these parameters on settlement of MSW is known, it is essential to obtain the values in quantitative terms in terms of time-settlement behavior for different parameters and this is possible by conducting parametric studies using suitable models or the approaches that consider all the above factors. The following sections provide an overview of the constitutive model of Babu et al. (2009a) and provides results of parametric study, including comparisons with two reported models- Marques (2001) and Marques (2003).

## CONSTITUTIVE MODEL (Babu et al. 2009a)

Considering elastic and plastic behavior as well as mechanical creep and biological decomposition, the total volumetric strain of the MSW under loading is expressed as:

$$d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p + d\varepsilon_v^c + d\varepsilon_v^b \quad (1)$$

where  $d\varepsilon_v^e$ ,  $d\varepsilon_v^p$ ,  $d\varepsilon_v^c$  and  $d\varepsilon_v^b$  are the increments of volumetric strain due elastic, plastic, time dependent mechanical creep and biodegradation

effects. The elastic volumetric strain  $d\varepsilon_v^e$  can be written as

$$d\varepsilon_v^e = -\frac{d\varepsilon_v^e}{1+e} = \frac{\kappa}{1+e} \frac{dp'}{p'} \quad (2)$$

And, increment in plastic volumetric strain can be written as

$$d\varepsilon_v^p = \left( \frac{\lambda - \kappa}{1+e} \right) \left[ \frac{dp'}{p'} + \frac{2\eta d\eta}{M^2 + \eta^2} \right] \quad (3)$$

The above formulations for increments in volumetric strain due to elastic and plastic are well established in critical state soil mechanics literature.

The mechanical creep is a time dependent phenomenon in exponential function given by

$$\varepsilon_c = b\Delta p' (1 - e^{-ct'}) \quad (4)$$

where  $b$  is the coefficient of mechanical creep;  $\Delta p'$  is the change in mean effective stress,  $c$  is the rate constant for mechanical creep; and  $t'$  is the time since application of the stress increment. The time dependent biodegradation is given by

$$\varepsilon_b = E_{DG}(1 - e^{-dt''}) \quad (5)$$

where  $E_{DG}$  is the total amount of strain that can occur due to biological decomposition;  $d$  is the rate constant for biological decomposition; and  $t''$  is the time since placement of the waste in the landfill. From Eq. (4), increment in volumetric strain due to creep is written as:

$$d\varepsilon_v^c = cb\Delta p' e^{-ct'} dt' \quad (6)$$

From Eq. (5), increment in volumetric strain due biodegradation effect is written as:

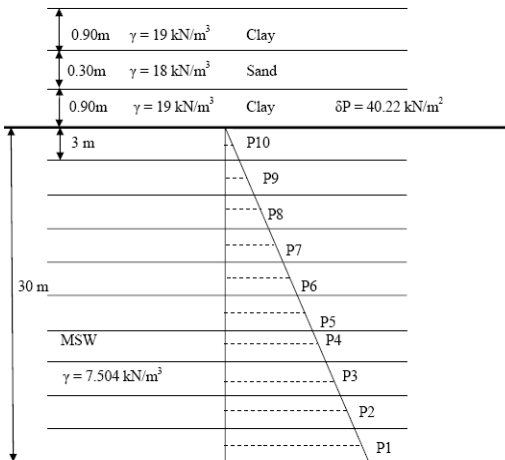
$$d\varepsilon_v^b = E_{DG} e^{-dt''} dt'' \quad (7)$$

In the present case  $t'$  time since application of the stress increment and  $t''$  time since placement of the waste in the landfill are considered equal to ' $t$ '. Using Eqs. (2), (3), (6), and (7) and substituting in Eq. (1), total increment in strain is given by

$$d\varepsilon_v = \frac{\kappa}{1+e} \frac{dp'}{p'} + \left( \frac{\lambda - \kappa}{1+e} \right) \left[ \frac{dp'}{p'} + \frac{2\eta d\eta}{M^2 + \eta^2} \right] + cb \Delta \sigma e^{-ct} dt + E_{DG} e \tag{8}$$

**PARAMETRIC STUDY**

A simple example of MSW landfill of 30 m height has been considered which is assumed to be filled in ten layers each of 3 m deep as shown in Fig. 1.



**Fig. 1** MSW landfill scenario for estimation of settlement versus time.

At the top of landfill, a final cover system has been assumed to be constructed which consists of composite liner (compacted clay and geomembrane) overlain by a sand drainage layer and then a vegetative cover soil layer. The objective of this study is to examine the results of variation of landfill settlement with time for a typical layer (for example, P6 layer) considering the variations of different ranges of parameters

over 30 years (10,950 days, which is typically the landfill closure time specified) and draw inferences with regard to time-settlement response of MSW.

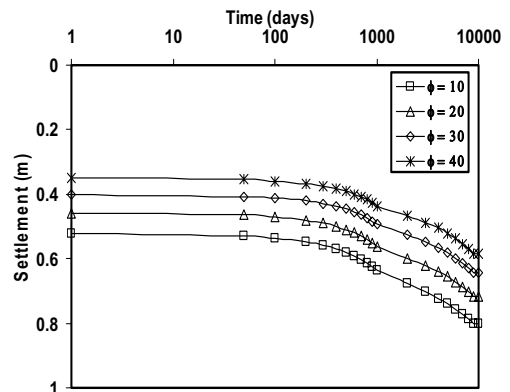
**Influence of shear strength parameters**

For the calculation purpose, it is assumed that only friction angle is variable and other parameters remain constant and values in the range of 10° to 40° are used. The model parameters used for the calculations and ultimate settlement at the end of 30 years are given in Table 1.

**TABLE 1.** Influence of friction angle ( $\phi$ ) on MSW settlement

$(\phi)$	$(C_c)$	$(E_{DG})$	$(d)(\text{day}^{-1})$	(m)
10°	0.20	0.159	0.00114	0.71
20°	0.20	0.159	0.00114	0.64
30°	0.20	0.159	0.00114	0.58
40°	0.20	0.159	0.00114	0.53

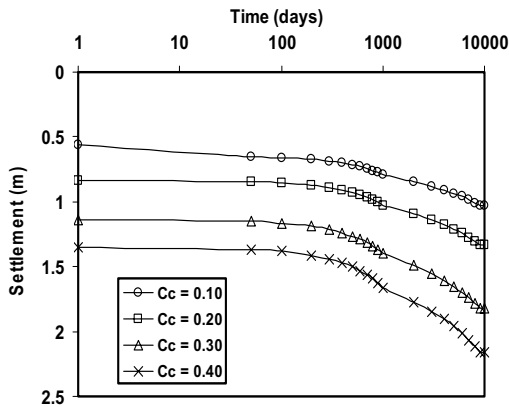
Fig. 2 shows the time-settlement response for MSW for friction angles. It can be noted that the settlement at the end of 30 years (10,950 days) corresponding to 10° is 0.71 m, whereas for MSW represented by higher friction angle 40°, the settlement is 0.53 m. For a particular time period, the settlement is less for MSW with higher friction angle. This shows that the shear strength in terms of friction angle of the MSW influences the ultimate settlement of the landfill considerably.



**Fig. 2** Time vs. settlement response of MSW for different values of friction angle

### Influence of Compression Index ( $C_c$ )

Using different values of compression index from 0.10 to 0.40, time-settlement responses are predicted keeping the other parameters constant. Comparison of values of maximum settlement from all the three models and the model parameters used are given in Table 2. It can be noted that the higher values of compression index are associated with larger settlements.



**Fig. 3** Time vs. settlement response of MSW for different values of compression index ( $C_c$ ).

The general observation from all three models is that with increasing values of compression index, the predicted value of ultimate settlement is more.

**TABLE 2.** Comparison of maximum values of settlement (in m) from different models keeping constant parameters ( $\phi = 20^\circ$ ,  $E_{DG} = 0.159$ ,  $d = 0.00114 \text{ day}^{-1}$ )

$(C_c)$	Marques (2001)	Marques et al. (2003)	Proposed model
0.10	1.017	1.075	1.030
0.20	13.47	1.417	1.334
0.30	1.676	1.759	1.821
0.40	2.005	2.101	2.164

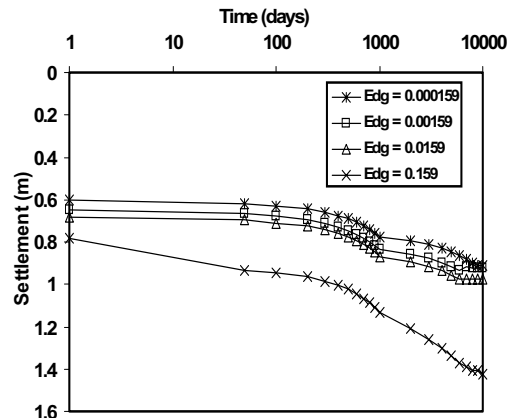
### Influence of Biodegradation

To assess the settlement behavior of MSW with respect to biodegradation effect, the three models are used with different values of total biodegradable strain,  $E_{DG}$  (0.000159 - 0.159) and the biodegradation rate is kept constant. Marques et al. (2003) reported the values of  $E_{DG}$  varying from 0.214 to 0.131 with an average value of 0.159. In this study, this value as well as the values lowered by 10, 100 and less have been used to discern the effect. Comparison of time-settlement response for different values of total biodegradable strain ( $E_{DG}$ ) is shown in Figure 4..

The maximum settlements corresponding to different values of  $E_{DG}$  and additional constant parameters are presented in Table 3.

**TABLE 3.** Comparison of maximum values of settlement (in m) from different models keeping constant parameters ( $\phi = 20^\circ$ ,  $C_c = 0.20$ ,  $d = 0.00114 \text{ day}^{-1}$ )

$(E_{DG})$	Marques (2001)	Marques et al. (2003)	Proposed model
0.000158	0.872	0.993	1.038
0.00158	0.876	0.9374	1.058
0.0158	0.919	0.981	1.120
0.158	1.348	1.417	1.408



**Fig. 4** Time vs. settlement response of MSW for different values of total biodegradable strain ( $E_{DG}$ ).

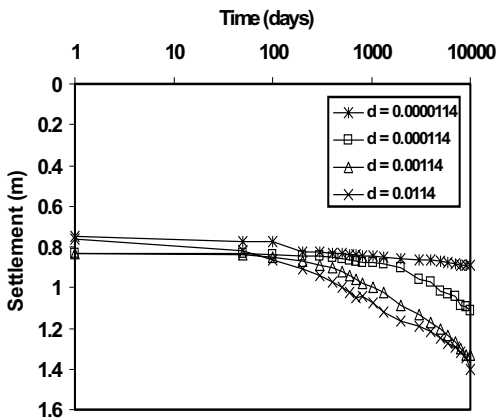
From the plotted results, it is observed that the model predictions of final settlements are influenced by the values of total biodegradable strain.

### ***Influence of Rate of Biodegradation ( $d$ )***

Comparison of the settlement response for different values of  $d$  is shown in Fig. 5. It shows that higher values of biodegradation rate constant cause higher settlement and hence enhancement of biodegradation rates using leachate recirculation helps in accelerated settlements. The maximum settlements observed corresponding to different values of  $d$  are presented and constant parameters used are presented in Table 4.

TABLE 4. Comparison of maximum values of settlement (in m) from different models keeping constant parameters ( $\phi = 20^\circ$ ,  $C_c = 0.20$ ,  $E_{DG} = 0.159$ )

$(d)$	Marques (2001)	Marques et al. (2003)	Constitutive model
0.0000114	0.923	0.974	0.891
0.000114	1.196	1.219	1.112
0.00114	1.348	1.417	1.328
0.0114	1.349	1.418	1.344



**Fig. 5** Time vs. settlement response of MSW for different values of biodegradation ( $d$ ).

## **CONCLUDING REMARKS**

In this paper, results of parametric study on the influence of strength, compressibility, biodegradation parameters on the settlement time response of MSW are presented. As expected, shear strength parameters, the total biodegradation content and rate of biodegradation influence the settlement response which can be predicted by the model proposed. This aspect can be advantageously used in the design of bioreactor landfills to accelerate settlement and also improve strength response.

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