



## Modeling of Volumetric Response of Cemented Sand Under Cyclic Loading

Paper No. 1.63

Krishna R. Reddy  
University of Illinois  
Chicago, IL, USA

**SYNOPSIS** This paper first describes cyclic tests which were conducted by Iwabuchi (1986) on both uncemented and cemented sands using a true triaxial apparatus. The paper then presents a thermodynamic or endochronic model which was used to predict the volumetric behavior of uncemented and cemented sands under cyclic loading conditions. The predicted results using the thermodynamic model and the test results using the true triaxial apparatus were found to be in good agreement.

### INTRODUCTION

The cementation of sand deposits for the purpose of improving engineering properties of the sand has been in practice for a few decades. Although cementation is generally believed to strengthen sands, no systematic study was performed until 1983 to quantify the beneficial effects of the cementation of sands. An extensive research program was initiated at that time which included performing conventional static triaxial, conventional cyclic triaxial, resonant column, and true (or cubical) triaxial tests. These tests were performed in order to understand the behavior of cemented sands subjected to various types of loadings under both drained and undrained conditions. The research results based on the static triaxial, cyclic triaxial, and resonant column tests have been presented and discussed by Saxena et al. (1988 a,b,c). The test results using the true triaxial apparatus on cemented sand under three-dimensional static loadings have been explained by Reddy et al. (1992) and Reddy and Saxena (1993). The constitutive modeling of cemented sand for static loading conditions was also investigated by Reddy and Saxena (1992). This paper presents a thermodynamic or endochronic model to predict the volumetric response of cemented sands under cyclic loading.

### EXPERIMENTAL RESULTS

Iwabuchi (1986) used a true triaxial apparatus for performing cyclic tests on both uncemented and cemented sands. This apparatus is similar to that developed by Reddy (1990) and it can accommodate a two inch (5.1 cm) cubical sample and also can apply cyclic loading. The materials used for sample preparation were the same as those used in previous investigations by Reddy (1990); namely Monterey No. 0/30 sand and Portland cement Type I. A brief summary of these test results is presented below. These test results were utilized for modeling purposes in this paper.

The cyclic tests were performed using the true triaxial apparatus under drained conditions on uncemented sand as well as on cemented sand which contained 2% cement based on the dry weight of sand. In these tests, cubical samples were first saturated

and then consolidated isotropically under an effective confining pressure ( $\bar{\sigma}_c$ ) of 104 kPa. Next, a sinusoidal stress with a frequency of one cycle per second was superimposed in the vertical direction while a constant effective pressure of 104 kPa in the lateral directions was maintained. Several tests were performed by imposing varying maximum amplitude of sinusoidal stress,  $(\Delta\sigma_1)_{max}$ , in vertical direction on the soil samples. These stress values were always a fraction of the effective confining pressure. The stress ratio ( $S_r$ ) for these tests is defined as:

$$S_r = \frac{(\Delta\sigma_1)_{max}}{2\bar{\sigma}_c}$$

While applying the cyclic loading, the deformation of each face of the test sample was measured. These deformations were then used to calculate the volumetric strain for each loading cycle. Figure 1 shows the results of the cyclic tests on uncemented sand which were conducted with  $S_r$  equal to 0.2, 0.3 and 0.35, respectively. Forty loading cycles were used for each test. Figure 2 shows the cyclic tests on cemented sand performed with  $S_r$  equal to 0.3, 0.4 and 0.45, respectively. Seventy loading cycles were used for each of these tests. From these test results, it can be seen that for both uncemented and cemented sands, the rate of volumetric strain increase is high during the initial stages of loading. After the initial loading stages, the volumetric strain rate is significantly reduced. The volumetric strain for uncemented sand was found to be consistently higher than the volumetric strain for cemented sand. The volumetric strain for both uncemented and cemented sands was found to be greater at higher values of stress ratio.

### MODELING PROCEDURE AND RESULTS

Valanis and Peters (1988, 89) proposed a thermodynamic or endochronic model based on the Second Law of Thermodynamics. This model is particularly suited for cyclic loading conditions because: (1) it does not require a priori definition of yield function, (2) it does not require a complicated unload-reload criteria, and (3) it takes into account initial anisotropy, stress-induced anisotropy, compressibility or pressure dependency and stress-history dependency of soils.

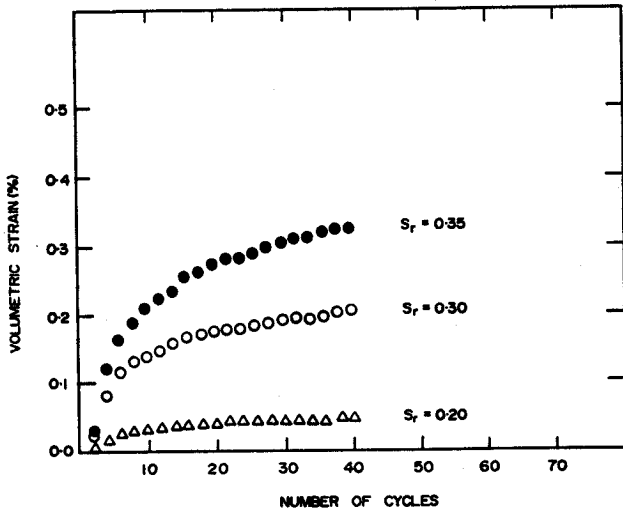


Figure 1- Cyclic Test Results for Uncemented Sand

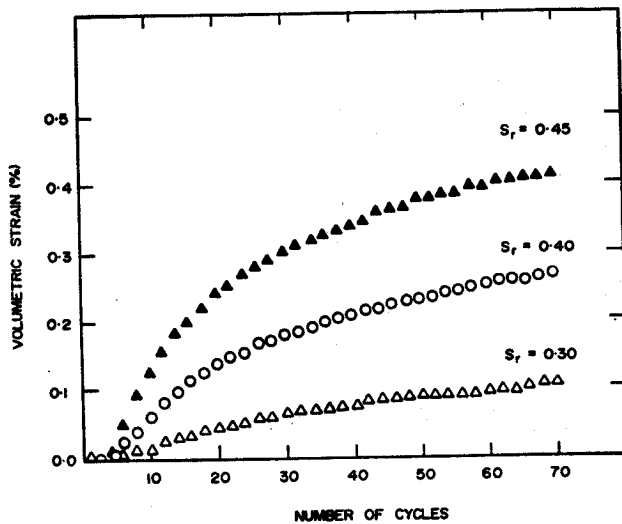


Figure 2- Cyclic Test Results for Cemented Sand

Reddy and Saxena (1992) found that the model can predict stress-strain and volume change behavior of uncemented and cemented sands under different monotonic stress paths. In the present investigation, the objective is to use the same model and investigate if it can also predict volumetric response measured during cyclic loading for uncemented and cemented sands.

### Overview of Thermodynamic Model

Based on thermodynamic laws, the deviatoric and hydrostatic stresses are expressed by following integral equations:

$$\bar{s} = \int_0^{Z_D} \rho(Z_D - Z') \frac{\partial e^p}{\partial Z'} dZ' \quad (1)$$

and

$$\sigma = \frac{\sigma_{kk}}{3} = \int_0^{Z_H} \phi(Z_H - Z') \frac{\partial e^p}{\partial Z'} dZ' + \int_0^{Z_H} \Gamma(Z_H - Z') s \frac{\partial e^p}{\partial Z'} dZ' \quad (2)$$

where

$$\sigma = \frac{\sigma_{kk}}{3} \quad (3)$$

$$e = e_{kk} \quad (4)$$

$$\bar{s} = \bar{\sigma} - \sigma \delta \quad (5)$$

$$d\bar{e} = d\bar{e} - \frac{1}{3} de\delta \quad (6)$$

In above equations, the stress tensor  $\bar{\sigma}$  and the strain tensor  $\bar{e}$  are decomposed into hydrostatic and shear components. Using the theory of elasticity:

$$d\bar{e}^p = d\bar{e} - \frac{1}{2G} d\bar{s} \quad (7)$$

$$de^p = de - \frac{1}{K} d\sigma \quad (8)$$

where  $G$  is the elastic shear modulus and  $K$  is the elastic bulk modulus. The superscript  $p$  refers to the plastic strain. The material time scale  $Z$  is defined as below:

$$dZ^2 = dZ_D^2 + k^2 dZ_H^2 \quad (9)$$

where

$$dZ_D = \frac{dZ}{F_D} = |d\bar{e}^p| \quad (10)$$

$$dZ_H = \frac{dZ}{kF_H} = |de^p| \quad (11)$$

$F_D$  and  $F_H$  are the hardening parameters for the deviatoric and hydrostatic conditions, respectively, while  $k$  is a coupling term used to couple hydrostatic and deviatoric responses. If  $F_D$  and  $F_H$  increase with  $Z$ , the material will harden, and when  $F_D$  and  $F_H$  decrease with  $Z$ , the material will exhibit softening behavior. However,  $F_D$  and  $F_H$  can not be negative quantities. All the

parameters involved in this model can be determined using the conventional hydrostatic and triaxial compression tests. More details on the theory, computational procedure, and determination of model parameters are given by Reddy and Saxena (1992) and Sengupta (1989).

### Application of Thermodynamic Model for Uncemented Sand

Iwabuchi (1986) prepared the uncemented sand samples using Monterey No. 0/30 sand at a unit weight of 15.07 kN/m<sup>3</sup>. The static triaxial tests performed by Reddy (1990) included tests on uncemented sand samples which were prepared using Monterey No.0/30 sand with a unit weight of 14.98 kN/m<sup>3</sup>. These test results were used to determine all the required parameters for the thermodynamic model (Reddy and Saxena, 1992). The values of these parameters are summarized in Table 1, and were used in this study for prediction of volumetric changes under cyclic loading conditions.

Table 1: Summary of Uncemented and Cemented Sand Parameter Values for Thermodynamic Model

Parameter	Uncemented Sand	Cemented Sand
K (kPa)	61906	80751
G (kPa)	57879	147933
$\Gamma_0$	0.002416 $\sigma + 1.2235$	1.6501
k	0.7363	0.4194
$\phi_1$	0.4048	0.4003
$\phi_0$	39.64 $\sigma^{0.672}$	14.63 $\sigma^{0.9324}$
$\eta$	0.0033	0.0033
$\delta_0$	0.0000001	0.00001
$A_1$	3857.868	2272.6981
A	135000	228572.09
$\alpha_1$	15.72	13.51
$\alpha_2$	417.156	605.302

With the model parameters shown in Table 1, the thermodynamic model predicted a linear variation of volumetric strain with the number of cycles for a specified stress ratio. When the stress ratio is changed, the volumetric strain was increased linearly. However, the experimental results shown in Figure 1, indicate a nonlinear variation of volumetric strain with the number of cycles. These experimental results necessitated that modifications be made to the thermodynamic model.

The effects of elastic properties, specifically, the bulk modulus (K) and the shear modulus (G) were investigated first. K and G were expressed in both strain dependent and stress dependent form. However, the influence of K and G values on the predicted results was found to be negligible. In cases of cyclic loading involving large strains, Sengupta (1989) showed that the parameter  $\phi_0$  in the hydrostatic hardening function  $F_H = (1 - \phi_1)\phi_0 + \phi_1\sigma$  should be considered in the functional form, in terms of plastic volumetric strain or initial effective hydrostatic stress. Based on this,  $\phi_0$  was represented as a function of the plastic volumetric strain; however, the plastic strains being very small under the cyclic loading, the predictions did not improve. The experimental results indicate increase in plastic work with increase in number of cycles as a result of particle rearrangement or degradation. Based on this premise, an alternate functional form for  $\phi_0$  expressed in terms of the stress ratio ( $S_c$ ) and the number of cycles (N) provided better

predictions. This functional form of  $\phi_0$  is defined as:

$$\phi_0 = \frac{898.63N}{13.86S_c + 0.01N - 1.89} \quad (12)$$

Figure 3 shows a comparison of the predicted results using the above equation in the model and the experimental results. From Figure 3, it can be seen that the model predictions are in reasonable agreement with the test results.

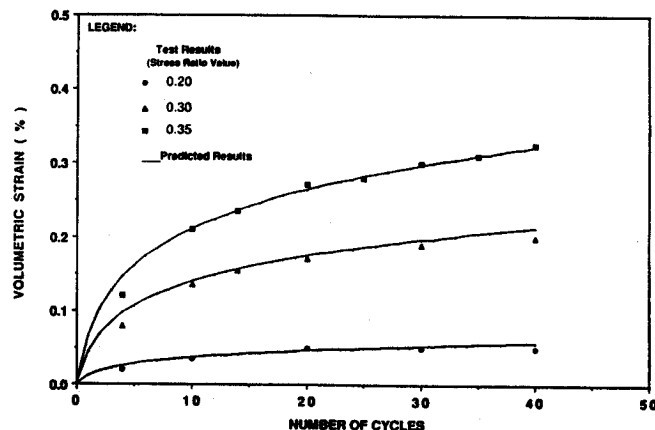


Figure 3- Comparison of Model Predictions and Test Results for Uncemented Sand

### Application of Thermodynamic Model for Cemented Sand

The cemented sand samples tested by Iwabuchi (1986) were prepared using Monterey No. 0/30 sand and Portland cement Type I. The unit weight and unconfined compressive strength of samples were 15.07 kN/m<sup>3</sup> and 59 kPa, respectively. These test conditions were the same as those used by Reddy (1990) for performing static triaxial tests. The results of these static triaxial tests were used to determine all the required parameters for the thermodynamic model for cemented sand (Reddy and Saxena, 1992). These parameter values are summarized in Table 1, and were used in predicting volumetric changes under cyclic loading conditions.

Predictions were made for the cyclic tests with stress ratio ( $S_c$ ) equal to 0.3, 0.4 and 0.45, respectively, up to 70 cycles. The predicted volumetric strain versus the number of cycles was found to be linear, which indicates that the volume change per cycle is constant. However, the experimental results given in Figure 2 show a nonlinear variation of volume change with an increase in number of cycles. As in the case of uncemented sand, a new

functional form of  $\phi_0$  for cemented sand was proposed as below:

$$\phi_0 = \frac{111.54N}{13.57S_r + 0.1N - 4.03} \quad (13)$$

Figure 4 shows a comparison of the model prediction incorporating the new functional form for  $\phi_0$  and the test results. From Figure 4, it can be concluded that the predicted results are in close agreement with the experimental results.

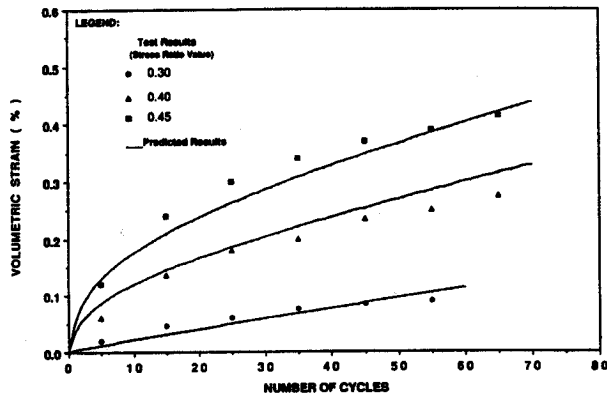


Figure 4- Comparison of Model Predictions and Test Results for Cemented Sand

The present study demonstrates the potential of a thermodynamic or endochronic model to predict cyclic test results with minor modifications. Since this model does not require complicated unload-reload criteria and is not based on the concept of yield surface, it is equally suited for monotonic as well as cyclic loadings.

## CONCLUSION

In order to solve boundary value geotechnical engineering problems using sophisticated mathematical techniques, a constitutive model is required to define the stress-strain and volume change characteristics of soils. This paper examined the applicability of a thermodynamic model to predict volume changes for both uncemented and cemented sands under cyclic loading conditions. In earlier studies, it was found that the model can predict constitutive behavior of both uncemented and cemented sands under static monotonic loading conditions. Using the new functional forms for  $\phi_0$ , the predicted volume change with the corresponding number of cycles was in agreement with the measured volume changes for both uncemented and cemented sands.

## REFERENCES

1. Iwabuchi, J., The Influence of Cementation on Liquefaction Resistance of Sands, Ph.D. Thesis, Department of Civil Engineering, Virginia Polytechnic and State University, Blacksburg, Virginia, 1986.
2. Reddy, K.R., Behavior of Cemented Sands Under Three-Dimensional Loadings, Ph.D. Thesis, Department of Civil Engineering, Illinois Institute of Technology, Chicago, Illinois, May, 1990.
3. Reddy, K.R., Saxena, S.K., and Budiman, J.S., "Development of a True Triaxial Apparatus," *Geotechnical Testing Journal*, ASTM, No.2, Vol.15, June, 1992, pp.89-105.
4. Reddy, K.R., and Saxena, S.K., "Effect of Cementation on Stress-Strain and Strength Characteristics of Sands," *Soils and Foundations*, JSSMFE, Vol.33, No.4, December, 1993, pp.123-136.
5. Reddy, K.R., and Saxena, S.K., "Constitutive modeling of Cemented Sand," *Mechanics of Materials*, Vol.14, December, 1992, pp.155-178.
6. Saxena, S.K., Reddy, K.R., and Avramidis, A., "Static Behavior of Artificially Cemented Sand," *Indian Geotechnical Journal*, Vol. 18, No. 2, April, 1988a, pp. 111-141.
7. Saxena, S.K., Avramidis, A., and Reddy, K.R., "Dynamic Moduli and Damping Ratios for Cemented Sands at Low Strains," *Canadian Geotechnical Journal*, Vol. 25, No. 2, May, 1988b, pp. 353-368.
8. Saxena, S.K., Reddy, K.R., and Arvamidis, A., "Liquefaction Resistance of Artificially Cemented Sand," *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 114, NO. GT12, December, 1988c, pp. 1395-1413.
9. Sengupta, A., Application of a Thermodynamic Model to Soils and Concrete, Ph.D. Thesis, Dept. of Civil Engineering, Illinois Institute of Technology, Chicago, Illinois, May, 1989.
10. Valanis, K.C., and Peters, J.F., "An Endochronic Plasticity Theory with Shear-volumetric Coupling," *Intl. J. Num. Ana. Methods in Geomechanics*, January, 1989.
11. Valanis, K.C., and Peters, J.F., *Thermodynamics of Frictional Materials, Report I, A Constitutive Theory of Soil with Dilatant Capability*, U.S. Army Engineers Waterways Experiment Station Report, July, 1988.