ChE 201 Fall 2000
Exam 1 Solutions

Problem 1. 25 pts.

A large storage tank with a cross sectional area of $A_{tank} = 2\ ft^2$ intially contains 1248 lb_m of water. A hole develops in the bottom and the water starts to leak out. When the tank if full, the water gushes out of the leak. As the tank drains (as the water level lowers) there is less pressure at the bottom of the tank, and the leak rate slows down. A mathematical expression to describe this changing leak rate is

m_{out} (in lb_m/min) = 2 z ,

where z is the height of water in the tank in feet, and 2 is a constant.

a) $\frac{lb_m}{min} [=] \left(\frac{lb_m}{min\ ft}\right)(ft)$; $m_{out} = 2z$

a) what must be the units of the constant "2" in the above expression? → $\frac{lb_m}{min\ ft}$

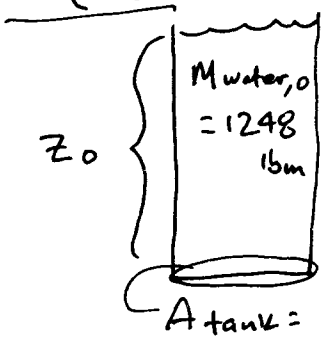
b) use a mass balance to derive the height of the water in the tank versus time.

c) how many minutes will it take for the tank to become 90% drained?

useful facts: $\rho_{water} = 62.4\ lb_m/ft^3$

$M_{water} = \rho_{water} V_{water} = \rho_{water} (A_{tank}\, z)$

t = 0:



$M_{water,0} = \rho_{water} A_{tank} z_0$

$z_0 = \frac{M_0}{\rho A} = \frac{1248\ lb_m}{62.4 \frac{lb_m}{ft^3}(2 ft^2)} = 10\ ft$

b) acc = in − out

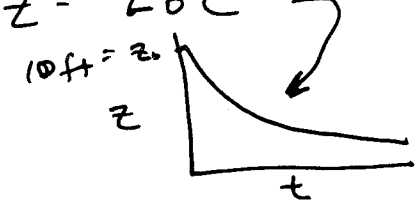
$\frac{dM}{dt} = -M_{out}$

$\frac{d(\rho A z)}{dt} = -2z$

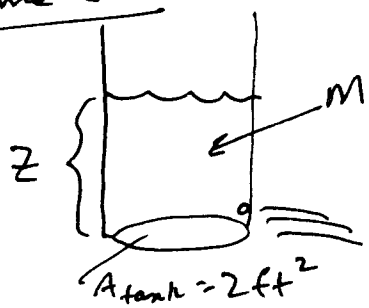
$\int_{z_0}^{z} \frac{dz}{z} = -\frac{2}{\rho A}\int_0^t dt$

$\ln \frac{z}{z_0} = -\frac{2}{\rho A} t$

$z = z_0 e^{-\left(\frac{2}{\rho A}\right)t}$



t = time t:



c) $t = -\frac{\rho A}{2} \ln \frac{z}{z_0} = \frac{\rho A}{2} \ln \frac{z_0}{z}$

$= \frac{(62.4 \frac{lb_m}{ft^3})(2 ft^2)}{2 \frac{lb_m}{min\ ft}} \ln 10$

$= 144\ min$