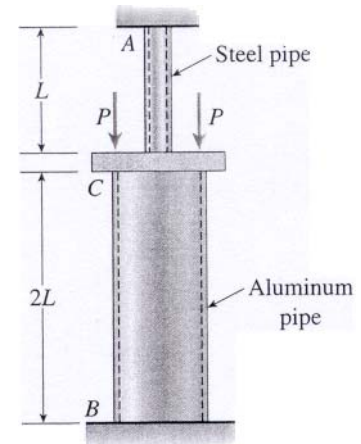


NAME: _____ UIN: _____

I pledge my honor that I have not sought unfair advantage over other students, including, but not limited to giving or receiving unauthorized aid during this exam. _____

signature

(1) The aluminum and steel pipes shown in the figure are fastened to rigid supports at ends A and B and to a rigid plate C at their junction. The aluminum pipe is twice as long as the steel pipe. Two equal and symmetrically placed loads P act on the plate at C. (a) Draw a free-body for the rigid plate C and obtain the equation of equilibrium. (b) Derive the equation of compatibility. (c) Obtain formulas for the axial stresses σ_a and σ_s in the aluminum and steel pipes, respectively, in terms of P, L, cross-sectional area of aluminum pipe A_a , cross-sectional area of steel pipe A_s , modulus of elasticity of aluminum E_a , and modulus of elasticity of steel E_s .



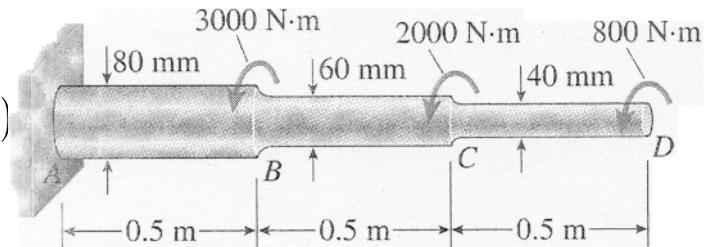
Axial Deformation

$$\sigma = \frac{P}{A}, \quad \epsilon = \frac{\sigma}{E} = \frac{P}{EA}, \quad \delta = \frac{PL}{EA} \text{ (for uniform deformation)}$$

$$\epsilon = \frac{d\delta}{dx}, \quad \epsilon = \frac{\delta}{L} \text{ (for uniform deformation)}$$

$$\sigma = E\epsilon, \quad u = \frac{1}{2}E\epsilon^2 = \frac{1}{2E}\sigma^2$$

(2) A stepped shaft ABCD consisting of solid circular segments is subjected to three torques, as shown in the figure. The torques have magnitudes 3000 N.m, 2000 N.m. and 800 N.m. The length of each segment is 0.5 m and the diameters of the segments are 80 mm, 60 mm, and 40 mm. The material is steel with shear modulus of elasticity $G = 80$ GPa. (a) Draw the internal torque diagram. (b) Calculate the maximum shear stress τ_{max} in the shaft ABCD. (c) Calculate the angle of twist ϕ_D (in degrees) at end D, relative to the fixed support at A.



Torsion

$$\phi = \frac{TL}{GI_p} \text{ (uniform torsion), } I_p = \frac{\pi}{2}(r_o^4 - r_i^4)$$

$$\theta = \frac{d\phi}{dx} = \frac{T}{GI_p}, \quad \gamma = \theta\rho = \frac{T\rho}{GI_p}$$

$$\tau = G\gamma = G\theta\rho = \frac{T\rho}{I_p}$$

$$u = \frac{1}{2}G\gamma^2 = \frac{1}{2G}\tau^2$$

