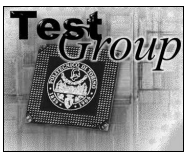
Lecture 6.3

Some examples



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Goal

- This lecture guides the students through the solution of some simple examples of manual synthesis of combinational networks.

Prerequisites

- Lectures 6.1 and 6.2

Homework

- Students are recommended to try to solve the exercise by themselves, before looking at the proposed solutions.

Further readings

- No particular suggestion

Exercise #6.3.1

Design a combinational circuit with 1 output U and an input X (3 downto 0) that, when it receives on X an unsigned *hexadecimal* digit X, provides, on U, a logical value 1 iff:

$X < 4$ or $X > 8$.

Solution #1

- We are going to present a complete first solution, i.e., till the netlist of the target circuit.

Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00				
01				
11				
10				

U

Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00				
01				
11				
10				

HINT:
label each cell with its corresponding decimal value

Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

U

Solution #1

X < 4 or X > 8.

X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1 0	4	12	8
01	1 1	5	13	9
11	1 3	7	15	11
10	1 2	6	14	10

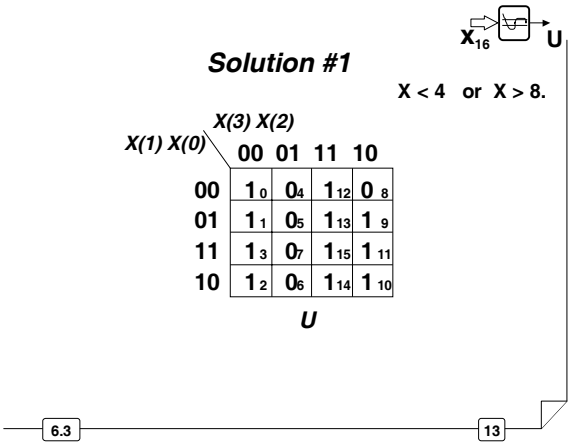
U

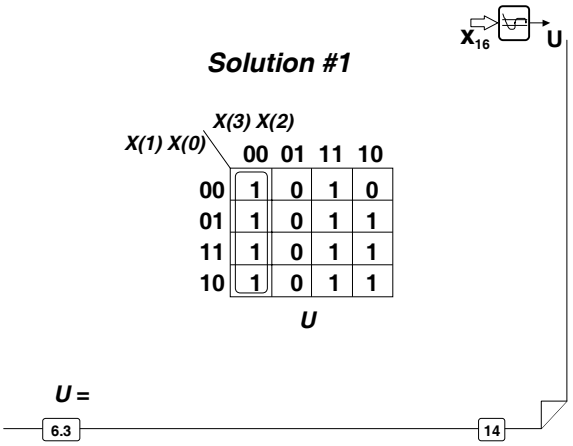
Solution #1

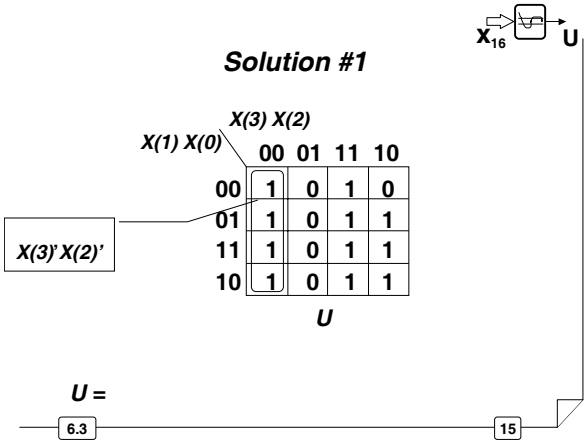
X < 4 or X > 8.

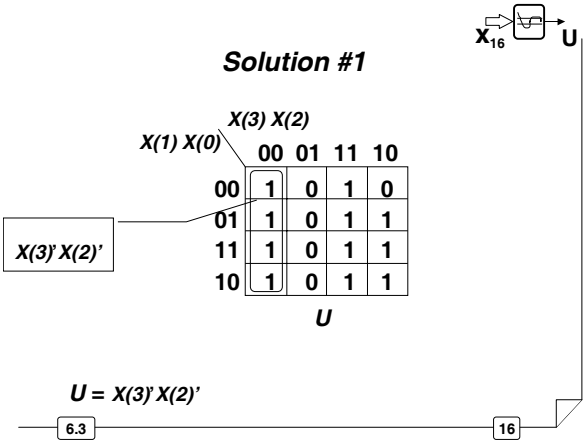
X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1 0	4	1 12	8
01	1 1	5	1 13	1 9
11	1 3	7	1 15	1 11
10	1 2	6	1 14	1 10

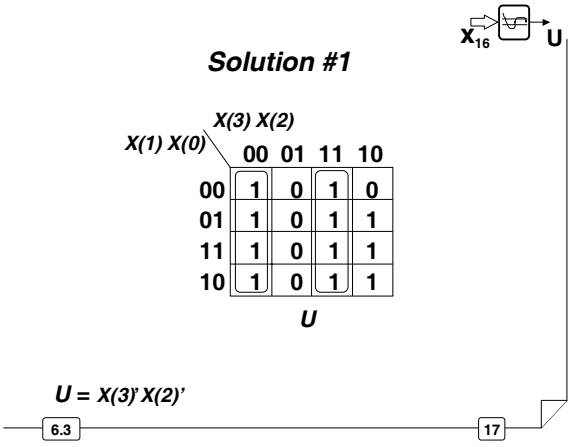
U

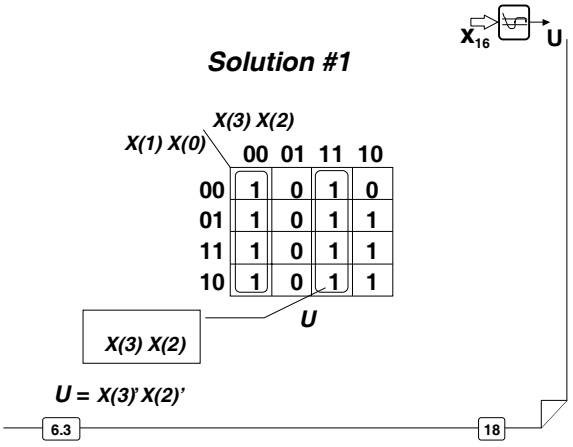


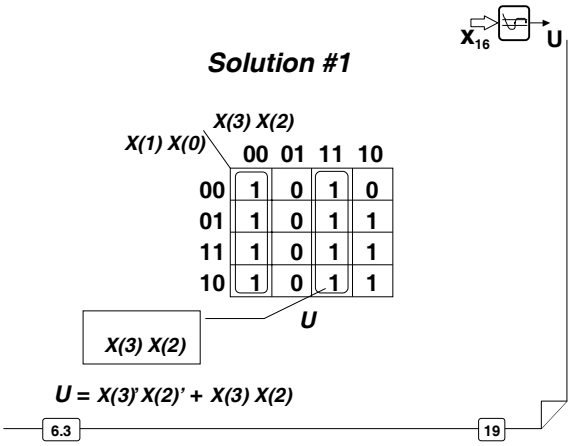


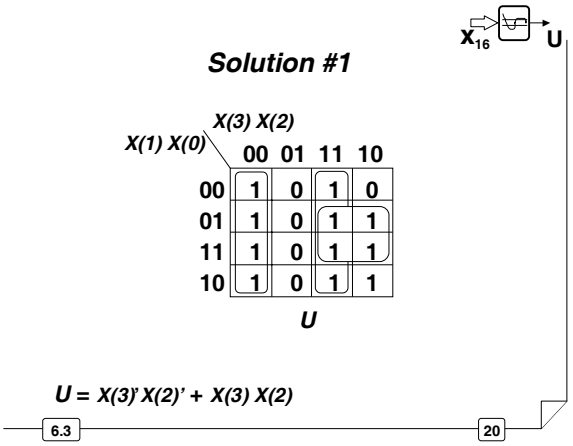


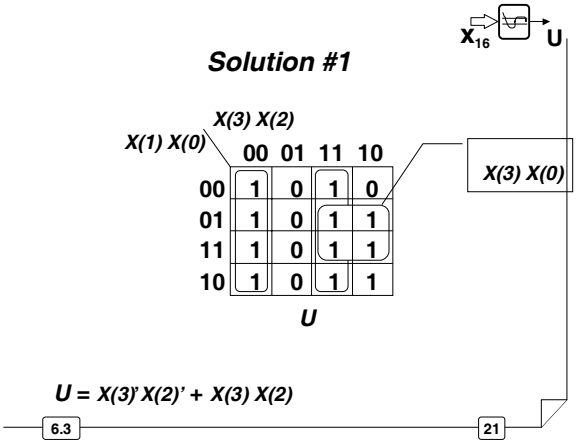


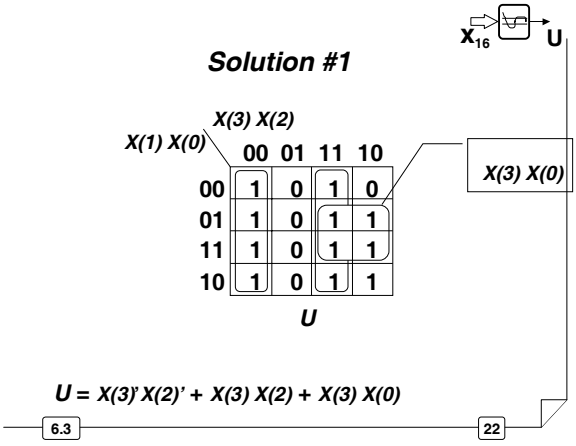


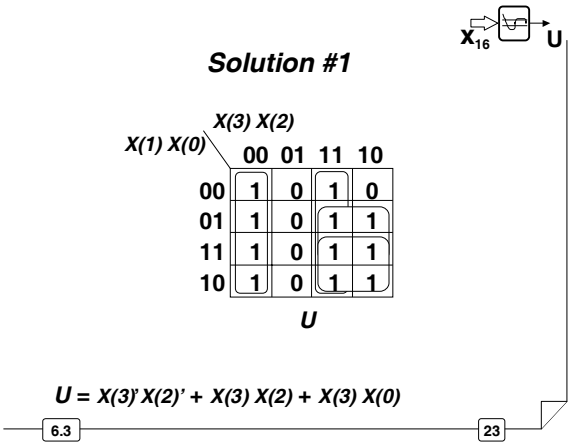


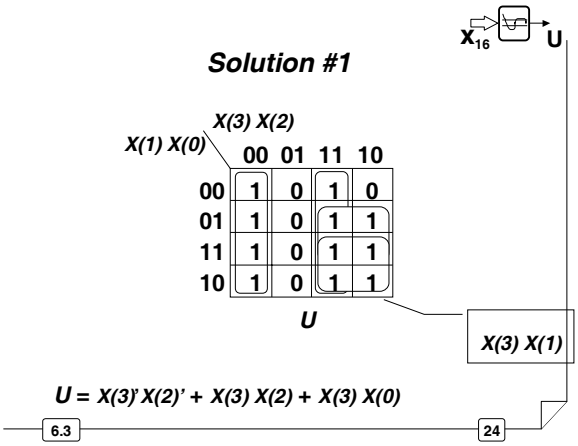




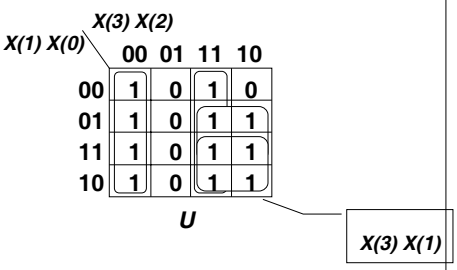








Solution #1



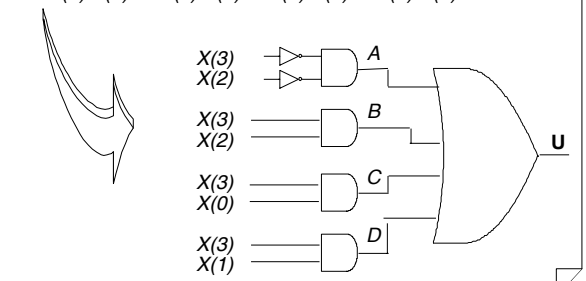
$U = X(3)'X(2)' + X(3)\,X(2) + X(3)\,X(0) + X(3)\,X(1)$

Solution #1

$U = X(3)'X(2)' + X(3)\,X(2) + X(3)\,X(0) + X(3)\,X(1)$

Solution #1

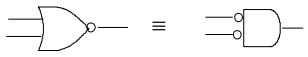
$U = X(3)'X(2)' + X(3)\,X(2) + X(3)\,X(0) + X(3)\,X(1)$



Solution #1

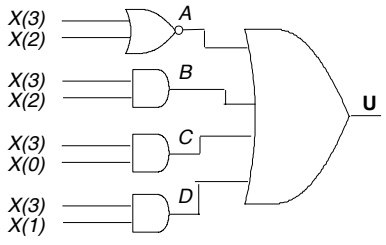
Further optimization:

- **by observing that:**



- **one gets:**

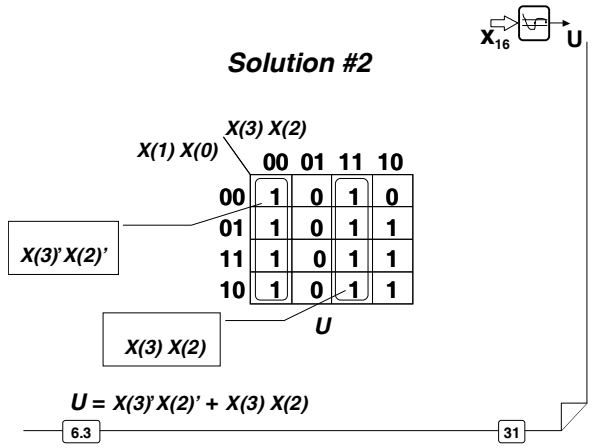
Solution #1



Solution #2

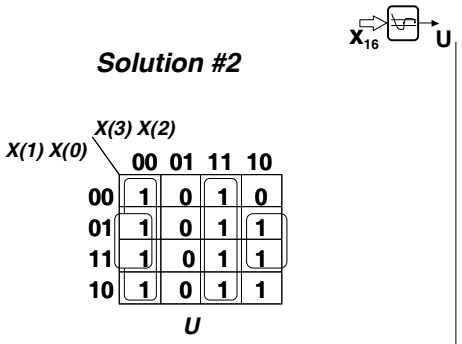
- **We are going to present now an alternative solution, due to a different cover of the K-map of the circuit.**

Solution #2



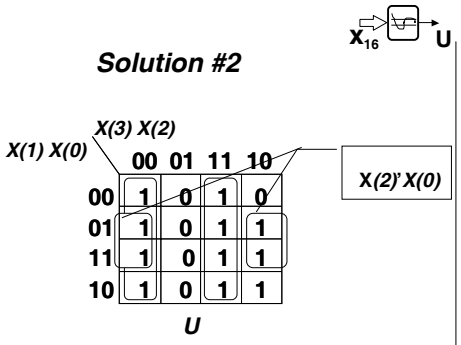
$U = X(3)'X(2)' + X(3)\,X(2)$

Solution #2



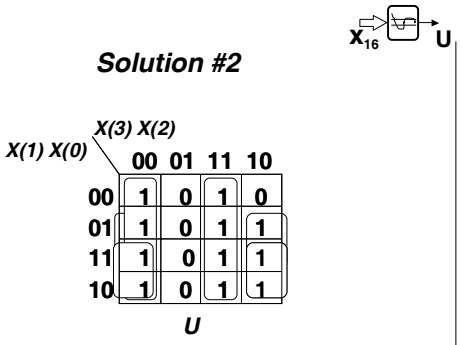
$U = X(3)'X(2)' + X(3)\,X(2)$

Solution #2



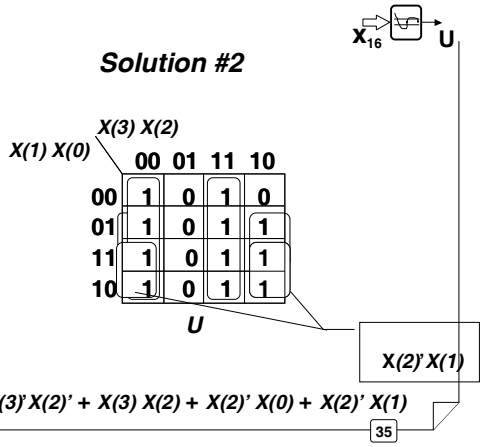
$U = X(3)'X(2)' + X(3)\,X(2) + X(2)'X(0)$

Solution #2



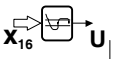
$U = X(3)'X(2)' + X(3)\,X(2) + X(2)'X(0)$

Solution #2



$U = X(3)'X(2)' + X(3)\,X(2) + X(2)'X(0) + X(2)'X(1)$

Solution #3



We shall now present a solution based on the *map composition* approach.

X(3) X(2) X(1) X(0) ≡ abcd

Solution #3

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	1	0	1	1
10	1	0	1	1

f

=

f =

Solution #3

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	1	0	1	1
10	1	0	1	1

f

=

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	1	0	1	0
10	1	0	1	0

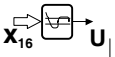
E

∨

cd \ ab	00	01	11	10
00	-	0	-	0
01	-	0	-	1
11	-	0	-	1
10	-	0	-	1

R

f = E + R



Solution #3

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	1	0	1	1
10	1	0	1	1

f

=

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	1	0	1	0
10	1	0	1	0

E

∨

cd \ ab	00	01	11	10
00	-	0	-	0
01	-	0	-	1
11	-	0	-	1
10	-	0	-	1

R

E = a'b' + ab = (a ⊕ b)'

R = ad + ac

f = E + R

Solution #3

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	1	0	1	1
10	1	0	1	1

f

=

cd \ ab	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	1	0	1	0
10	1	0	1	0

E

∨

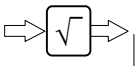
cd \ ab	00	01	11	10
00	-	0	-	0
01	-	0	-	1
11	-	0	-	1
10	-	0	-	1

R

E = a'b' + ab = (a ⊕ b)'

R = ad + ac

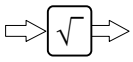
f = E + R = a'b' + ab + ad + ac



Exercise #6.3.2

Design a combinational circuit with a 4-bit input and a 3-bit output such that, when it receives on its input a *hexadecimal* digit X, provides, on its output:

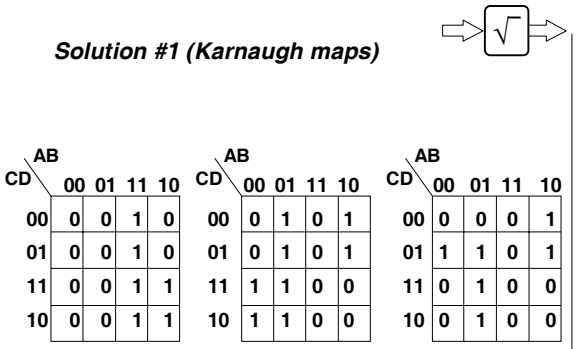
⌈ sqrt (X) ⌉



Solution #1 (truth table)

A B C D	⌈ sqrt (X) ⌉
0 0 0 0	0 0 0
0 0 0 1	0 0 1
0 0 1 0	0 1 0
0 0 1 1	0 1 0
0 1 0 0	0 1 0
0 1 0 1	0 1 1
0 1 1 0	0 1 1
0 1 1 1	0 1 1
1 0 0 0	0 1 1
1 0 0 1	0 1 1
1 0 1 0	1 0 0
1 0 1 1	1 0 0
1 1 0 0	1 0 0
1 1 0 1	1 0 0
1 1 1 0	1 0 0
1 1 1 1	1 0 0

Solution #1 (Karnaugh maps)

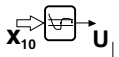


CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	1
10	0	0	1	1

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	0

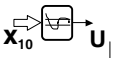
CD \ AB	00	01	11	10
00	0	0	0	1
01	1	1	0	1
11	0	1	0	0
10	0	1	0	0

⌈ sqrt (X) ⌉



Exercise #6.3.3

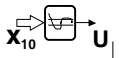
Similar to the example #1, with the only difference that the input digit X is, in this case, a *decimal* one.



Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

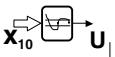
U



Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1 0	0 4	- 12	0 8
01	1 1	0 5	- 13	1 9
11	1 3	0 7	- 15	- 11
10	1 2	0 6	- 14	- 10

U



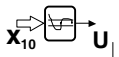
Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1	0	-	0
01	1	0	-	1
11	1	0	-	-
10	1	0	-	-

X(3)' X(2)'

U

$U = X(3)' X(2)'$



Solution #1

X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1	0	-	0
01	1	0	-	1
11	1	0	-	-
10	1	0	-	-

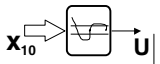
X(3)' X(2)'

X(3) X(0)

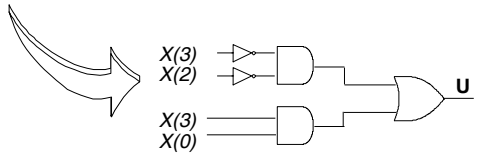
U

$U = X(3)' X(2)' + X(3) X(0)$

Solution #1



$U = X(3)' X(2)' + X(3)\ X(0)$



Solution #2

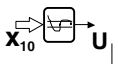
X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1	0	-	0
01	1	0	-	1
11	1	0	-	-
10	1	0	-	-

$X(3)' X(2)'$

U

$U = X(3)' X(2)'$

Solution #2



X(1) X(0) \ X(3) X(2)	00	01	11	10
00	1	0	-	0
01	1	0	-	1
11	1	0	-	-
10	1	0	-	-

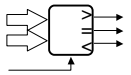
$X(3)' X(2)'$

$X(2)' X(0)$

U

$U = X(3)' X(2)' + X(2)' X(0)$

Exercise #6.3.4



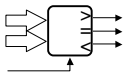
Design a comparator which, receiving in input:

- **`A(1 downto 0)` and `B(1 downto 0)`**
- **a signal `2C/~UM` specifying the coding used for `A` and `B` (1=*2's complement,* 0=*unsigned magnitude*)**

provides 3 outputs `A_GT_B`, `A_EQ_B` and `A_LT_B` such that:

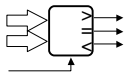
- **`A_GT_B` = 1 iff `A > B`**
- **`A_EQ_B` = 1 iff `A = B`**
- **`A_LT_B` = 1 iff `A < B`.**

Solution #1



B(1)B(0) \ A(1)A(0)	2C/~UM =0				2C/~UM =1			
	00	01	11	10	00	01	11	10
00								
01								
11								
10								

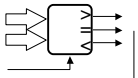
Solution #1



B_1B_0 \ A_1A_0	2C/~UM =0				2C/~UM =1			
	00	01	11	10	00	01	11	10
00	0	1	1	1	0	1	0	0
01	0	0	1	1	0	0	0	0
11	0	0	0	0	1	1	0	0
10	0	0	1	0	1	1	1	0

A_gt_B

Solution #1

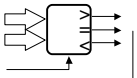


The implementation of `A_eq_B` is trivial:

B_1B_0 \ A_1A_0	2C/~UM =0				2C/~UM =1			
	00	01	11	10	00	01	11	10
00	1	0	0	0	1	0	0	0
01	0	1	0	0	0	1	0	0
11	0	0	1	0	0	0	1	0
10	0	0	0	1	0	0	0	1

`A_eq_B`

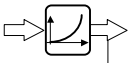
Solution #1



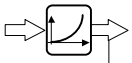
The 3rd output `A_lt_B` can be easily obtained as:

```
A_lt_B = (A_gt_B)'(A_eq_B)'
```

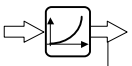

Exercise #6.3.5



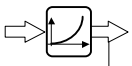
Design a combinational circuit with an input X (2 downto 0) and an output Y (2 downto 0) such that, when it receives on X an unsigned *decimal* digit X, provides, on Y, the 3 most significant bits of X^2.



$X_{(2)}$	$X_{(10)}$	$X^2_{(10)}$	$X^2_{(2)}$	Y
000				
001				
010				
011				
100				
101				
110				
111				



$X_{(2)}$	$X_{(10)}$	$X^2_{(10)}$	$X^2_{(2)}$	Y
000	0			
001	1			
010	2			
011	3			
100	4			
101	5			
110	6			
111	7			



$X_{(2)}$	$X_{(10)}$	$X^2_{(10)}$	$X^2_{(2)}$	Y
000	0	0		
001	1	1		
010	2	4		
011	3	9		
100	4	16		
101	5	25		
110	6	36		
111	7	49		

$X_{(2)}$	$X_{(10)}$	$X^2_{(10)}$	$X^2_{(2)}$	Y
000	0	0	000000	
001	1	1	000001	
010	2	4	000100	
011	3	9	001001	
100	4	16	010000	
101	5	25	011001	
110	6	36	100100	
111	7	49	110001	

6.3 61

$X_{(2)}$	$X_{(10)}$	$X^2_{(10)}$	$X^2_{(2)}$	Y
000	0	0	000000	000
001	1	1	000001	000
010	2	4	000100	000
011	3	9	001001	001
100	4	16	010000	010
101	5	25	011001	011
110	6	36	100100	100
111	7	49	110001	110

6.3 62

X	Y
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

6.3 63

$X_2X_1X_0$	$Y_2Y_1Y_0$
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

X_2 \ X_1X_0	00	01	11	10
0				
1				

Y_2

6.3 64

$X_2X_1X_0$	$Y_2Y_1Y_0$
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	0	0	1	1

Y_2

6.3 65

$X_2X_1X_0$	$Y_2Y_1Y_0$
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

X_2 \ X_1X_0	00	01	11	10
0				
1				

Y_1

6.3 66

$X_2X_1X_0$	$Y_2Y_1Y_0$
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Y_1

6.3 67

$X_2X_1X_0$	$Y_2Y_1Y_0$
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

X_2 \ X_1X_0	00	01	11	10
0				
1				

Y_0

6.3 68

$X_2X_1X_0$	$Y_2Y_1Y_0$
000	000
001	000
010	000
011	001
100	010
101	011
110	100
111	110

X_2 \ X_1X_0	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Y_0

6.3 69

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	0	0	1	1

Y_2

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Y_1

X_2 \ X_1X_0	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Y_0

6.3 70

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	0	0	1	1

Y_2

X_2 \ X_1X_0	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Y_1

X_2 \ X_1X_0	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Y_0

$Y_2 = X_1X_2$

$Y_1 = X_1'X_2 + X_0X_2$

$Y_0 = X_0X_1X_2' + X_0X_1'X_2$

6.3 71

Remark

A remark on Y_0.

The expression an be factorized as it follows:

$$Y_0 = X_0X_1X_2' + X_0X_1'X_2 = X_0(X_1X_2' + X_1'X_2) = X_0(X_1 \oplus X_2)$$

6.3 72

Remark (cont'ed)

The same expression could be easily obtained by a proper cover of the Y_0 K-map:

Remark (cont'ed)

X_2 \ $X_1 X_0$	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Y_0

Remark (cont'ed)

X_2 \ $X_1 X_0$	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Y_0 =

X_2 \ $X_1 X_0$	00	01	11	10
0	1	0	1	0
1	0	1	0	1

A ∧

X_2 \ $X_1 X_0$	00	01	11	10
0	0	1	1	0
1	0	1	1	0

B

Remark (cont'ed)

X_2 \ $X_1 X_0$	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Y_0 =

X_2 \ $X_1 X_0$	00	01	11	10
0	1	0	1	0
1	0	1	0	1

A ∧

X_2 \ $X_1 X_0$	00	01	11	10
0	0	1	1	0
1	0	1	1	0

B

$$Y_0 = A\,B = (X_0 \oplus X_1 \oplus X_2)\,X_0 = (X_1 \oplus X_2)\,X_0$$