

**EECS 417 - Digital Signal Processing I**  
**Spring 2001 Semester**  
**Final Examination Answers**

1. The sequence  $h(n) = (0.5)^n u(n-1)$  may also be written as:

$$h(n) = (0.5)^n u(n) - \delta(n)$$

2. Which of the following may correspond to the Z-Transform of a two sided sequence  $h(n)$ ? (ROC not specified)

$$H(z) = \frac{(z-1)}{(z+\frac{1}{2})(z+2)}$$

3. Given:  $H(z) = \frac{z^4 - 1}{(z + \frac{1}{2})(z - \frac{1}{2})(z - 2)}$  (ROC not specified)

How many different sequences  $h(n)$  may have this  $H(z)$ , only each with a different region of convergence?

3

4. Given:  $H(z) = \frac{2}{(2z-1)(z+2)}$

Specify the ROC when  $h(n)$  is anticausal:

$$|z| < \frac{1}{2}$$

5. If  $x(n)$  is right-sided, then:

$X(z)$  may or may not have pole(s) at infinity

6. Given:  $H(z) = \frac{z^2 - 0.90}{z^2 - 0.81}$

What is the ROC when  $h(n)$  is causal?

$$|z| > 0.90$$

7. What is the region of convergence of  $H(z) = z - z^{-1}$ ?

$$0 < |z| < \infty$$

8. Find the inverse Z-Transform of:

$$\frac{z}{z - \frac{1}{2}} - \frac{z}{z + \frac{3}{2}}, \quad \text{ROC: } \frac{1}{2} < |z| < \frac{3}{2}$$

$$\left(\frac{1}{2}\right)^n u(n) + \left(-\frac{3}{2}\right)^n u(-n-1)$$

9. Find the inverse Z-Transform of:

$$\frac{2z^2 + 5z}{z^2 + 5z + 6}, \quad \text{ROC: } |z| < 2$$

$$-(-2)^n u(-n-1) - (-3)^n u(-n-1)$$

10. Given:  $H(z) = \frac{z^2 - 1}{z^2 - 2z}$

What is the difference equation that realizes  $H(z)$  as a causal filter?

$$y(n) = 2y(n-1) + x(n) - x(n-2)$$

11. Given:  $y(n] = \frac{1}{4}x(n+1) + \frac{1}{2}x(n) + \frac{1}{4}x(n-1)$

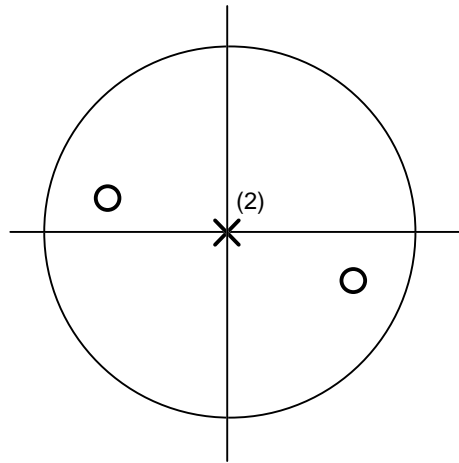
The digital filter implemented with this difference equation is:  
FIR

12. Given:  $H(z) = \frac{4 + z^2}{1 + 4z^2}$

We proceed to find right-sided  $h(n)$  by the method of long division.  
As a result:

$$h(2) = \frac{15}{16}$$

13. Which one of the following pole/zero plots correctly describes  $H(z)$ , when it is known that it is a stable and causal minimum-phase system?



14. Which of the following is the impulse response of an FIR filter?

$$h(n) = 0.5^n (u(n+2) - u(n-2))$$

15. Which of the following system functions exhibits linear phase?

$$H(z) = 2 + 2z^{-1}$$

16. Estimate the magnitude frequency response of

$$H(z) = \frac{4z^2 - 4}{4z^2 - 1} \quad \text{at } \omega = -\frac{\pi}{2}:$$

$$\frac{32}{5}$$

17. Estimate the phase of  $H(z) = -10\left(\frac{z+1}{z}\right)$  at  $\omega = \frac{\pi}{2}$ :

$$\frac{3\pi}{4} \text{ radians}$$

18. Given denominator polynomial  $D(z) = z^4 + z - 1$ . What numerator polynomial will yield an allpass filter response with magnitude gain = 2 at all  $\omega$ ?

$$N(z) = 2z^4 - 2z^3 - 2$$

19. Given:  $H(z) = \frac{z^5 + 2}{z^4 - \frac{1}{2}}$ . What is this filter's d.c. gain value?

6 *(this answer was not one of the choices; problem not graded)*

20. Given:  $x_c(t)$  is a continuous-time sinusoid at 15 KHz frequency. After sampling this signal to yield sequence  $x(n) = x_c(nT_s)$ , we find there are impulses at  $\omega = \pm\frac{\pi}{2}$  in  $X(e^{j\omega})$ . Which of the following was the sampling frequency if we know that aliasing had not occurred?

$$f_s = 60 \text{ KHz}$$

21. In Question #20 above, which of the following may have been the sampling frequency if we know that aliasing had occurred?

$$f_s = 20 \text{ KHz}$$

22. Given:  $h(n) = \delta(n) + \delta(n-1)$ . What is the frequency response of a digital filter having this impulse response?

$$H'(\omega) = 1 + e^{-j\omega}$$

23. A 1000-pt. sequence is to be convolved with a 100-pt. sequence using the FFT method. Of the following, which FFT size is the smallest that may be used for this purpose?

$$N = 2048$$

24. A four point sequence is operated on by the Discrete Fourier Transform to yield:

$$DFT(x(n)) = X(k) = [ 1, 2, 3, 4 ]$$

Let  $y(n) = x(n)e^{j(3n\pi/2)}$ ,  $0 \leq n \leq 3$ . What is  $DFT(y(n))$ ?

$$[ 2, 3, 4, 1 ]$$

25. Given:  $h(n) = -\delta(n) + 2\delta(n-1) - \delta(n-2)$

Which type of linear-phase FIR filter is described by this impulse response sequence?

Type I

26. We wish to implement a notch filter that has unity gain at frequencies  $\omega = 0, \pm\pi$  and zero gain at frequencies  $\omega = \pm\frac{\pi}{2}$ . Which type of linear-phase FIR filter is best suited for this purpose?

Type I

27. How much delay is introduced by the digital filter having impulse response  $h(n) = \frac{1}{4}(\delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3))$ ?

$\frac{3}{2}$  samples

28. When using the bilinear transformation, where does  $s = 0$  map to in the  $z$ -plane?

$z = 1$

29. When using the bilinear transformation with constant  $T = 2$ , where does the frequency  $\Omega = 1$  map to on the  $\omega$  scale?

$\omega = \frac{\pi}{2}$

30. When using the impulse invariant transformation with sampling frequency  $f_s = 2$  Hz, where does a pole at  $s = j2\pi$  map to in the  $z$ -plane?

$z = -1$

31. The impulse invariant transformation, matched- $z$  transform and bilinear transformation methods of digital filter design all have the following in common:

they map poles from the  $s$ -plane to the  $z$ -plane

32. When designing a linear-phase causal FIR lowpass filter by Hamming-windowing a delayed sinc sequence, we are told that the maximum tolerable delay of this filter is  $1 \times 10^{-2}$  sec. What is the narrowest transition bandwidth possible in Hz (the time between samples =  $2 \times 10^{-5}$  sec)?

165 Hz

33. When designing an FIR lowpass filter by windowing  $\text{sinc}(\omega_c n)$ , and striving to obtain at least 40 dB stopband attenuation with the narrowest transition band possible, which of the following windows should be used?

Hanning

34. Which of the following window functions should be used to window  $\text{sinc}(\omega_c n)$  when designing a lowpass filter with at least 20 dB stopband attenuation, at most  $\pm 0.01$  dB passband ripple, and the narrowest transition band possible?

Blackman

35. IIR digital filters are advantageous over FIR digital filters in the respect that:

they typically require lower filter orders for the same magnitude frequency response specifications