

# **SAMPLE SIZE AND POWER**

## **Formulation of the Problem**

**The problem is to determine the number of subjects required in order to be  $100 \times (1-\beta)\%$  of identifying an effect size equal to  $R$  or greater at the  $100 \times (1-\alpha) \%$  level of statistical significance.**

**The effect size  $R$ , depends on the particular study design and epidemiologic issue. It could be a difference between two means, a difference between two proportions, and odds ratio, a relative risk, etc.**

**The entity,  $100 \times (1-\beta)\%$ , represents the degree of certainty that the study will identify the risk factor, if it is as large or larger than  $R$  (Generally,  $100 \times (1-\beta)\%$  is set at 80% or 90%)**

**The entity,  $100 \times (1-\alpha)\%$ , represents the desired level of significance (Generally,  $100 \times (1-\alpha)\%$  set at 95%)**

## SAMPLE SIZE ESTIMATION FOR CASE CONTROL STUDIES

- The sample size issue for unmatched case control studies having dichotomous exposures, dichotomous disease status, and no stratification is as follows: One needs to determine the number of cases and controls necessary to have  $100 - (1 - b)\%$  confidence of identifying a Relative Risk (exposure odds ratio) of  $R$  or larger at the  $100 - (1 - a)\%$  level of statistical significance if the rate of exposure in the controls is equal to  $p_0$ .
- It can be shown that the exposure rate,  $p_1$  among cases would then be equal to:

$$p_1 = \frac{p_0 R}{[1 + p_0 (R - 1)]}$$

- With this in mind, we have the following equation for the necessary cases and controls (corresponds to PEPI results without continuity correction factor)

$$n = \frac{\left( z_a \sqrt{2\bar{p}\bar{q}} + z_b \sqrt{p_1 q_1 + p_0 q_0} \right)^2}{(p_1 - p_0)^2}$$

where

$$\bar{p} = \frac{p_1 + p_0}{2}$$

$$\bar{q} = 1 - \bar{p}$$

$$q_1 = 1 - p_1$$

$$q_0 = 1 - p_0$$

**For 95% Significance, two tailed use  $z_a=1.96$**

**For 80% Power, use  $z_b=.84$**

## **Example of Sample Size Determination**

**Suppose you want to conduct a case control study to evaluate the nature of the putative association between exposure to cellular phones and incidence of acoustic neuroma (a benign but often troublesome tumor on the auditory nerve). The study should be designed in such a way that one would be 80% certain of estimating a relative risk (or odds ratio) that is at least twofold in magnitude, at a significance level of 95% (using a two tailed test).**

**The cases will be chosen from a registry of acoustic neuroma, and the controls will be selected by random digit dialing. The number of cases and controls will be the same.**

**It is projected that 10% of the population will be exposed to cellular phones.**

**FOR THIS EXAMPLE:**

$$p_0 = .10$$

$$p_1 = \frac{.10 \times 2}{1 + (.10 \times 1)} = .1818$$

$$\bar{p} = \frac{.1818 + .10}{2} = .1409$$

$$\bar{q} = .8591$$

$$q_1 = .8182$$

$$q_0 = .90$$

$$n = \frac{\left[ 1.96 \sqrt{2 \times .1409 \times .8591} + .84 \sqrt{(.1818 \times .8182) + (.10 \times .9)} \right]^2}{(.1818 - .10)^2} =$$

$$\therefore \frac{1.8902}{.0067} = 283$$



**NOW TRY ORIGINAL DATA, BUT USE OR of 3**

AA

Significance level (two-tailed test) = 0.05  
Power = 80%  
Ratio of sample sizes (B:A) = 1  
Proportion in population B = 0.10  
Odds ratio of A to B = 3

AA

**REQUIRED SAMPLE SIZE:**

Total 200: 100 in group A ("cases"), 100 in B ("controls")  
or (with continuity correction):  
Total 226: 113 in group A ("cases"), 113 in B ("controls")

**EXPECTED PRECISION of observed difference between proportions:**  
Approximate 95% C.I. = observed difference plus or minus 0.106

**NOW TRY OR OF 1.5**

AA

Significance level (two-tailed test) = 0.05  
Power = 80%  
Ratio of sample sizes (B:A) = 1  
  
Proportion in population B = 0.10  
Odds ratio of A to B = 1.5

AA

**REQUIRED SAMPLE SIZE:**

Total 1,822: 911 in group A ("cases"), 911 in B ("controls")  
or (with continuity correction):  
Total 1,914: 957 in group A ("cases"), 957 in B ("controls")

**EXPECTED PRECISION of observed difference between proportions:**  
Approximate 95% C.I. = observed difference plus or minus 0.030

**NOW TRY OR OF 1.5 BUT WITH TWO CONTROLS PER CASE**

AA

Significance level (two-tailed test) = 0.05  
Power = 80%  
Ratio of sample sizes (B:A) = 2  
Proportion in population B = 0.10  
Odds ratio of A to B = 1.5

AA

**REQUIRED SAMPLE SIZE:**

Total 2,007: 669 in group A ("cases"), 1,338 in B ("controls")  
or (with continuity correction):  
Total 2,112: 704 in group A ("cases"), 1,408 in B ("controls")

**EXPECTED PRECISION of observed difference between proportions:**  
Approximate 95% C.I. = observed difference plus or minus 0.030

## Formulas for Multiple Controls per Case

**c=# of controls per case**

$$n = \frac{\left[ z_a \sqrt{\left(1 + \frac{1}{c}\right) \bar{p}' \bar{q}'} + z_b \sqrt{p_1 q_1 + \frac{p_0 q_0}{c}} \right]^2}{(p_1 - p_0)^2}$$

**where**

$$\bar{p}' = \frac{(p_1 + cp_0)}{1 + c}$$

$$\bar{q}' = 1 - \bar{p}'$$

$$p_1 = \frac{p_0 R}{[1 + p_0(R-1)]}$$

**n=# of cases needed**

**c x n=# of controls needed**

## **EFFECT OF CONFOUNDING ON REQUIRED SAMPLE SIZES**

**Presence of confounding variables will require a larger sample size. Since the confounding variables must be controlled for in the analysis, a more complex statistical model must be fit, thus requiring a larger sample to achieve the stated specifications.**

## **CALCULATING STUDY POWER**

**If financial constraints limit the study size (i.e. the number of cases and controls is fixed), can determine study power using varying values of R or  $z_{\alpha}$**

**Change values in SAMPLES—i.e. change significance level, ratio of controls to cases, Odds Ratio you want to detect, etc**

**Can also use POWER to determine if in a study already completed, what the power of your study was to detect different levels of R at a fixed significance level.**

**If you found a non-significant elevated OR or RR, you can determine if your study had adequate POWER to detect such a risk. Failure to reject the Null could be due to low power (i.e. inadequate sample size)**

**FORMULA FOR CALCULATING POWER—  
Equal number of cases, controls  
From Schlesselman**

$$z_b = \frac{\left[ \sqrt{n(p_1 - p_0)^2} - z_a \sqrt{2\bar{p}\bar{q}} \right]}{\sqrt{p_1q_1 + p_0q_0}}$$

$p_0$  = **proportion exposed in controls (use population estimate)**

$$p_1 = \frac{p_0 R}{1 + p_0(R - 1)}$$

$$\bar{p} = \frac{p_1 + p_0}{2}$$

$$\bar{q} = 1 - \bar{p}$$

$$q_1 = 1 - p_1$$

$$q_0 = 1 - p_0$$

**n=number of cases, controls in each group**

**Find power corresponding to  $1 - Z_B$**

**EXAMPLE FROM SCHLESSELMAN, p.149:**

**For study with  $n=50$ ,  $p_0=30$  and  $\alpha=.05$  (two sided), what is power to detect a twofold increased risk?**

**See p. 149**

**Power= 38%**

## **POWR MODULE IN PEPI**

**Use Option 1 for Independent samples (i.e. not matched)—this is for case control studies and cohort studies with exposed and non-exposed groups**

**Enter level of significance desired**

**Choose Option A to enter size of sample A and ratio of B to A  
(For case control studies, A is number of cases, B is ratio of controls to cases)**

**(For cohort studies, A is size of exposed group, B is ratio of exposed to unexposed)**

**Enter size of sample A and ratio**

**Enter proportion in Sample B—**

**(For case control studies, proportion is proportion exposed in controls)**

**(For cohort studies, proportion is proportion of non-exposed with disease)**

**Then choose option A for cohort studies (rate ratio)**

**Option B for case control studies (odds ratio)**

**And enter effect that you want to detect (e.g. Odds ratio of 1.5)**







## **EFFECT OF VARYING SPECIFICATIONS**

**1. All else being equal, it will take a larger (smaller) sample to detect a smaller (larger) effect size.**

**Number, n, of cases and controls in each group required for various values of Odds Ratio, R, for  $\alpha=.10$ ,  $\beta=.20$ , and  $p_0=.10$  (for equal number of cases, controls)**

<b>Odds Ratio, R</b>	<b>Number, n, of cases and controls</b>
<b>1.20</b>	<b>3850</b>
<b>1.50</b>	<b>718</b>
<b>1.70</b>	<b>401</b>
<b>2.00</b>	<b>223</b>
<b>2.50</b>	<b>119</b>
<b>3.00</b>	<b>79</b>
<b>4.00</b>	<b>46</b>
<b>5.00</b>	<b>32</b>
<b>10.00</b>	<b>14</b>

**2. All else being equal, the greater the background exposure prevalence or prevalence of non-exposure, the smaller the sample size required**

**Odds ratio=2,  $\alpha=.10$ ,  $\beta=.20$**

**Background prevalence      Number, n, of cases and controls**

<b>.01</b>	<b>1889</b>
<b>.05</b>	<b>406</b>
<b>.10</b>	<b>223</b>
<b>.20</b>	<b>135</b>
<b>.30</b>	<b>111</b>
<b>.40</b>	<b>105</b>
<b>.50</b>	<b>108</b>
<b>.60</b>	<b>120</b>
<b>.70</b>	<b>147</b>

## **Effect of Other Parameters on Required Sample Size**

**Raising the Confidence Level Parameter  $100 \times (1-\beta)\%$  will increase the sample size required (the more power required, the larger the sample size required)**

**Lowering the significance level,  $\alpha$ , (e.g., from .05 to .01) will cause an increase in the necessary sample size.**

**If number of cases is limited (i.e. not a common disease) can increase number of controls per case to increase power. Incremental gain after 4 controls per case however is limited.**

**EFFECT OF NUMBER OF CONTROLS PER CASE ON  
THE RELIABILITY OF THE RESULTING ESTIMATES**

<b>Controls Per Case</b>	<b>Reliability of Resulting Odds Ratio (Relative to One Control Per Case)</b>	<b>Incremental Gain</b>
<b>1</b>	<b>1.00</b>	<b>-</b>
<b>2</b>	<b>1.33</b>	<b>33%</b>
<b>3</b>	<b>1.50</b>	<b>17%</b>
<b>4</b>	<b>1.60</b>	<b>10%</b>
<b>5</b>	<b>1.67</b>	<b>7%</b>
<b>6</b>	<b>1.71</b>	<b>4%</b>
<b>7</b>	<b>1.76</b>	<b>3%</b>

**VERY SMALL INCREMENTAL GAIN AFTER 4  
CONTROLS PER CASE**

**EFFECT OF NUMBER OF CONTROLS PER CASE ON  
POWER—USING  $\alpha=.10$ ,  $\beta=.20$ , Odds Ratio=2**

<b>Controls Per Case</b>	<b>POWER</b>
<b>1</b>	<b>76.2</b>
<b>2</b>	<b>86.5</b>
<b>3</b>	<b>89.9</b>
<b>4</b>	<b>91.5</b>
<b>5</b>	<b>92.4</b>
<b>6</b>	<b>93.0</b>
<b>7</b>	<b>93.4</b>

## **SAMPLE SIZE FOR MATCHED STUDIES**

**To determine number of total pairs required, need to know probability of an exposure-discordant pair.**

**Can estimate using estimated proportion of exposed controls in target population, and estimated proportion of exposed cases.**

**Need to estimate similarity of findings based on matching –i.e. what is the extent to which matching will make the findings in the two samples similar, necessitating larger sample sizes (i.e. if there are a lot of concordant pairs, you will need a larger number of matched pairs in order to get a sufficient number of discordant pairs). The matching factor is seldom more than 2.5.**

**Can use SAMPLES in PEPI—USE OPTION 2 for MATCHED STUDIES**



## **SAMPLE SIZE FOR COHORT STUDIES**

**Difference in disease proportions between two groups**

**See handout for formulas**

**FOR PEPI SAMPLES:**

**Need proportion with disease in non-exposed,  
proportion exposed, and ratio of exposed group to non-  
exposed**

**USE option 1**

**Enter Significance level, power, proportion of disease in  
unexposed group, ratio of exposed to unexposed**

**Enter 1 when prompted for difference to be detected  
(rate ratio)**

**Enter Relative Risk to be detected**



IN A COHORT STUDY, FOR FIXED NUMBER OF EXPOSED, CAN INCREASE POWER BY INCREASING RATIO OF UNEXPOSED TO EXPOSED

POWR - Power of a Test Comparing Two Samples  
Tuesday, 8th October 2002.

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 1
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 80.20% (no continuity correction)  
or 75.92% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 2
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 90.68% (no continuity correction)  
or 88.53% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 3
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 93.67% (no continuity correction)  
or 92.18% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 4
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 94.99% (no continuity correction)  
or 93.80% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 5
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 95.71% (no continuity correction)  
or 94.70% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 6
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 96.17% (no continuity correction)  
or 95.26% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Sample size, population A	= 200
Ratio of sample B to sample A	= 7
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 96.48% (no continuity correction)  
or 95.64% (with continuity correction)

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FOR FIXED TOTAL NUMBER OF EXPOSED AND  
UNEXPOSED, NO GAIN IN POWER BY INCREASING  
NUMBER OF UNEXPOSED, REDUCING NUMBER OF  
EXPOSED

POWR - Power of a Test Comparing Two Samples  
Tuesday, 8th October 2002.

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 1
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 88.16% (no continuity correction)  
or 85.40% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 2
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 85.29% (no continuity correction)  
or 82.06% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 3
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 80.17% (no continuity correction)  
or 76.03% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 4
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 74.98% (no continuity correction)  
or 69.94% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 5
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 70.20% (no continuity correction)  
or 64.36% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 6
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 65.92% (no continuity correction)  
or 59.40% (with continuity correction)

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Significance level (two-tailed test)	= 0.05
Combined sample size	= 500
Ratio of sample B to sample A	= 7
Proportion in population B	= 0.10
Rate ratio of A to B	= 2

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POWER = 62.14% (no continuity correction)  
or 55.03% (with continuity correction)

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