UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Problem Set #2

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Reading: Montgomery, Ch. 3

1. On page 86, Montgomery writes, "The *sample* mean and variance $\bar{x}$ and S^2 are unbiased estimators of the *population* mean and variance μ and σ^2, respectively. One should note, however, that the sample standard deviation S is *not* an unbiased estimator of the population standard deviation σ."
Why is S not an unbiased estimator of σ?

2. (Montgomery 3-6, 3-10, 3-24) A machine is used to fill containers with a liquid product. Fill volume can be assumed to be normally distributed. A random sample of 10 containers is selected:

12.03 oz	12.01 oz
12.04	12.02
12.05	11.98
11.96	12.02
12.05	11.99

(a) Construct a 95% two-sided confidence interval on the mean fill volume.
(b) Construct a 95% two-sided confidence interval on the variance of fill volume.
(c) Suppose the manufacturer wants to be "sure" that the mean net content exceeds 12.0 oz. What conclusions can be drawn from these data? (Use $\alpha = 0.01$.)

3. (Montgomery 3-40) Consider the hypotheses

$$H_0: \quad \mu = \mu_0$$
$$H_1: \quad \mu \neq \mu_0$$

where σ^2 is known. Derive a general expression for determining the sample size for detecting a true mean of $\mu_1 \neq \mu_0$ with probability $1 - \beta$ if the type I error is α.

4. (Montgomery 3-41. Yes, the answer is in the book. I want a coherent derivation.) Suppose we are testing the hypotheses

$$H_0: \quad \mu_1 = \mu_2$$
$$H_1: \quad \mu_1 \neq \mu_2$$

where ${\sigma_1}^2$ and ${\sigma_2}^2$ are known. Resources are limited, so we can only take $N = n_1 + n_2$ samples. How should we allocate these N observations between the two populations to obtain the most powerful test?

5. (Montgomery 3-46) Nonconformities occur in glass bottles according to a Poisson distribution. A random sample of 100 bottles contains a total of 11 nonconformities. Test the hypothesis that the mean occurence rate of nonconformities is $\lambda = 0.15$. Use $\alpha = 0.01$.