

UNIVERSITY OF ILLINOIS AT CHICAGO  
Mechanical Engineering

**IE 446**  
**Problem Set #4**

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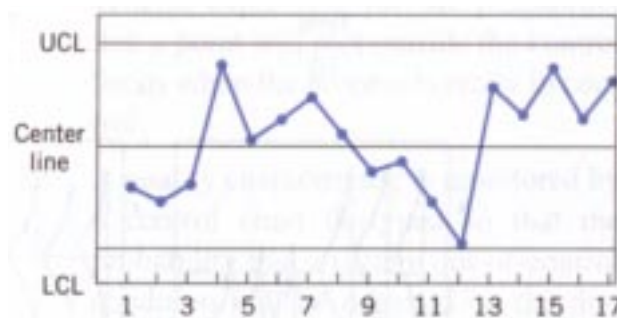
Issued: 2 Feb 2000  
Due: 9 Feb 2000

Reading: Montgomery, Ch. 4

1. Send email to [mjscott@uic.edu](mailto:mjscott@uic.edu) with your email address, so that I can assemble a mailing list for announcements. Note that I won't grade the rest of this homework if I haven't received an email from you.
2. Consider the Western Electric Rules for Shewhart control charts, which conclude that a process is out of statistical control if one of the following occurs:
  1. One point plots outside one of the  $3\text{-}\sigma$  control limits.
  2. Two out of three consecutive points plot beyond one of the  $2\text{-}\sigma$  warning limits.
  3. Four out of five consecutive points plot at or beyond one of the  $1\text{-}\sigma$  lines.
  4. Eight consecutive points plot on one side of the center line.

Answer the following questions, assuming that a process is normally distributed and is in statistical control, and that a sample of size  $n = 5$  is taken:

- (a) What is the probability that each of the four Western Electric Rules will be activated?
  - (b) What is the probability that at least one of the four will be activated? (Hint: You may approximate this answer, but tell whether, and why, your approximation is an over-estimate or an under-estimate of the true probability.)
3. Now assume that the process is out of statistical control, and that the true value of the sampled quantity is  $\sigma$  from the desired value. What is the probability that each of the four Western Electric Rules will be activated?
  4. (Montgomery 4-18)  
Consider the control chart below:



Does the pattern appear random? Are any of the Western Electric criteria for declaring the process out of control satisfied?

5. (Montgomery 4-11)

A manufacturing process produces 500 parts per hour. A sample part is selected every half-hour, and after five parts are obtained, the average of these five measurements is plotted on an  $\bar{x}$  control chart.

- (a) Is this an appropriate sampling scheme to detect an instantaneous upward shift in the mean that is of very short duration?
- (b) If your answer is no, propose an alternative procedure.  
(NOTE: no collaboration on this part of the problem!)

6. (Montgomery 4-16, more or less)

Is the average run length performance of a control chart a more meaningful measure of performance than the type I and type II error probabilities? What information is conveyed by each that is not conveyed by the other?