UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Midterm #1

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1. The P-value for a hypothesis test is the smallest value of α for which the null hypothesis is rejected; it is thus the breakpoint between accepting and rejecting the null hypothesis. In general, when we reject the null hypothesis, we discard that lot. The P-value tells us how much of a risk we run of discarding a good lot in order to use the test to discard a bad lot.

2. There are two main reasons why the normal distribution is used so often in quality control. One is that many processes are well approximated by a normal distribution. The other is the Central Limit Theorem, which states that the sum of a number of random variables approaches being normally distributed, even if the random variables themselves are not. Therefore, many sample statistics are well approximated by a normal distribution.
   There is another reason, and not such a good one: the normal distribution is well-understood and tabulated, so it's easy to use it to get answers, even if the process isn't actually normally distributed.

3. The test statistic is
$$t_0 = \frac{|\mu_2 - 2.00|}{\frac{S_2}{\sqrt{n}}} = \frac{0.04}{\frac{0.06}{3}} = 2$$
   Since $t_0 < t_{0.025,8} = 2.306$, we accept the null hypothesis: the mean position of the slot is on target.

4. The P-value is the value of α such that $t_{\frac{\alpha}{2},8} = 2$. The table says $t_{0.05,8} = 1.86$, so $\frac{\alpha}{2}$ is between 0.025 and 0.05, so α is between 0.05 and 0.10.

5. The 95% confidence interval on σ_1^2 is given by
$$\begin{aligned} \frac{(n-1)S_1^2}{\chi^2_{0.025,8}} \leq & \quad \sigma_1^2 \quad \leq \frac{(n-1)S_1^2}{\chi^2_{0.975,8}} \\ \frac{(8)0.0016}{17.53} \leq & \quad \sigma_1^2 \quad \leq \frac{(8)0.0016}{2.18} \\ 0.00073 \leq & \quad \sigma_1^2 \quad \leq 0.0059 \end{aligned}$$

6. One test statistic is
$$F_0 = \frac{S_2^2}{S_1^2} = 2.25$$
   which we reject if $F_0 > F_{0.025,8,8} = 4.43$. Therefore we accept the hypothesis that the variances for the two machines are equal.

7. Since the variances for the two machines are equal, we can use the pooled variance
$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(8)0.0016 + (8)0.0036}{16} = 0.0026$$
   The test statistic is
$$t_0 = \frac{0.02}{S_p\sqrt{\frac{1}{9}+\frac{1}{9}}} = 0.832$$
   which is less than $t_{0.025,16} = 2.12$. Therefore we accept the hypothesis that the means are equal.

8. Note that $t_{0.25,16} = 0.690$, and $t_{0.1,16} = 1.337$. The P-value is thus between $\alpha = 0.20$ and $\alpha = 0.50$.

9. We assume both operations generate normal distributions. Certainly the easiest, if not the most accurate, estimate is provided by using the test statistics as the values for the true means and variances. In that case:

$$\begin{array}{lll} \mu_1 = 1.98 & \sigma_1^2 = 0.0016 & \sigma_1 = 0.04 \\ \mu_2 = 1.96 & \sigma_2^2 = 0.0036 & \sigma_2 = 0.06 \end{array}$$

   (a) The center position of the hole is then normally distributed with mean 1.98 and standard deviation 0.04. The interval $[1.95, 2.05]$ corresponds to the interval $[\mu_1 - 0.75\sigma_1, \mu_1 + 1.75\sigma_1]$. From the cumulative distribution table, $\Phi(-0.75) = 0.22663$, and $\Phi(1.75) = 0.95994$. The percentage that falls *outside* the interval is thus

$$1 - (\Phi(1.75) - \Phi(-0.75)) = 0.2667$$

   Expect 267 parts of 1000 to be rejected on the basis of the hole position.

   (b) For the slot, $[1.95, 2.05]$ corresponds to $[\mu_2 - 0.167\sigma_2, \mu_2 + 1.5\sigma_2]$. The percentage that falls outside the interval is thus

$$1 - (\Phi(1.5) - \Phi(-0.17)) = 1 - (0.93319 - 0.43251) = 0.4993$$

   Expect 499 parts of 1000 to be rejected on the basis of the slot position.

   (c) Assume the difference between the center position of the hole and the center position of the slot is normally distributed with $\mu = 0.02$ and $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.072$. The interval of interest is $[-0.03, 0.03]$, which corresponds to $[\mu - 0.69\sigma, \mu + 0.14\sigma_1]$. The percentage that falls outside the interval is thus

$$1 - (\Phi(0.14) - \Phi(-0.69)) = 1 - (0.55567 - 0.24510) = 0.6894$$

   Expect 689 parts of 1000 to be rejected on the basis of the difference in slot and hole positions.

Those answers are all approximate, of course. We've ignored interactions, and we've taken the sample statistics to be equal to the population statistics.