UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Midterm #2

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1. For $n = 4$ (samples of size 4), $B_4 = 2.266$, and $B_3 = 0$. The parameters of the S chart are given by:

UCL:	$B_4\bar{S} = 2.9458$
Center line:	$\bar{S} = 1.3$
LCL:	$B_3\bar{S} = 0$

For the $\bar{\bar{x}}$ chart, we use $A_3 = 1.628$. The parameters of the $\bar{\bar{x}}$ chart are given by:

UCL:	$\bar{\bar{x}} + A_3\bar{S} = 14.1364$
Center line:	$\bar{\bar{x}} = 12.02$
LCL::	$\bar{\bar{x}} - A_3\bar{S} = 9.9036$

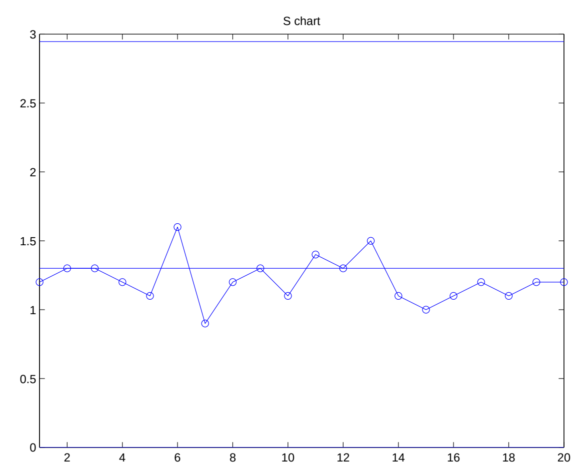
There is one out of control point, the fourth (14.3). If we assume that an assignable cause is found for it, we need to get the new value of $\bar{\bar{x}}$ that ignores that point. That value is $\bar{\bar{x}} = 11.9$ (easily calculated by $\frac{12.02(20)-14.3}{19}$). The values of A_3 and $\bar{S}$ are the same, so we subtract 0.12 from each control limit:

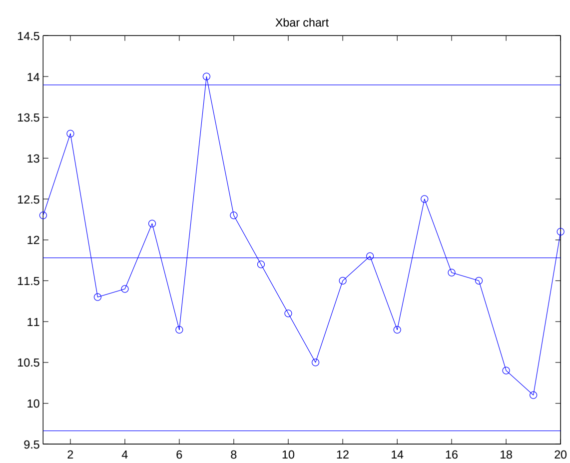
UCL:	14.1364 - 0.12 = 14.0164
Center line:	11.9
LCL::	9.9036 - 0.12 = 9.7836

Now there is one more out of control point, the fifteenth (14.1). If we assume that an assignable cause is found for it, we need to get the new value of $\bar{\bar{x}}$ that ignores that point. That value is $\bar{\bar{x}} = 11.78$ (easily calculated by $\frac{11.9(19)-14.1}{18}$). The new control limits are:

UCL:	13.8964
Center line:	11.9
LCL::	9.9036 - 0.12 = 9.6636

2. For this problem it would suffice to draw a sketch of the control chart by hand. The charts look like this:





The seventh point (14.0) is out of control.

3. For a tabular cusum chart, we do need to have an estimate for σ (this is a mistake in the problem statement; however, I will accept any estimate for σ). We also need to decide how big a shift we want to detect; again, I accept any reasonable offer.

   I take $\bar{S} = 1.3$ to be the estimate for σ, and I assume we wish to detect a shift of 1 σ. Using $k = 0.5$ and $h = 4.77$, it is reasonable to use $K = 0.65$ and $H = 6.20$. The headings of the chart would be:

$$i \quad x_i \quad x_i - 12.67 \quad C_i^+ \quad N^+ \quad 11.37 - x_i \quad C_i^- \quad N^-$$

4. The development of the chart is as follows. $H = 6.20$ signals out of control:

i	x_i	$x_i - 12.67$	C_i^+	N^+	$11.37 - x_i$	C_i^-	N^-
1	12.3	-0.37	0	0	-0.93	0	0
2	13.3	0.63	0.63	1	-1.93	0	0
3	11.3	-1.37	0	0	0.07	0.07	1
4	11.4	-1.27	0	0	-0.03	0.04	2
5	12.2	-0.47	0	0	-0.83	0	0
6	10.9	-1.77	0	0	0.47	0.47	1
7	14.0	1.33	1.33	1	-2.63	0	0
8	12.3	-0.37	0.96	2	-0.93	0	0
9	11.7	-0.97	0	0	-0.33	0	0
10	11.1	-1.57	0	0	0.27	0.27	1
11	10.5	-2.17	0	0	0.87	1.14	1
12	11.5	-1.17	0	0	-0.13	1.01	2
13	11.8	-0.87	0	0	-0.43	0.58	3
14	10.9	-1.77	0	0	0.47	1.05	4
15	12.5	-0.17	0	0	-1.13	0	0
16	11.6	-1.07	0	0	-0.23	0	0
17	11.5	-1.17	0	0	-0.13	0	0
18	10.4	-2.27	0	0	0.97	0.97	1
19	10.1	-2.57	0	0	1.27	2.24	2
20	12.1	-0.57	0	0	-0.73	1.51	3

   The process certainly seems to be in control. The other chart caught an out of control signal.

5. With $p = 0.3$ and $n = 60$, the control limits are:

$$\begin{aligned} \text{UCL:} \quad & \bar{p} + 3\sqrt{\tfrac{\bar{p}(1-\bar{p}}{n}} = 0.4755 \\ \text{Center line:} \quad & 0.3 \\ \text{LCL::} \quad & \bar{p} - 3\sqrt{\tfrac{\bar{p}(1-\bar{p}}{n}} = 0.1225 \end{aligned}$$

   (a) The probability that a shift to $\bar{p} = 0.4$ will be detected is the cumlative binomial distribution to 0.1225*60 (rounds to 7) plus one minus the cumlative binomial distribution to 0.4755*60 (rounds to 29), or about 7.46%. The probability of detection by the end of the third sample is thus $0.0746 + (0.0746)(1 - 0.0746) + (0.0746)(1 - 0.0746)^2 = 0.2075$, or 20.75%.

   (b) For a nonzero control limit, we require

$$n > \frac{9(0.7)}{0.3} = 21$$

   so we need n to be at least 22.

   The chance of detecting a shift to 0.1 is essentially the probability of zero errors, or `binocdf(0,22,0.1)`, which is 9.85%.