UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Problem Set #1

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1. (a) The sample average is:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{5199}{40} = 129.975$$

The standard deviation is:

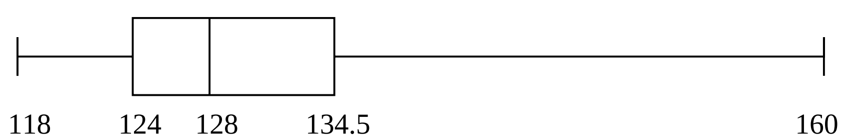
$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = 8.914$$

(b) Your histogram should have time to failure on the horizontal axis, and frequency on the vertical axis.

(c) Stem-and-leaf plot:

Stem	Leaf	Frequency
11	8 9	2
12	0 1 1 2 3 3 4 4 [1st] 4 4 5 5 5 5 5 6 7 8 [median] 8 9 9 9	22
13	0 1 1 1 3 3 [3rd] 6 7 7	9
14	0 0 1 2 2	5
15	1	1
16	0	1

(d) Note that the median (128), the first quartile (124), and the third quartile ($\frac{133+136}{2} = 134.5$) are noted in the stem-and-leaf plot above.



2. If the probability distribution of x is $f(x) = ke^{-x}$, with $0 \leq x \leq \infty$, then we find k using the identity

$$\int_0^{\infty} ke^{-x} dx = 1.$$

(discrete probability distributions always sum to 1, continuous probability distributions always integrate to 1). Now,

$$\int_0^{\infty} ke^{-x} dx = k\left[-e^{-x}\right]_0^{\infty} = k\left[0 - -1\right] = 1$$

so $k = 1$.
The mean and variance could be calculated directly from the formulae

$$\mu = \int_0^{\infty} xe^{-x} dx$$
$$\sigma^2 = \int_0^{\infty} (x-\mu)^2 e^{-x} dx$$

but it is simpler to note that this is an exponential distribution with parameter $\lambda = 1$. (In fact, by noting it is an exponential distribution you can fix k without any integration.) Therefore:

$$\begin{aligned} \mu &= \frac{1}{\lambda} = 1 \\ \sigma^2 &= \frac{1}{\lambda^2} = 1 \end{aligned}$$

3. Assuming that the fraction of nonconforming product manufactured is $p = 0.03$, the number of nonconforming units is described by a binomial distribution with $n = 100$ and x unknown. $P\{\hat{p} \leq 0.04\}$, the probability that $\hat{p} \leq 0.04$, is equal to the sum

$$P\{x = 0\} + P\{x = 1\} + P\{x = 2\} + P\{x = 3\} + P\{x = 4\}$$

Using the binomial distribution, this is:

$$\binom{100}{0} 0.03^0 0.97^{100} + \binom{100}{1} 0.03^1 0.97^{99} + \binom{100}{2} 0.03^2 0.97^{98} + \binom{100}{3} 0.03^3 0.97^{97} + \binom{100}{4} 0.03^4 0.97^{96}$$

Recalling that $\binom{n}{x} = \frac{n!}{(n-k)!k!}$ (there was an error in the notes), these probabilities are:

$$0.04755 + 0.14707 + 0.22515 + 0.22747 + 0.17061 = 0.81785$$

4. Normal distribution, $\mu = 40$, $\sigma = 8$. To find how what fraction of parts fail to meet the minimum tensile strength of 34 lb, we *could* integrate

$$\int_{-\infty}^{34} \frac{1}{8\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-40}{8}\right)^2}$$

However, it is much easier to note that $\mu - 34 = 6 = 0.75\sigma$, and "integrate"

$$\int_{-\infty}^{0.75} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$

by looking at the table in the Appendix of Montgomery. The answer is 0.77337, which means that 0.22663 is the fraction that fail to meet the minimum. Since there are 50,000 parts total, $50000(0.22663) = 11332$ parts fail to meet the 34 lb specification.
To find how many parts exceed 48 lb, note that $48 - \mu = 8 = \sigma$, or exactly one standard deviation, so $50000(1 - 0.84134) = 7933$ parts exceed 48 lb tensile strength.

5. If x is a discrete random variable with Poisson distribution $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, \ldots$, then the mean of its distribution is

$$\begin{aligned} \mu &= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} \\ &= 0 + \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} \\ &= \lambda \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \end{aligned}$$

Let $y = x - 1$, then

$$\mu = \lambda \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^y}{y!}$$

Since we know that $\sum_{y=0}^{\infty} p(y) = 1$, we conclude

$$\mu = \lambda$$

The variance of the distribution is

$$\begin{aligned}
\sigma^2 &= \sum_{x=0}^{\infty} (x-\lambda)^2 \frac{e^{-\lambda}\lambda^x}{x!} \\
&= \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!} - 2\lambda \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} + \lambda^2 \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!}
\end{aligned}$$

Using the results of the last problem,

$$\begin{aligned}
\sigma^2 &= \left(\sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!} \right) - 2\lambda^2 + \lambda^2 \\
&= \left(\lambda \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \right) + \lambda^2
\end{aligned}$$

Setting $y = x - 1$ as before,

$$\begin{aligned}
\sigma^2 &= \left(\lambda \sum_{y=0}^{\infty} (y+1) \frac{e^{-\lambda}\lambda^y}{y!} \right) - \lambda^2 \\
&= \left(\lambda \sum_{y=0}^{\infty} y \frac{e^{-\lambda}\lambda^y}{y!} + \lambda \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^y}{y!} \right) - \lambda^2 \\
&= \lambda^2 + \lambda - \lambda^2 \\
&= \lambda
\end{aligned}$$