UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Problem Set #11

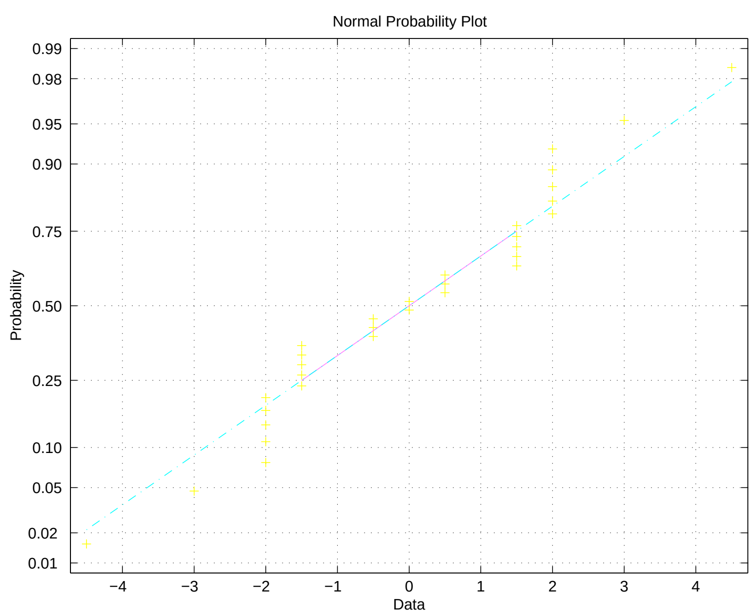
Michael J. Scott Spring 2000

1. (Montgomery 11-4) The effects and their significance levels are:

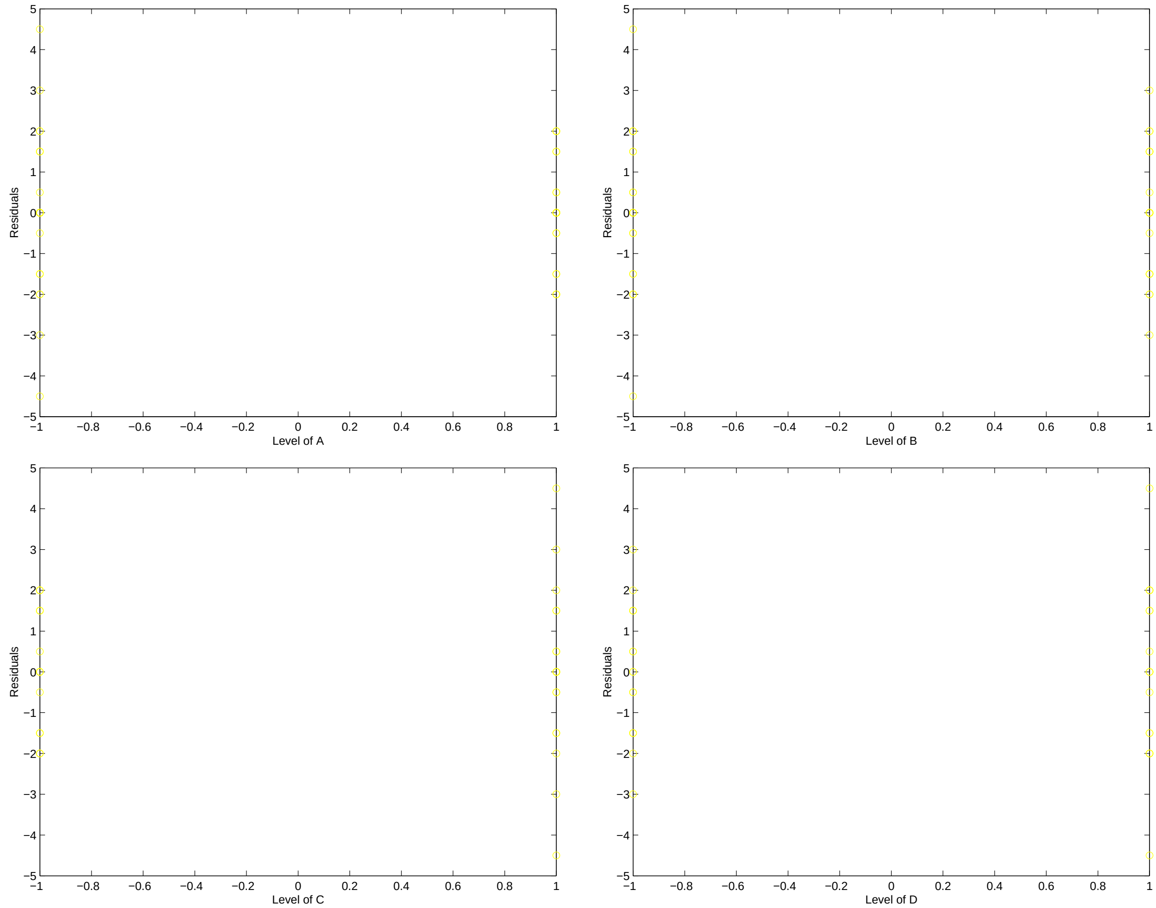
Variable	Effect	α
A	-9.0625	0.0000
B	-1.3125	0.1984
C	-2.6875	0.0143
D	3.9375	0.0010
AB	4.0625	0.0007
AC	0.6875	0.4923
AD	-2.1875	0.0399
BC	-0.5625	0.5733
BD	-0.1875	0.8504
CD	1.6875	0.1038
ABC	-5.1875	0.0001
ABD	4.6875	0.0002
ACD	-0.9375	0.3522
BCD	-0.9375	0.3522
ABCD	2.4375	0.0241

Therefore A, C, D, AB, AD, ABC, ABD, and ABCD are significant at a level of $\alpha = 0.5$; only A, D, AB, ABC, and ABD are significant at a level of $\alpha = 0.1$.

2. (Montgomery 11-5) The normal probability plot of the residuals looks like this:



There are a few outliers at the left side of the plot, but otherwise normality seems justified. We have to calculate the residuals and then plot them for each factor at both high and low levels. The four plots look like this:



A high and C low seem to have lower variablility.

3. (Montgomery 11-6) The error estimate based on the sum of squares was $\hat{\sigma}^2 = 0.7656$; the standard error is given by

$$\sqrt{\frac{\hat{\sigma}^2}{n2^k}} = \sqrt{\frac{0.7656}{32}} = 0.4891$$

To analyze using standard error, we take the effects and divide by two to get the coefficients, then divide the coefficients by the standard error and compare to $t_{\alpha,16}$:

Variable	Coefficent/std. error	α
A	-9.2639	0.0000
B	-1.3417	0.1984
C	-2.7472	0.0143
D	4.0250	0.0010
AB	4.1528	0.0007
AC	0.7028	0.4923
AD	-2.2361	0.0399
BC	-0.5750	0.5733
BD	-0.1917	0.8504
CD	1.7250	0.1038
ABC	-5.30285	0.0001
ABD	4.79175	0.0002
ACD	-0.95835	0.3522
BCD	-0.95835	0.3522
ABCD	2.49175	0.0241

identical to Montgomery 11-4.

4. (Montgomery 11-7) If we use only the first run from Montgomery 11-4, we get the following sums of squares:

Variable	Coeff. Est.	Sums of Squares
A	-5.0000	328.5156
B	-0.3750	6.8906
C	-0.3750	28.8906
D	2.5000	62.0156
AB	2.2500	66.0156
AC	0.2500	1.8906
AD	-1.8750	19.1406
BC	-0.6250	1.2656
BD	-0.7500	0.1406
CD	0.7500	11.3906
ABC	-3.0000	107.6406
ABD	2.3750	87.8906
ACD	-0.1250	3.5156
BCD	-1.0000	3.5156
ABCD	1.6250	23.7656

Again, A, C, D, AB, AD, ABC, ABD, and ABCD are the most important.

5. Using the fractional factorial design and only the first run of samples, we get the following effects:

Coefficent	Alias	Effect
A	BCD	-12
B	ACD	-1
C	ABD	4
D	ABC	-1
AB	CD	6
AC	BD	-8.5
AD	BC	5

The big difference is that D/ABC does not seem significant here. However, in the full effects table above we can see that these two, which are aliased, probably cancel each other out.