UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

# IE 446
# Solutions to Problem Set #4

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2. (a) Taking the rules one at a time:

   1. One point plots outside either one of the 3-$\sigma$ control limits with a probability of $p_1 = 0.0027$ (use normal CDF: $1 - \Phi(3) = 0.00135$.)

   2. One point plots outside *a particular* one of the 2-$\sigma$ warning limits with a probability of 0.02275 (from $1 - \Phi(2)$). There are four ways that two out of three consecutive points can plot outside:

   | | Point 1 | Point 2 | Point 3 |
   |---|---|---|---|
   | 1 | outside | outside | outside |
   | 2 | inside | outside | outside |
   | 3 | outside | inside | outside |
   | 4 | outside | outside | inside |

   The probability of way 1 is $0.02275^2 = 0.000011775$. The probability of each of ways 2–4 is $0.02275^2(1 - 0.02275) = 0.00050579$. Adding up all four ways gives a probability of 0.0015 for each of the two warning lines, or a probability of $p_2 = 0.0031$ considering both lines.

   3. One point plots outside a particular one of the 1-$\sigma$ warning limits with a probability of 0.15866 (from $1 - \Phi(1)$). There are six ways that four out of five consecutive points can plot outside. One way has all five outside with a probability of $0.15866^5 = 0.00010052$. The other four ways each have probability $0.15866^4(1 - 0.15866) = 0.00053308$. Adding up and multiplying by two gives a total probability of $p_3 = 0.0013$.

   4. Eight consecutive points on one side of the center line, times two: $p_4 = 2(0.5^8) = 0.0078$.

   (b) The probability that at least one rule will be activated is complicated because the rules interact. The calculation is tedious. But if we add all four probabilities we get 0.0128. This is probably an underestimate of the chance that one rule will be triggered within eight points, since each of rules 1–3 has several chances to be triggered. The calculation:

   $$8p_1 + \frac{8}{3}p_2 + \frac{8}{5}p_3 + p_4 = 0.0342$$

   is probably better. Even this is probably an underestimate. Montgomery, page 150, says that the average run length (ARL) for the Western Electric Rules is 91.25, corresponding to $p = 0.0110$ for each point (which would be 0.0877 for 8 points).

3. We suppose that the sampled mean lies on one of the 1-$\sigma$ warning lines. Then the probabilities are:

   1. $p_1 = (1 - \Phi(2)) + (1 - \Phi(4)) = 0.02278$.
   2. $p_2 = (1 - \Phi(1))^3 + 3(1 - \Phi(1))^2(\Phi(1)) + (1 - \Phi(3))^3 + 3(1 - \Phi(3))^2(\Phi(3)) = 0.02517$.
   3. $p_3 = (0.5)^5 + 5(0.5)^4(0.5) + (1 - \Phi(2))^5 + 5(1 - \Phi(2))^4(\Phi(2)) = 0.06250$.
   4. $p_3 = \Phi(1)^5 + (1 - \Phi(1))^5 = 0.42167$.

4. Yes, the pattern appears random. None of the Western Electric Rules are satisfied.

5. (a) No, averaging five measurements with one taken each half-hour is unlikely to catch an instantaneous upward shift in the mean that is of very short duration, even if one of the five samples is taken during the shift.

   (b) No one right answer, no good ideas refused.