

UNIVERSITY OF ILLINOIS AT CHICAGO  
Mechanical Engineering

**IE 446**  
**Solutions to Problem Set #5**

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1. It turns out that, regardless of how the individual tables are set up, the  $P$ -value is always one minus the value of the cumulative distribution function.

(a)  $Z_{0.015778} = 2.15$ .

```
>> 1-normcdf(2.15)
ans =
    0.01577760739109
```

Possible hypothesis:  $\mu = \mu_0$ , normally distributed,  $\sigma^2$  known.

Possible test: Is  $Z_0 = \frac{(\mu - \mu_0)\sqrt{n}}{\sigma} > Z_\alpha$ ?

(b)  $t_{0.037553,6} = 2.15$ .

```
>> 1-tcdf(2.15,6)
ans =
    0.03755313773720
```

Possible hypothesis:  $\mu = \mu_0$ , normally distributed,  $\sigma^2$  not known (use  $S^2$  instead).

Possible test: Is  $t_0 = \frac{(\mu - \mu_0)\sqrt{n}}{S} > t_{\alpha, n-1}$ ?

(c)  $\chi^2_{0.905399,6} = 2.15$ .

```
>> 1-chi2cdf(2.15,6)
ans =
    0.90539895148442
```

Possible hypothesis:  $\sigma^2 = \sigma_0^2$ , normally distributed.

Possible test: Is  $\chi^2_0 = \frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{\frac{\alpha}{2}, n-1}$ ?

Note that there is also the test: Is  $\chi^2_0 = \frac{(n-1)S^2}{\sigma_0^2} > \chi^2_{\frac{1-\alpha}{2}, n-1}$ ?

Therefore it is possible that  $\alpha = 0.094601$ , instead:

```
>> chi2cdf(2.15,6)
ans =
    0.09460104851558
```

(d)  $F_{0.126651,13,6} = 2.15$ .

```
>> 1-fcdf(2.15,13,9)
ans =
    0.12665141108948
```

Possible hypothesis:  $\sigma_1^2 = \sigma_2^2$ , normally distributed.

Possible test: Is  $F_0 = \frac{S_1^2}{S_2^2} > F_{\frac{\alpha}{2}, n_1-1, n_2-1}$ ?

The  $P$ -value may come from switching the numerator and denominator:

```
>> 1-fcdf(2.15,9,13)
ans =
    0.10182282394983
```

in which case it is  $\alpha = 0.101823$ .

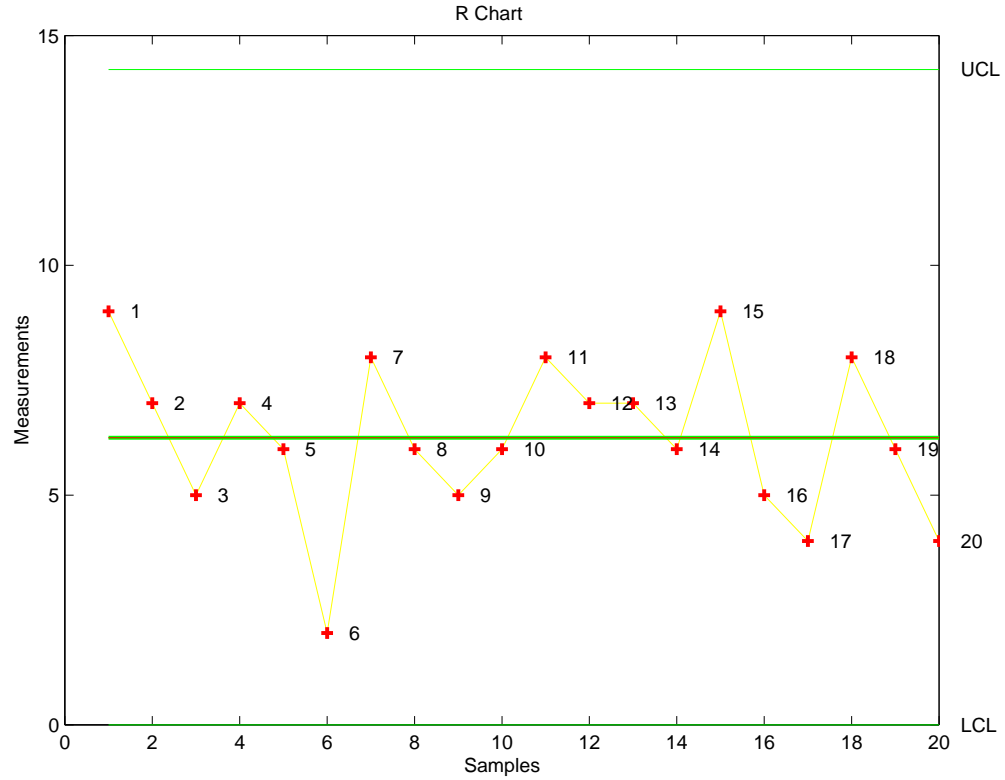
2. Running the numbers for the trial samples, we have the following:  $\bar{\bar{x}} = 10.375$ ,  $\bar{\bar{R}} = 6.25$ , and for  $n = 4$  the constants from Appendix VI are  $A_2 = 0.729$ ,  $D_3 = 0$ , and  $D_4 = 2.282$ .

- (a) Set up the  $R$  chart first, with centerline  $\bar{\bar{R}} = 6.25$  and control limits

$$D_4 \bar{\bar{R}} = 14.2625$$

$$D_3 \bar{\bar{R}} = 0$$

Plotting the given samples on the preliminary  $R$  chart looks like this (yes, that's `matlab`, but I had to tweak it a little):



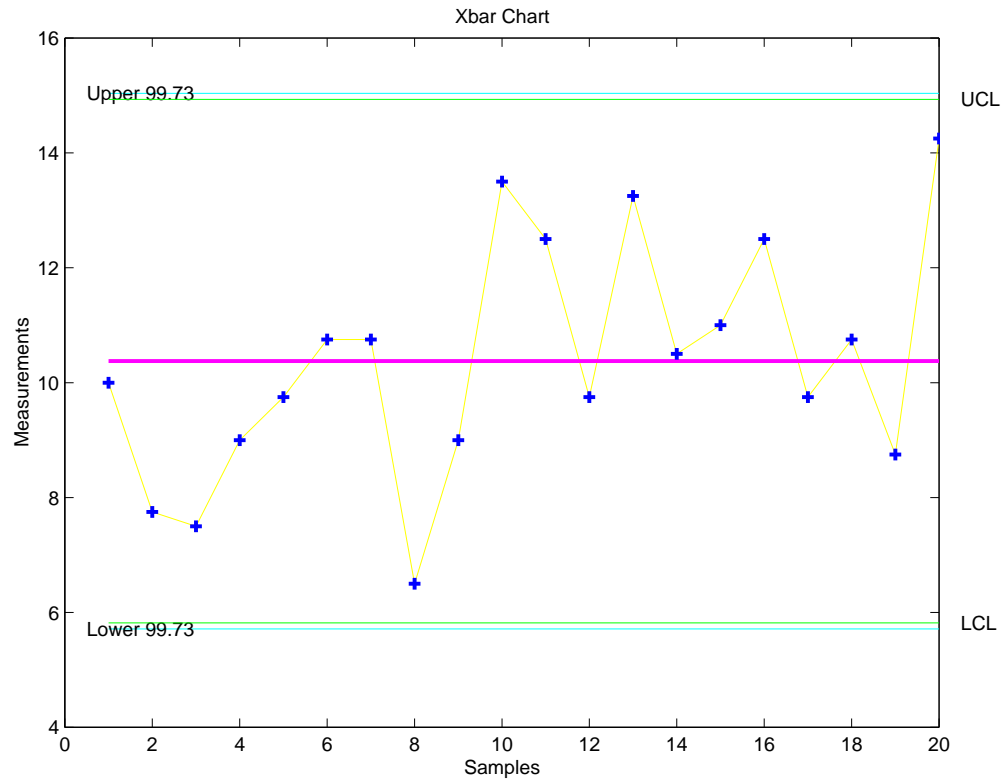
which is very much in control.

The  $\bar{x}$  chart has upper and lower control limits at:

$$\bar{\bar{x}} + A_2 \bar{\bar{R}} = 14.9312$$

$$\bar{\bar{x}} - A_2 \bar{\bar{R}} = 5.8188$$

The chart looks like this:

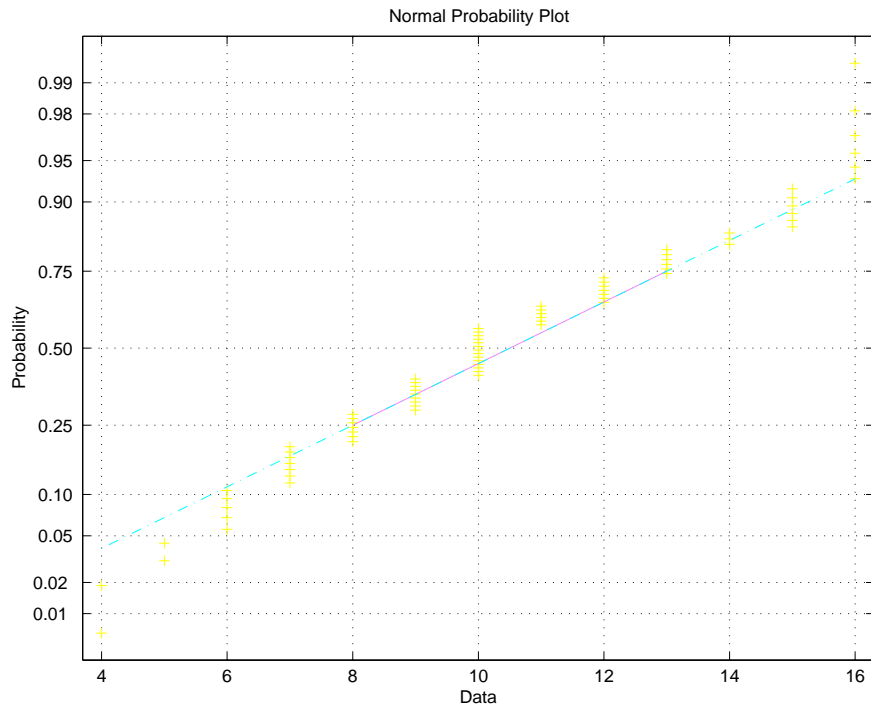


which, again, is very much in control.

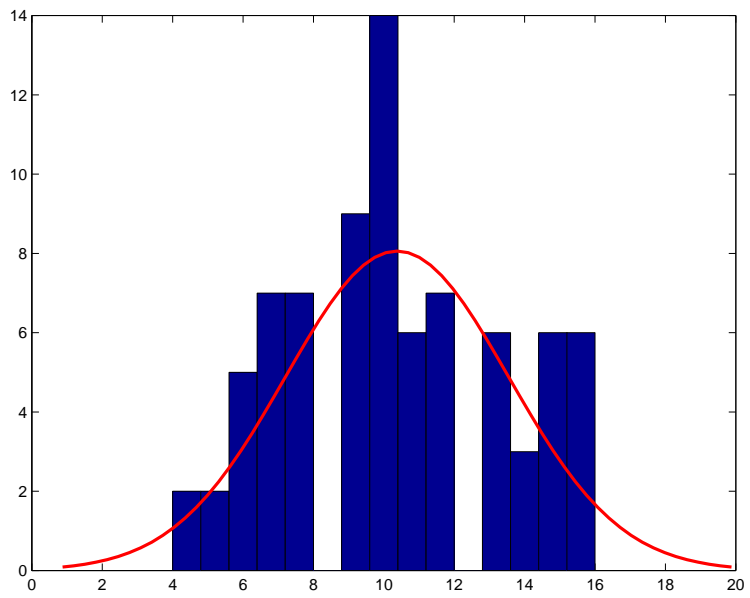
- (b) If the specification limits are at  $350 \text{ V} \pm 5 \text{ V}$ , then the specification limits on the  $\bar{x}$  chart are at  $\pm 50$ , several standard deviations outside the chart's control limits. Thus the in-control process will produce defective parts almost never (less than one in a billion). The PCR is  $\frac{100}{20.749} = 4.82$ , a very large number for a PCR.

Note that the process mean is *not* the specification mean, in fact, all sampled points are above the specification mean, but it does not much matter with such wide specification limits.

- (c) To check whether we have a normal distribution, we can plot the data on some graph where it will look wrong if it is not normally distributed. Two choices are the normal distribution plot shown here:



and a histogram with superimposed normal fit, shown here:



There is a small concentration of samples near the upper end of the range, but not enough to discard the assumption of normality.