UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Problem Set #6

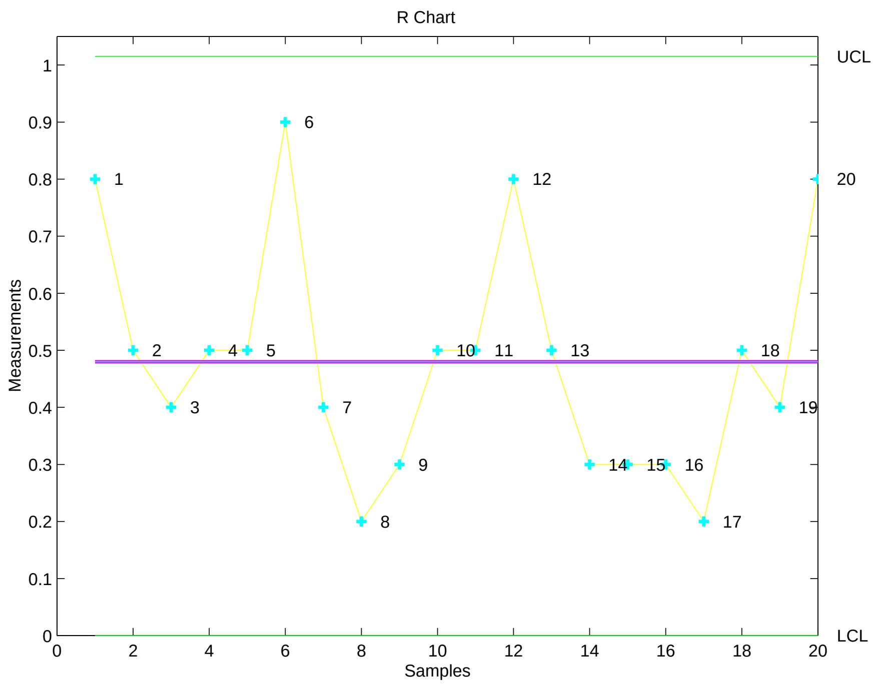
Michael J. Scott Spring 2000

1. Running the numbers for the trial samples, we have the following: $\bar{\bar{x}} = 16.278$, $\bar{R} = 0.48$, and for $n = 5$ the constants from Appendix VI are $A_2 = 0.577$, $D_3 = 0$, and $D_4 = 2.115$.

   (a) Set up the R chart first, with centerline $\bar{R} = 6.25$ and control limits

$$D_4\bar{R} = 1.0152$$
$$D_3\bar{R} = 0$$

Plotting the given samples on the preliminary R chart looks like this:

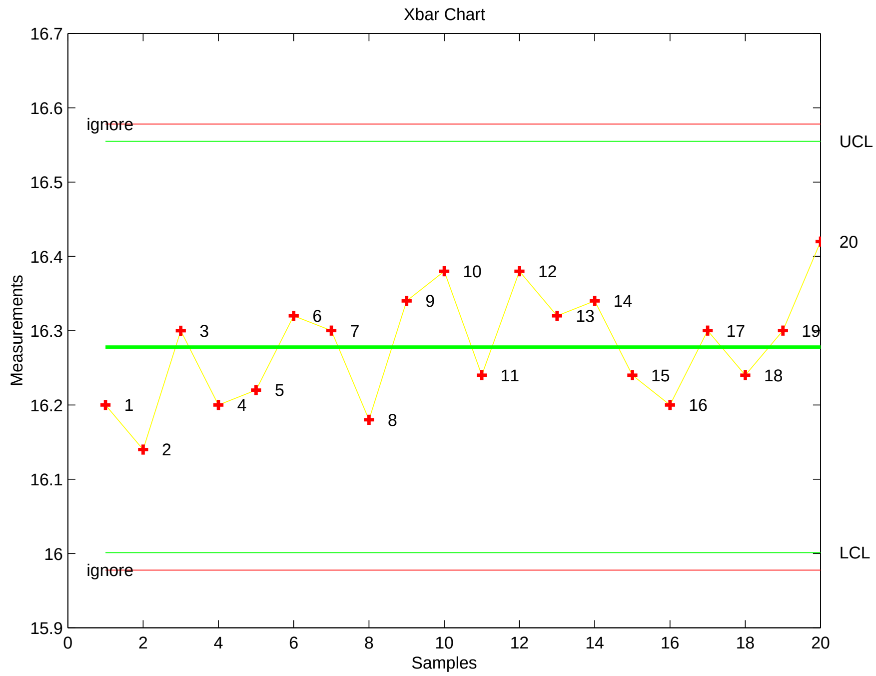


which is in control.

The $\bar{x}$ chart has upper and lower control limits at:

$$\bar{\bar{x}} + A_2\bar{R} = 16.555$$
$$\bar{\bar{x}} - A_2\bar{R} = 16.001$$

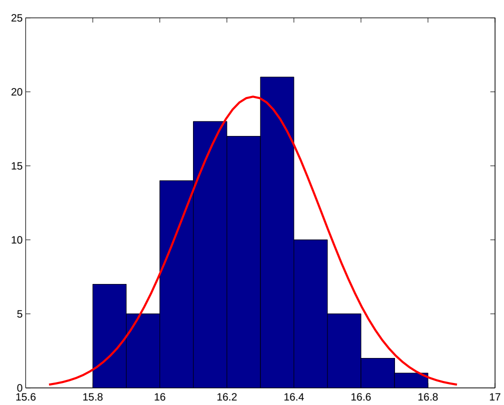
The chart looks like this:



which, again, is in control.

(b) $\bar{\bar{x}} = 16.278$ is an estimate for μ.
$\frac{\bar{R}}{d_2} = \frac{0.48}{2.326} = 0.2064$ is an estimate for σ.

(c) Test the normal distribution assumption with a histogram:



which looks pretty good.

(d) The specification range is exactly 1, so the PCR is $\frac{1}{6\sigma} = 0.8076$; the process uses up more than the specification range, so expect more than 27 in 10000 non-conforming. (But see next item.)

(e) Expect 25 of 10000 below 15.7, but 2 of 100 above 16.7:

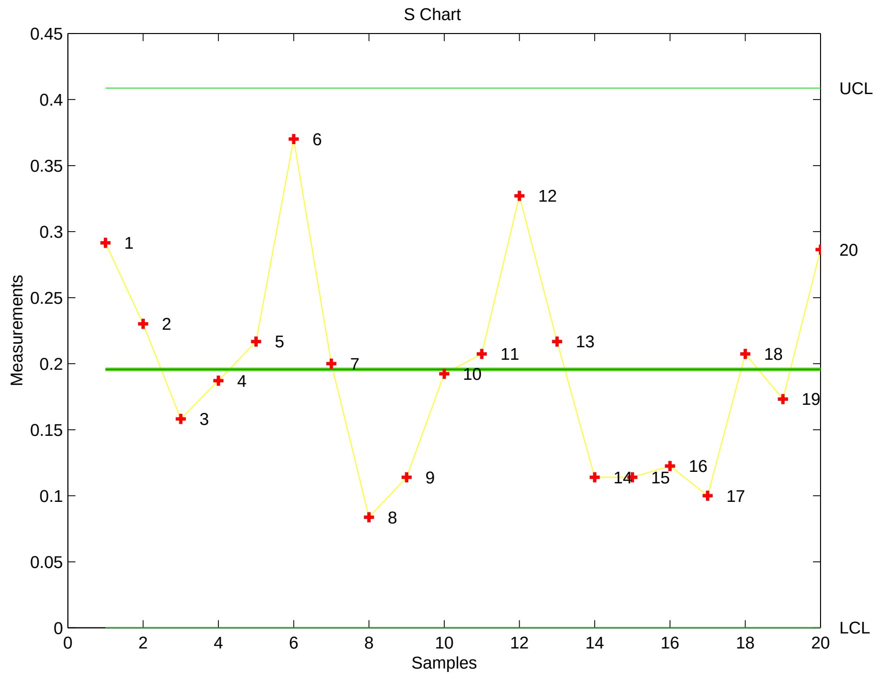
```
normcdf(15.7,xbarbar,sigma) = 0.0025
1-normcdf(16.7,xbarbar,sigma) = 0.0204
```

2. $B_4 = 2.089$, $B_3 = 0$, and $A_3 = 1.427$.

(a) Set up the S chart first, with centerline $\bar{S} = 0.1956$ and control limits

$$B_4\bar{S} = 0.4087$$
$$B_3\bar{S} = 0$$

Plotting the given samples on the preliminary S chart looks like this:

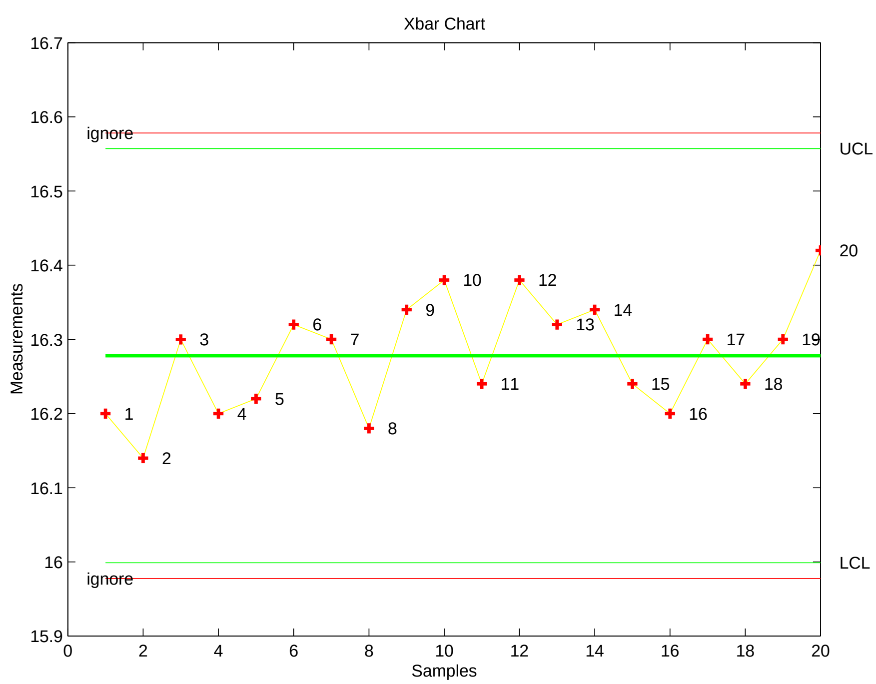


which is in control.

The $\bar{x}$ chart now has upper and lower control limits at:

$$\bar{\bar{x}} + A_3\bar{S} = 16.557$$
$$\bar{\bar{x}} - A_3\bar{S} = 15.999$$

The chart looks like this:



which, again, is in control.

(b) Same estimated process mean.

$\frac{\bar{S}}{c_4} = \frac{0.1956}{0.94} = 0.208$ is the new estimate for σ.

(c) No difference.

(d) Here the PCR is $\frac{1}{6\sigma} = 0.8008$, almost identical to the earlier answer.

(e) Expect 27 of 10000 below 15.7, but 2.1 of 100 above 16.7:

```
normcdf(15.7,xbarbar,sigmas) = 0.0027
1-normcdf(16.7,xbarbar,sigmas) = 0.0213
```

Note that this is slightly less optimistic than the estimate from the R chart.

3. We have $\bar{\bar{x}} = 40$, $\bar{R} = 5$, and for $n = 8$ the constants from Appendix VI are $A_2 = 0.373$, $d_2 = 2.847$, $D_3 = 0.136$, and $D_4 = 1.864$.

(a) The control limits for the R chart are 9.32 and 0.68.
For the $\bar{x}$ chart they are 41.865 and 38.135.

(b) The estimate for σ is 1.7652. The tolerance limits are thus 45.2687 and 34.7313.

(c) The PCR is 0.9490.

(d) Expect 11 of 1000 below 36, and 3 of 10000 above 46:

```
normcdf(36,xbarbar,sigma) = 0.0114
1-normcdf(46,xbarbar,sigma) = 0.000317
```

Therefore 0.03% can be reworked while 1.1% must be scrapped.

(e) The process could be improved by shifting the mean higher so that more of the non-conforming parts could be reworked rather than scrapped.

4. For the S chart part of the question, need the constants (for $n = 8$):
$A_3 = 1.099$, $B_3 = 0.185$, $B_4 = 1.815$, and $c_4 = 0.965$. Also, note that $\bar{S} = 1.6$.

(a) Control limits for S chart are 2.904 and 0.2960.
Control limits for $\bar{x}$ chart are 41.7584 and 38.2416.

(b) Here the estimate for σ is 1.6580, so the tolerance limits are 44.9741 and 35.0259.

(c) Here the PCR is 1.005, which indicates slightly more process capability than does the R chart.

(d) Expect 0.8% below 36, and 0.0015% above 46:

```
normcdf(36,xbarbar,sigmas) = 0.0079
1-normcdf(46,xbarbar,sigmas) = 0.000148
```

(e) Same suggestions as with the R chart.

The two approaches are sufficiently close that the assumptions seem reasonable. The estimated standard deviations are within 6.5% of each other, within the efficiency of R for $n = 8$ given on page 185.