UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Problem Set #7

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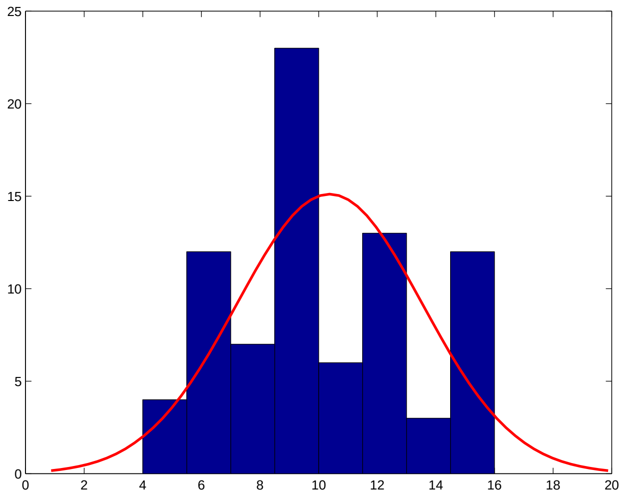
1. For the 80 sample points, we have $\mu = 10.3750$ and $\sigma = 3.1678$. A histogram with 8 bins looks like this:



We test the hypothesis that the data are actually normally distributed by testing

$$X_0 = \sum_{i=1}^{8} \frac{(O_i - E_i)^2}{E_i}$$

against ${\chi^2}_{\alpha,k-p-1}$. Here, $\alpha = 0.05$, $k = 8$ (the number of bins), and $p = 2$ (the number of parameters that describe a normal distribution).

The O_i are the observed frequencies in the bins, or $(4, 12, 7, 23, 6, 13, 3, 12)$. The E_i are the expected frequencies, each of which can be calculated by something like:

```
>> (normcdf(high,10.375,3.1678)-normcdf(low,10.375,3.1678))*80
```

where `high` and `low` are the upper and lower limits of the bins.

Performing the calculations, we find that $X_0 = 3.7530$. Since ${\chi^2}_{0.05,5} = 11.07$ is much larger, we accept the hypothesis that the data are normally distributed. In fact, we would require $\alpha = 0.5855$ to get ${\chi^2}_{\alpha,5} = 3.753$ (that's the P-value for the test).

2. Montgomery 5-42. $n = 4$.

   (a) Since $d_2 = 2.059$, estimate $\sigma = 4$.

   (b) Using the estimated σ and $B_5 = 0$, $B_6 = 2.088$, and $c_4 = 0.9213$, the S chart has limits 0 and 8.352, and centerline 3.685.

(c) Estimated mean is 620. Expect 10.56% to be greater than 625, and 0.00000002% to be less than 595.

(d) If the mean could be shifted to 610, less than 0.02% would be nonconforming (half above, half below).

(e) A new mean 610 is 2.5σ from the original mean. The chart is a 3-σ chart. Then

$$\beta = \Phi(3 - 2.5\sqrt{4}) - \Phi(-3 - 2.5\sqrt{4}) = 0.0228$$

is the probability of not detecting a shift, so there is a 97.72% chance of detecting the shift on the first time.

(f) The probability of detecting by at least the third sample is

$$0.9772 + (0.0228)0.9772 + (0.0228)^2 0.9772 = 0.999988$$

which is close enough to 1 for our purposes.

3. Montgomery 5-44. $n = 4$.

(a) Estimate $\mu = 700$, $\sigma = 7.979/c_4 = 8.66$.

(b) 1.05% above USL; 12.4% below LSL.

(c) $\alpha = 0.228$, or 22.8% type I error. (Note LSL = LCL; double the percentage from the last problem.)

(d) A total of about 31%. Note that the *process* σ is 12, but the sample σ is 6.

```
>> normcdf(690,693,6) + 1 - normcdf(710,693,6)
ans =
   0.31084080485768
```

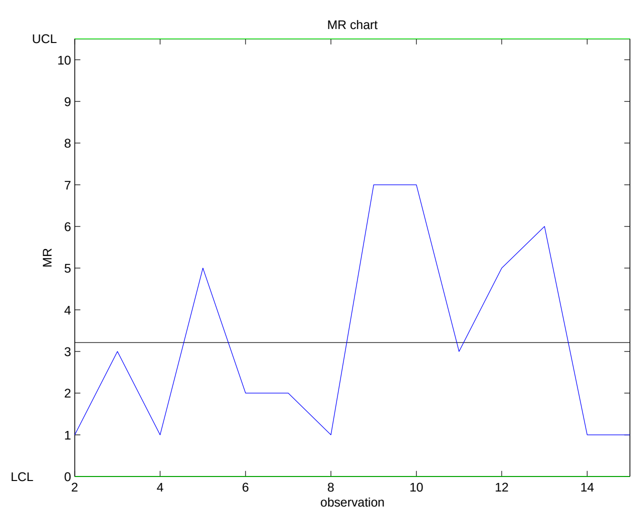
(e) $1 - \beta = 0.31$, so ARL is $\frac{1}{1-\beta} = 3.23$.

4. Make a function `beta(L,k,n)`. Then we wish to choose n for $k = 1$ and $L = 3$ so that $\beta = 0.4$. This will work:

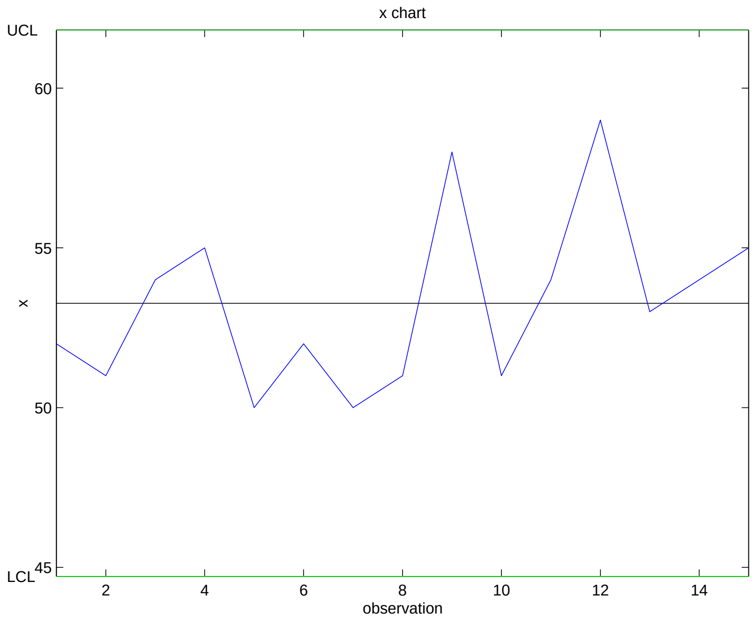
```
>> fzero('beta(3,1,x)-0.4',6)
ans =
  10.58426737009903
```

so the sample size must be at least 11.

5. Montgomery 5-46. Moving range average is 3.2; LCL is 0; UCL is 10.5. Plot:



$\bar{x} = 53.3$; UCL is 61.8; LCL is 44.7. Plot:



Both charts appear to be in control.

A fitted histogram of the data shows that the assumption of normality is not reasonable: