UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

IE 446
Solutions to Problem Set #8

Michael J. Scott — Spring 2000

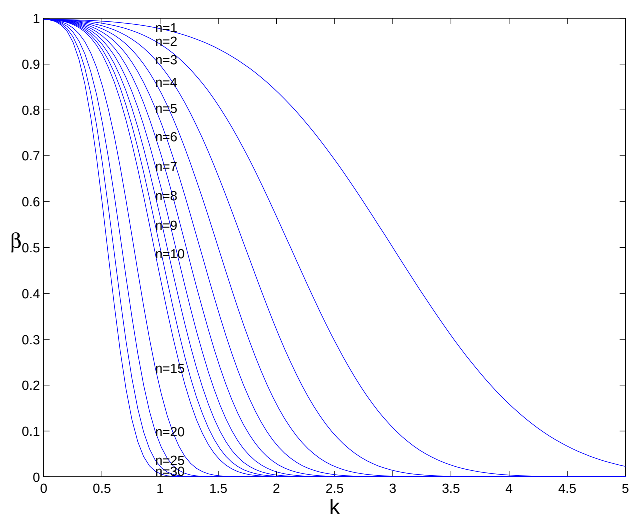
1. I used the file `beta.m` to provide the function beta:

```
function out = beta(l,k,n)
%BETA(L,K,N) gives beta in terms of L (usually 3), K, and N
out = normcdf(l-k*sqrt(n)) -normcdf(-l-k*sqrt(n));
```

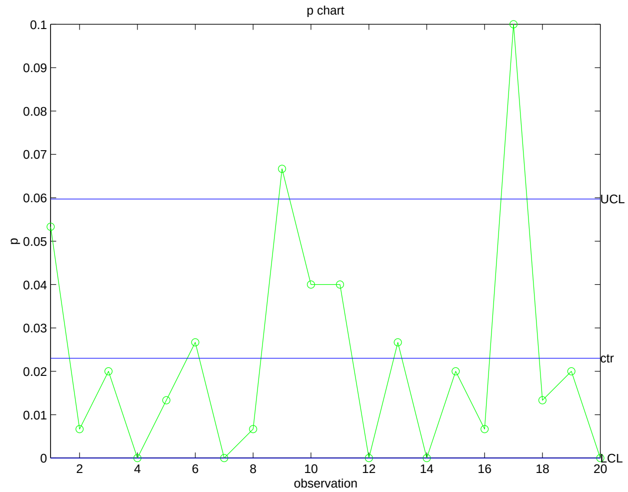
Then the code to generate the plot looks like this, including labelling:

```
x = linspace(0,5);
hold off;
for i=1:10
  y = beta(3,x,i);
  plot(x,y);
  text(x(20),y(20),sprintf('n=%d',i));
  hold on;
end
  for i=15:5:30
  y = beta(3,x,i);
  plot(x,y);
  text(x(20),y(20),sprintf('n=%d',i));
  hold on;
end
xlabel('k','FontSize',16);
ylabel('b','FontName','Symbol','FontSize',16,'Rotation',0);
```

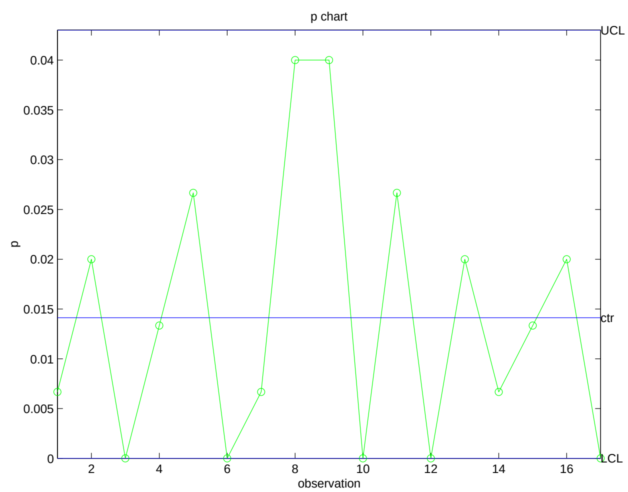
This produces the following plot:



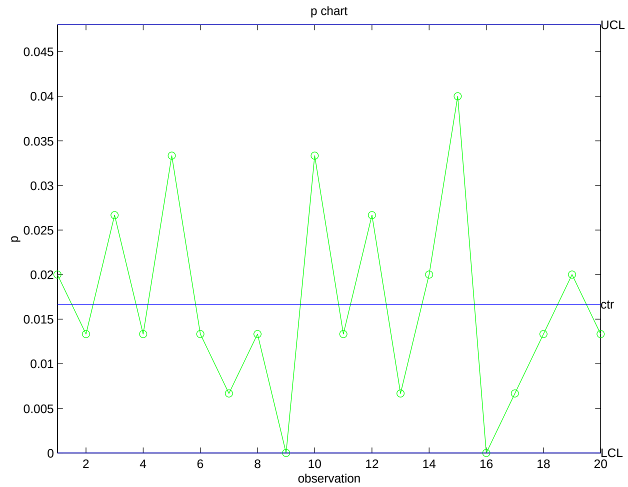
2. (Montgomery 6-2) Given $n = 150$. Calculate $p = 0.0230$. Then UCL is 0.0597, LCL is 0. The plotted data look like this:



Removing points 9 and 15, get the new limits $p = 0.0163$, UCL at 0.0473, LCL still 0. But then point 1 is out of control; remove it to get $p = 0.0141$, UCL of 0.0430, LCL of 0. The new control chart looks like this:



3. (Montgomery 6-4) Calculate $p = 0.0167$, UCL at 0.0480, LCL at 0. The chart looks like:



For a non-zero control limit, we require:

$$p > 3\sqrt{\frac{p(1-p)}{n}}$$

or

$$n > \frac{9(1-p)}{p} = 531$$

So the sample size must be at least 531.

4. (Montgomery 6-10) $np = 16.0$, with $n = 100$, so $p = .160$ (n and p are the binomial distribution parameters). The control limits are at 5.0018 and 26.9982.

   (a) If the mean shifts to $np = 20$ ($p = 0.2$), then the chart calls out of control if the average is at least 27 or at most 5:

```
>> 1-binocdf(27,100,0.2) + binocdf(5,100,0.2)
 ans = 0.0342
>> ans + ans*(1-ans) + ans*(1-ans)*(1-ans)
 ans = 0.0990
```

   so there is a 3.42% chance of detecting the shift the first day, and a 9.9% chance of detecting by the third day.

   (b) For a non-zero control limit, we require:

$$np > 3\sqrt{np(1-p)}$$

   or

$$n > \frac{9(1-p)}{p} = 47.25$$

   So the sample size must be at least 48.

5. (Montgomery 6-12) $n = 100$, $p = 0.080$.

   (a) The number nonconforming chart multiplies everything by 100: center line at 8.00, with UCL at 16.1, and LCL at 0.

   (b) The actual type I error is given by

```
>> 1-binocdf(17,100,.08)
 ans = 9.3086e-04
```

The Poisson approximation to the binomial uses $\lambda = np = 8$.

```
>> 1-poisscdf(17,8)
 ans = 0.0016
```

which is not such a good approximation.

(c) You've got a computer; use it.

```
>> binocdf(17,100,.2)
 ans = 0.2712
```

which gives a type II error of 27%.

(d) The chance to detect the shift by the fourth sample is given by

```
>> (1-ans) + ans*(1-ans) + ans^2*(1-ans) + ans^3*(1-ans)
 ans = 0.9946
```