UNIVERSITY OF ILLINOIS AT CHICAGO
Mechanical Engineering

# IE 446
# Solutions to Problem Set #9

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1. (Montgomery 6-44)

   (a) An inspection unit is 4 transmissions; sample size n is 1 unit. The average number of nonconformities is $\bar{u} = 1.6875$, so the centerline is at 1.6875. The upper control limit is at $\bar{u} + 3\sqrt{\bar{u}} = 5.585$, the lower control limit is at 0.

   (b) The process is in control.

   (c) Now $n = 2$ inspection units. So the centerline is at $n\bar{u} = 3.375$. The upper control limit is at $n\bar{u} + 3\sqrt{n\bar{u}} = 8.886$. Lower control limit still 0.

2. (Montgomery 6-60) The $c$ chart is for nonconformities per unit of 6 clocks, thus $\bar{c} = 6(0.75) = 4.5$. UCL is $\bar{c} + 3\sqrt{\bar{c}} = 10.86$, LCL is 0.

3. (Montgomery 7-4)

   (a) $H = h\sigma = 0.2385$, and $K = k\sigma = 0.025$. The chart is developed as below:

| $i$ | $x_i$ | $x_i - 8.045$ | $C_i^+$ | $N^+$ | $7.995 - x_i$ | $C_i^-$ | $N^-$ |
|---|---|---|---|---|---|---|---|
| 1 | 8.00 | -0.045 | 0 | 0 | -0.005 | 0 | 0 |
| 2 | 8.01 | -0.035 | 0 | 0 | -0.015 | 0 | 0 |
| 3 | 8.02 | -0.025 | 0 | 0 | -0.025 | 0 | 0 |
| 4 | 8.01 | -0.035 | 0 | 0 | -0.015 | 0 | 0 |
| 5 | 8.00 | -0.045 | 0 | 0 | -0.005 | 0 | 0 |
| 6 | 8.01 | -0.035 | 0 | 0 | -0.015 | 0 | 0 |
| 7 | 8.06 | 0.025 | 0.025 | 1 | -0.065 | 0 | 0 |
| 8 | 8.07 | 0.035 | 0.06 | 2 | -0.075 | 0 | 0 |
| 9 | 8.01 | -0.035 | 0.025 | 3 | -0.015 | 0 | 0 |
| 10 | 8.04 | 0.005 | 0.03 | 4 | -0.045 | 0 | 0 |
| 11 | 8.02 | -0.025 | 0.005 | 5 | -0.025 | 0 | 0 |
| 12 | 8.01 | -0.035 | 0 | 0 | -0.015 | 0 | 0 |
| 13 | 8.05 | 0.015 | 0.015 | 1 | -0.055 | 0 | 0 |
| 14 | 8.04 | 0.005 | 0.02 | 2 | -0.045 | 0 | 0 |
| 15 | 8.03 | -0.045 | 0 | 0 | -0.05 | 0 | 0 |
| 16 | 8.05 | 0.015 | 0.015 | 1 | -0.055 | 0 | 0 |
| 17 | 8.06 | 0.025 | 0.04 | 2 | -0.065 | 0 | 0 |
| 18 | 8.04 | 0.005 | 0.045 | 3 | -0.045 | 0 | 0 |
| 19 | 8.05 | 0.015 | 0.06 | 4 | -0.055 | 0 | 0 |
| 20 | 8.06 | 0.025 | 0.085 | 5 | -0.065 | 0 | 0 |
| 21 | 8.04 | 0.005 | 0.09 | 6 | -0.045 | 0 | 0 |
| 22 | 8.02 | -0.025 | 0.065 | 7 | -0.025 | 0 | 0 |
| 23 | 8.03 | -0.045 | 0.02 | 8 | -0.05 | 0 | 0 |
| 24 | 8.05 | 0.015 | 0.035 | 9 | -0.055 | 0 | 0 |

   which never signals out of control, even though the process is clearly biased above the nominal mean.

(b) This is an individual measurements chart, so the best available estimate of $\sigma$ is $\frac{\bar{\text{MR}}}{d_2} = \frac{\bar{\text{MR}}}{1.128}$. Since MR = 0.187, that estimate is $\hat{\sigma} = 0.0166$. So the assumption that $\sigma = 0.05$ is not a good assumption.

4. (Montgomery 7-5) For $k = 0.25$, $h = 8.01$, we have $K = 0.0125$, $H = 0.4005$, and the chart changes to:

| $i$ | $x_i$ | $x_i - 8.0325$ | $C_i^+$ | $N^+$ | $8.0075 - x_i$ | $C_i^-$ | $N^-$ |
|---|---|---|---|---|---|---|---|
| 1 | 8.00 | -0.0325 | 0 | 0 | 0.0075 | 0.0075 | 1 |
| 2 | 8.01 | -0.0225 | 0 | 0 | -0.0025 | 0 | 0 |
| 3 | 8.02 | -0.0125 | 0 | 0 | -0.0125 | 0 | 0 |
| 4 | 8.01 | -0.0225 | 0 | 0 | -0.0025 | 0 | 0 |
| 5 | 8.00 | -0.0325 | 0 | 0 | 0.0075 | 0.0075 | 1 |
| 6 | 8.01 | -0.0225 | 0 | 0 | -0.0025 | 0 | 0 |
| 7 | 8.06 | 0.0275 | 0.0275 | 1 | -0.0525 | 0 | 0 |
| 8 | 8.07 | 0.0425 | 0.07 | 2 | -0.0625 | 0 | 0 |
| 9 | 8.01 | -0.0225 | 0.0475 | 3 | -0.0025 | 0 | 0 |
| 10 | 8.04 | 0.0075 | 0.065 | 4 | -0.0325 | 0 | 0 |
| 11 | 8.02 | -0.0125 | 0.0525 | 5 | -0.0125 | 0 | 0 |
| 12 | 8.01 | -0.0225 | 0.03 | 6 | -0.0025 | 0 | 0 |
| 13 | 8.05 | 0.0175 | 0.0475 | 7 | -0.0425 | 0 | 0 |
| 14 | 8.04 | 0.0075 | 0.055 | 8 | -0.0325 | 0 | 0 |
| 15 | 8.03 | -0.0025 | 0.0525 | 9 | -0.0225 | 0 | 0 |
| 16 | 8.05 | 0.0175 | 0.07 | 10 | -0.0425 | 0 | 0 |
| 17 | 8.06 | 0.0275 | 0.0975 | 11 | -0.0525 | 0 | 0 |
| 18 | 8.04 | 0.0075 | 0.105 | 12 | -0.0325 | 0 | 0 |
| 19 | 8.05 | 0.0175 | 0.1225 | 13 | -0.0425 | 0 | 0 |
| 20 | 8.06 | 0.0275 | 0.15 | 14 | -0.0525 | 0 | 0 |
| 21 | 8.04 | 0.0075 | 0.1575 | 15 | -0.0325 | 0 | 0 |
| 22 | 8.02 | -0.0125 | 0.145 | 16 | -0.0125 | 0 | 0 |
| 23 | 8.03 | -0.0025 | 0.1425 | 17 | -0.0225 | 0 | 0 |
| 24 | 8.05 | 0.0175 | 0.16 | 18 | -0.0425 | 0 | 0 |

Once again the process seems clearly biased above the nominal mean, but the chart is in control.

As for theoretic performances of the two charts, the $\text{ARL}_0$ calculations based on Siegmund's approximation (p. 323) are identical.

5. (Montgomery 7-6) For the FIR chart, we use $H = 0.2385$, $K = 0.0125$, and $C_0^+ = C_0^- = H/2 = 0.119$. Note that the new target is $\mu = 8.00$. The chart develops:

| $i$ | $x_i$ | $x_i - 8.0125$ | $C_i^+$ | $N^+$ | $7.9875 - x_i$ | $C_i^-$ | $N^-$ |
|---|---|---|---|---|---|---|---|
| 1 | 8.00 | -0.0125 | 0.1065 | 1 | -0.0125 | 0.1065 | 1 |
| 2 | 8.01 | -0.0025 | 0.104 | 2 | -0.0225 | 0 | 0 |
| 3 | 8.02 | 0.0075 | 0.1115 | 3 | -0.0325 | 0 | 0 |
| 4 | 8.01 | -0.0025 | 0.1090 | 4 | -0.0225 | 0 | 0 |
| 5 | 8.00 | -0.0125 | 0.0965 | 5 | -0.0125 | 0 | 0 |
| 6 | 8.01 | -0.0025 | 0.0940 | 6 | -0.0225 | 0 | 0 |
| 7 | 8.06 | 0.0475 | 0.1415 | 7 | -0.0725 | 0 | 0 |
| 8 | 8.07 | 0.0575 | 0.1990 | 8 | -0.0825 | 0 | 0 |
| 9 | 8.01 | -0.0025 | 0.1965 | 9 | -0.0225 | 0 | 0 |
| 10 | 8.04 | 0.0275 | 0.2240 | 10 | -0.0525 | 0 | 0 |
| 11 | 8.02 | 0.0075 | 0.2315 | 11 | -0.0325 | 0 | 0 |
| 12 | 8.01 | -0.0025 | 0.2290 | 12 | -0.0225 | 0 | 0 |
| 13 | 8.05 | 0.0375 | 0.2615 | 13 | -0.0525 | 0 | 0 |

which is out of control at the thirteenth sample.