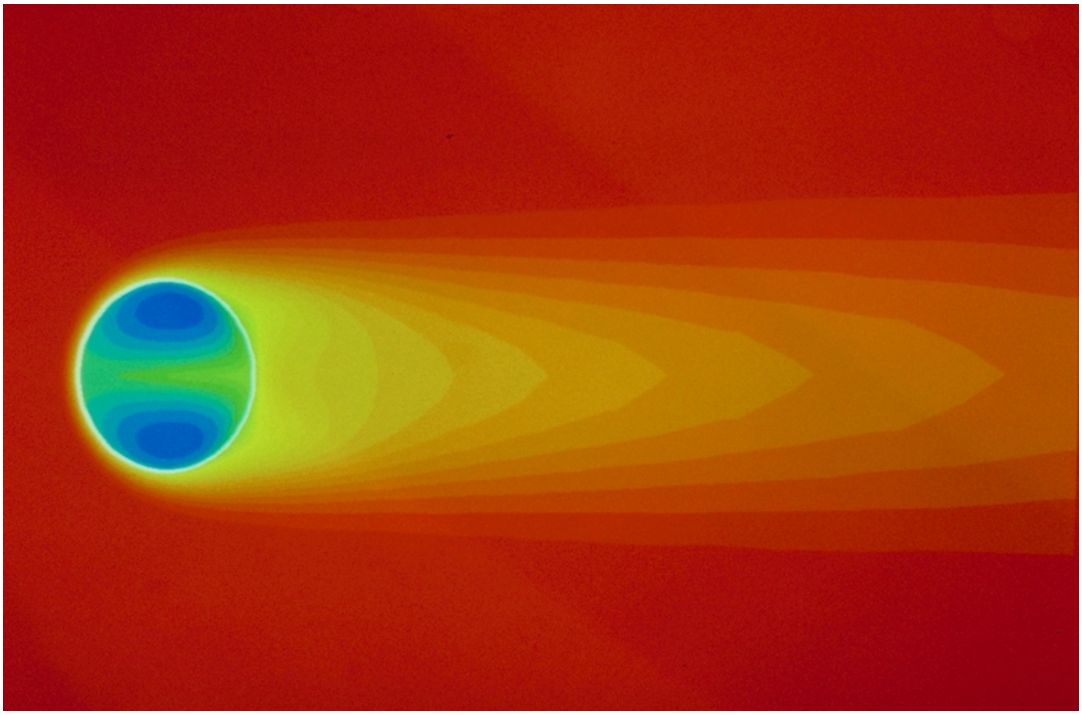
Laboratory Manual

FLUID MECHANICS

ME 211



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EXPERIMENT #4

INTEGRAL MOMENTUM EQUATION: IMPACT OF A JET

Objective

In this experiment, the force generated by a cylindrical water jet as it strikes a flat plate or a hemispherical cup is measured and compared with the force predicted using the integral momentum equation.

Theoretical background

Mechanical work can be produced by allowing fluid under pressure to strike the vanes of a turbine wheel. Rotational motion is then produced by the force generated as the jet strikes the vanes.

Let us consider a vane (Fig. 1) which is axisymmetric about the vertical (y) axis. If a fluid jet exits a circular nozzle with velocity v_1 along the y-axis and strikes the vane higher up, it is deflected by an angle β with respect to the horizontal direction (Fig. 1). The fluid leaves the vane with velocity v_3 along the angle of deflection. Because the jet is exposed to the ambient atmosphere, changes in static pressure after the fluid exits the nozzle can be neglected. Using the momentum equation along y, we can calculate the *force* F_y *exerted by the vane on the fluid* by considering the control volume shown in Fig. 1

$$-F_y = \int_{CS} v\rho \vec{V}.\vec{n}\, dA = v_2 \rho (-v_2) A_2 - v_3 \sin\beta \,\rho\, v_3 A_3 \qquad (1)$$

The mass conservation equation for the same control volume takes the form

$$\rho A_2 v_2 = \rho A_3 v_3 = \dot{M} \qquad (2)$$

Combining Eqs (1) and (2), we obtain

$$F_y = \dot{M}(v_2 + v_3 \sin\beta) \qquad (3)$$

Applying Bernoulli's equation between points 1–2 and 1–3, respectively

$$p_{atm} + \rho g h_1 + 1/2\rho v_1^2 = p_{atm} + \rho g h_2 + 1/2\rho v_2^2 \qquad (4)$$

$$p_{atm} + \rho g h_1 + 1/2\rho v_1^2 = p_{atm} + \rho g h_3 + 1/2\rho v_3^2 \qquad (5)$$

Finally, combining equations (3), (4) and (5) yields

$$F_y = \dot{M}[\sqrt{v_1^2 - 2g(h_2 - h_1)} + \sqrt{v_1^2 - 2g(h_3 - h_1)}\sin\beta] \qquad (6)$$

By measuring the mass flow rate and the elevation differences h_2–h_1 and h_3–h_1, and since

$$v_1 = \dot{M} / \rho A_1$$

the theoretical value of F_y can be determined using Eq. (6). Furthermore, Eq. (6) presents the functional relationship between F_y and the mass flow rate when the flow is ideal. Note that this relationship is *not* linear because of the dependence of v_1 on the flow rate.

Experiment outline

1. Review apparatus description and overall procedure.
2. Measure, calculate, or obtain the quantities listed in the attached table.
3. Plot the data as instructed.
4. Discuss possible causes for differences between experimental and theoretical results.
5. Follow the lab report write-up format given in this manual.

Apparatus (Fig. 2)

1. Water pumping table with a weighing device to measure the mass flow rate of the jet.
2. A round tapered nozzle used to form the vertical jet, which is deflected by either a flat plate or a hemispherical vane (see Figs. 1 and 2). The nozzle and vane are contained within a transparent cylinder.
3. A force balancing lever device for finding the resultant force on the vane (see Figs 2 and 3).
4. Timing mechanism.

Measurement method

As shown in Fig. 2, the vane (or the plate) is supported by a lever which carries a jockey weight. The lever can pivot around one end (fulcrum) and is restrained by a vertically mounted spring. With no flow, the lever is balanced by placing the jockey weight at its zero position, and then adjusting the nut above the spring. After the jet flow is turned on, the force generated by the impact of the jet causes the free end of the lever to move upward. The lever can be restored to its original balanced position by moving the jockey weight along the lever away from the spring. The impact force can then be measured using the analysis described below. The discussion refers to the schematic of Fig. 3, which depicts the forces acting on the lever.

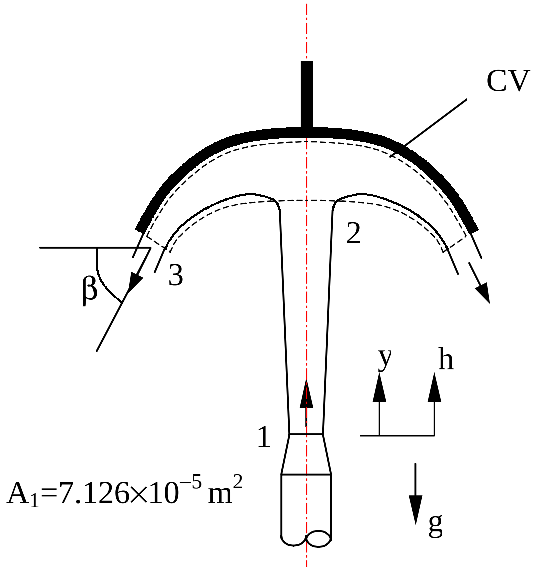

Fig. 1: Jet impingement detail showing the vane and the applicable control volume.

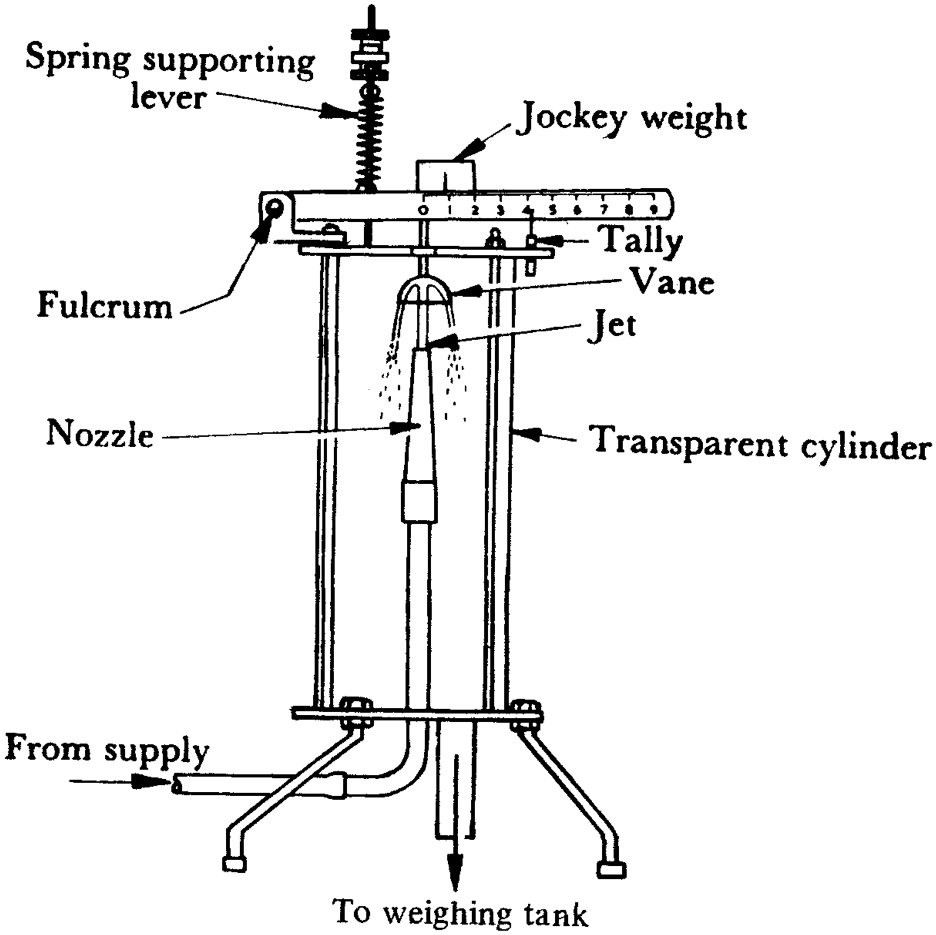

Fig. 2: Arrangement of jet impact apparatus

When the jet flow is absent and the lever is balanced, the jockey weight is at $x = 0$. In this case, the forces acting on the lever include the spring force F_s and the jockey weight F_w. In addition, the weight W_l of the lever (not shown in Fig. 3) is exerted at a distance l (unknown) from the point of pivot. The applicable moment equation is

$$F_s b = F_w a + W_l l \qquad (7)$$

When the jet flow is present, the lever balance is restored by moving the jockey weight at a distance x from its original position (see Fig. 3). The force exerted by the jet on the plate/vane is depicted by F'_y in Fig. 3. This force is equal and opposite to the force exerted by the plate/vane on the fluid, i.e., $F'_y = -F_y$. The moment equation takes the form

$$F_s b + F'_y a = F_w (a + x) + W_l l \qquad (8)$$

Combining Eqs (7) and (8) we obtain

$$F'_y = -F_y = F_w \frac{x}{a} = \frac{5.34N}{0.1524m} x = 35.04x \qquad (9)$$

with x in meters and F'_y (likewise F_y) in N. Thus, measurement of x produces (through Eq. 9) a value for the force exerted by the vane/plate on the fluid. This force can be compared to the theoretical value obtained from Eq. (6).

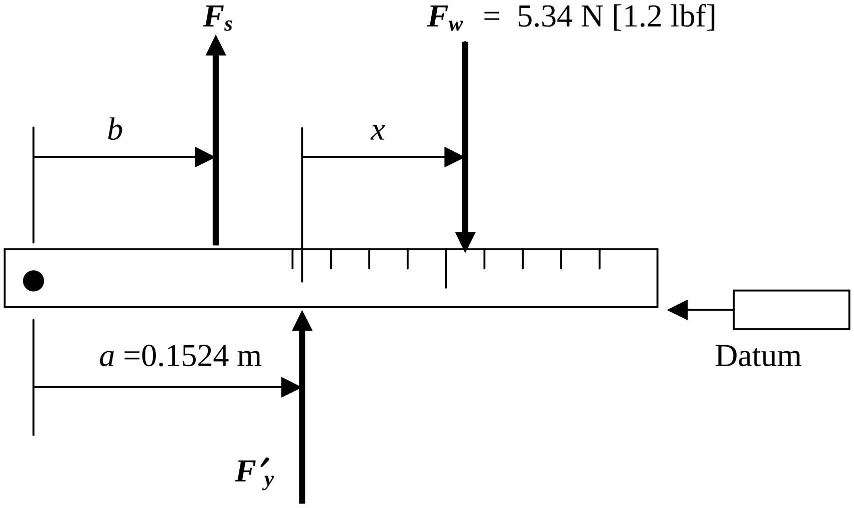

Fig. 3: Schematic of the forces exerted on the lever set.

Using the experimental measurements, record your data in Table 1 and plot the measured magnitude of F_y versus the mass flow rate. On the same plot, show the theoretical variation of F_y with flow rate. i.e., Eq. (6).

Procedure

1. Level the apparatus by first setting the lever to the balanced position (as indicated by the tally) with the jockey weight placed at its zero position.

2. Using the bench valve, increase the flow rate of water to its maximum, then restore the lever to its balanced position by moving the jockey weight towards the free end of the lever. Record the position x of the jockey weight at the new balanced lever configuration.

3. By decreasing the rate of flow, rebalance the lever by repositioning the jockey weight. Repeat this procedure for a series of about eight different flow rates and record the corresponding values of x. Try to attain roughly equally spaced positions of the jockey weight for these different flow rates.

4. Obtain measurements for both the hemispherical and flat vanes.

Questions

1. Discuss why the magnitude of the measured force F'_y is different from the theoretical value F_y for the same mass flow rate. Does the comparison indicate weaknesses in the theoretical model?

2. Comparing the relative error ($F_y - F'_y / F_y$) for the two vanes, can one deduce sources of error due to the shape of the vanes?

3. In Fig. 1, why is the jet cross sectional area at station 2 different from the area at station 1?

4. Comparing the force exerted on the hemispherical vane with the one on the flat plate, which one is greater? Why? Explain both in mathematical and physical terms.

5. If the distance between the vane and the nozzle changes, how do you expect F_y to be influenced by this change?

DATA TAKING SHEET

Conversions
1 lbm = 0.4536 kg **1 in. = 0.0254 m**

FLAT PLATE **$\beta = 0°$, $h_2 - h_1 = h_3 - h_1 = 0.06m$**					
Run	*W [lb]*	*M [kg]*	*t [s]*	*x*	
				[in]	*[m]*
1					
2					
3					
4					
5					
6					
7					
8					

HEMISPHERICAL CUP **$\beta = 90°$, $h_2 - h_1 = 0.06m$** **$h_3 - h_1 = 0.03m$**					
Run	*W [lb]*	*M [kg]*	*t [s]*	*x*	
				[in]	*[m]*
1					
2					
3					
4					
5					
6					
7					
8					

TABLE 1

FLAT PLATE $\beta = 0°$, $h_2 - h_1 = h_3 - h_1 = 0.06m$							
Run	*M[kg]*	*t [s]*	$\dot{M}$ *[kg / s]*	v_1 *[m/s]*	*Fy [N]* Eq. (6)	*x[m]*	*F'y [N]* Eq. (9)
1							
2							
3							
4							
5							
6							
7							
8							

HEMISPHERICAL CUP $\beta = 90°$, $h_2 - h_1 = 0.06m$; $h_3 - h_1 = 0.03m$							
Run	*M[kg]*	*t [s]*	$\dot{M}$ *[kg/s]*	v_1 *[m/s]*	*Fy [N]* Eq. (6)	*x[m]*	*F'y [N]* Eq. (9)
1							
2							
3							
4							
5							
6							
7							
8							