Example 1

Air is isobarically expanded from state 1 (P_1 = 1 bar, v_1 = 1 m^3 kg^{-1}), to state 2 (P_2 = 1 bar, v_2 = 3 m^3 kg^{-1}), and then compressed isometrically to state 3 (P_3 = 3 bar, v_3 = 3 m^3 kg^{-1}). Determine the final temperature and the net work.
Air is isometrically compressed from state 1 (P_1 = 1 bar, v_1 = 1 m^3 kg^{-1}), to state 4 (P_4 = 3 bar, v_4 = 1 m^3 kg^{-1}), and then expanded isobarically to state 3 (P_3 = 3 bar, v_3 = 3 m^3 kg^{-1}). Determine the final temperature and the net work.

Solution

The P–v diagram for this example is illustrated below. The final temperature T_3 is independent of the work path, and

$$T_3 = P_3v_3/R = 300\times3\div0.287 = 3136 \text{ K}.$$

The work along the two paths

$$w_{123} = P_1 (v_2 - v_1) = 1 \times 100 \times (3 - 1) = 200 \text{ kJ kg}^{-1}, \text{ and} \quad (A)$$

$$w_{143} = P_4 (v_3 - v_2) = 3 \times 100 \times (3 - 1) = 600 \text{ kJ kg}^{-1}. \quad (B)$$

Remarks

The net work in the second case, i.e., w_{143}, is larger compared to W_{123}. The temperature represents the state of the system, and its functional form, e.g., $T_3 = P_3v_3/R$, is *independent of the path* selected to reach that state. However, the work expressions w_{123} and w_{143} (Eqs. A and B) depend upon the path selected to reach the same final state, even though the expressions for work (contain variables that only represent properties. Therefore, the final temperature is path independent, but the net work is not.
The inexact differential W integrated between two identical states along dissimilar paths 1–2–3 and 1–4–3 yields different results. An inexact differential can only be integrated if its path is known.

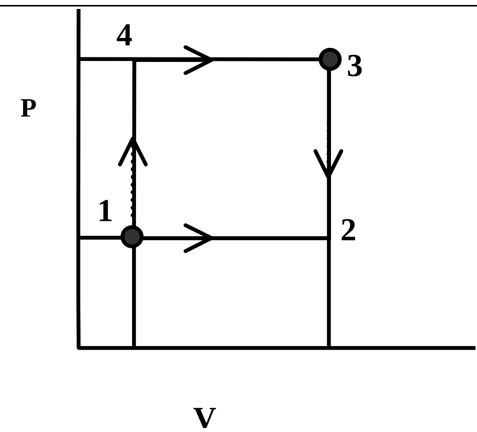

Figure: P–v diagram with P expressed in units of bar and v in m^3 kg^{-1}.

Example 2

One kilogram of water at a temperature $T = T_{ref} = T_{tp}$ = 0.01ºC is contained in an adiabatic piston cylinder assembly. The assembly resides in an evacuated chamber and a weight is placed on top of the piston such that $P = P_{ref}$ = 0.61 kPa. At these reference conditions, the specific volume $v(T_{ref},P_{ref})$ = 0.001 m^3 kg^{-1} is assumed to be independent of temperature. During an isobaric process, a current of 0.26 A provided at a potential of 110 V over a duration of 60.96 min raises the water temperature to 25ºC. Determine the enthalpy of water at that state if h_{ref} = 0.

Solution

We will use the energy conservation equation
$\delta Q - \delta W = dU$
and select the water mass as the system. In general, the work term will include a volumetric change component in addition to the electrical work so that
$\delta Q - PdV - \delta W_{elec} = dU$.
At constant pressure, $\delta Q_P - \delta W_{elec,P} = dU + PdV = dH$, and on a unit mass basis

$\delta q_P - \delta w_{elec,P} = du + Pdv = dh.$

Recalling that the system is adiabatic ($q_P = 0$), and integrating the latter expression

– $w_{elec,P} = h - h_{ref}$.

Example 3

Air is contained in an adiabatic piston cylinder assembly at P_1 = 100 kPa, V_1 = 0.1 m^3, and T_1 = 300 K. The piston is constrained with a pin, and its area A is 0.01 m^2. Vacuum surrounds the assembly. A weight Wt of 2 kN is rolled on to the piston, and the pin is released. Assuming that k_o ($=c_p/c_v$) = 1.4, and c_{vo} = 0.7 kJ kg^{-1} K^{-1},

Is the process 1–2 reversible or irreversible?

What are the final pressure, volume, and temperature?

Solution

We will select our system to include both the air and the weight rather than the air alone because the sudden process by which it changes state cannot be completely characterized. The process is clearly irreversible, since the system cannot be restored to its initial state unless the weight is lifted back to its original position, which requires extra work.

$$P_2 A = Wt, \text{ or } P_2 = Wt/A. \quad (A)$$

With Wt = 2 kN, $P_2 = 2 \div 0.01 = 200$ kPa.

Applying the First Law to the system,

$Q_{12} - W_{12} = 0 = E_2 - E_1$, or $E_2 = E_1$.

Neglecting the kinetic energy,

$$E_2 = U_2 + Wt\, Z_2, \text{ and } E_1 = U_1 + Wt\, Z_1. \quad (B)$$

Substituting Eq. (A) in (B), since $E_2 = E_1$,

$U_2 - U_1 = Wt\,(Z_2 - Z_1) = Wt\,(V_2 - V_1) \div A = P_2\,(V_1 - V_2)$, or

$$m\,(u_2 - u_1) = Wt\,(V_1 - V_2) \div A. \quad (C)$$

Treating the air as an ideal gas, Eq. (C) may be written in the form

$$m\, c_{vo}\,(T_2 - T_1) = Wt\,(V_1 - V_2) \div A. \quad (D)$$

The two unknowns in Eq. (D) are T_2, V_2, so that an additional equation is required to solve the problem. Invoking the ideal gas law for the fixed mass

$$P_1V_1/RT_1 = P_2V_2/RT_2, \quad (E)$$

Equations (D) and (E) provide the solution for V_2 and T_2. Substituting for V_2 from Eq. (E) in (D), we obtain a solution for T_2/T_1, namely,

$$T_2/T_1 = (P_2/P_1 + c_{vo}/R)/(1 + c_{vo}/R). \quad (F)$$

Using $R = \bar{R}/M = 8.314 \div 28.97 = 0.287$ kJ kg^{-1} K^{-1},

$T_2/T_1 = (200 \div 100 + 0.7 \div 0.287) \div (1 + 0.7 \div 0.287) = 1.29$, or

$T_2 = 387$ K.

Substituting this result in Eq. (E),

$$V_2/V_1 = (1 + (c_{vo}/R)(P_1/P_2))/(1 + c_{vo}/R). \quad (G)$$

$\therefore V_2/V_1 = (1 + 0.7 \times 100 \div (0.287 \times 200)) \div (1 + 0.7 \div 0.287) = 0.65$, and

$V_2 = 0.65 \times 0.1 = 0.065$ m^3.

Remarks

The potential energy of the weight is converted into thermal energy in air.

Once P_2 and T_2 are known, it is possible to determine k_o ($= c_{po}/c_{vo}$) for an ideal gas using Eq. (F). Furthermore, employing the identity $\bar{R} = \bar{c}_{p,o} - \bar{c}_{v,o}$ it is possible to calculate the molar specific heats. The gas molecular weight is required in order to ascertain the mass–based specific heats.

If the ambient pressure is finite, then Eq. (F) and (G) remain unaffected, but P_2= W/A + P_o.

Example 4

A rigid cylinder is charged with an ideal gas through a pressurized line, and the flow is choked. Determine:

The enthalpy in the tank at a time t » 0, assuming $m_{t=0}$ = 0.

A relation between the cylinder and line temperatures.

The cylinder temperature, pressure, and mass as a function of time.

Solution

For this problem, $\dot{m}_e = 0$ so that

$$dm_{c.v}/dt = \dot{m}_i . \quad \text{(A)}$$

Assuming that the gas charging occurs over a short duration, heat losses can be ignored, i.e., $\dot{Q}_{cv} = 0$. Furthermore, the kinetic and potential energies, and boundary and shaft work can also be assumed negligible, and, using Eq. (50),

$$(dU_{c.v}/dt) = \dot{m}_i h_i. \quad \text{(B)}$$

Assuming a uniform state in the c.v, $U_{c.v} = \int u\rho dV = u\rho\int dV = um$, and Eq. (B) may be written in the form

$$(d(mu)/dt) = \dot{m}_i h_i, \text{ or} \quad \text{(C)}$$

$$m\, du/dt + u\, dm/dt = \dot{m}_i h_i. \quad \text{(D)}$$

From Eqs. (A) and (D)

$$dm/dt\,(h_i - u) = m\, du/dt. \quad \text{(E)}$$

Assuming the inlet state to be at steady state, i.e., $h_i \neq h(t)$, Eq. (E) may be written as

$$dm/m = du/(h_i - u), \quad \text{(F)}$$

which upon integration, using the initial conditions $u = u_o$, $m = m_o$, at t = 0, assumes the form

$$m/m_o = (h_i - u_o)/(h_i - u). \quad \text{(G)}$$

From Eq. (G)

$$m(h_i - u) = m_o(h_i - u_o). \quad \text{(H)}$$

With $m = m_0 + m_i$ and simplifying

$$u= h_i -\{m_o /(m_0 + m_i)\} (h_i - u_o) \quad \text{(I)}$$

It should be noted that eq.(I) is valid whether the matter is ideal gas or not, Since $dh = c_{po}dT$ and $du = c_{vo}dT$, and c_{po} and c_{vo} remain constant, $h_i = c_{po}T_i$ and $u = c_{vo}T$. Using these relations in Eq. (I) we obtain,

$$T= \{m_o T_0 + m_i k_0 T_i \}/(m_0 + m_i) \quad \text{(J)}$$

where $k_o = c_{po}/c_{vo}$. The pressure P in tank is given by P(t)= m (t) RT/V; thus

$$P(t) = \{m_o T_0 + m_i \ k_0 T_i\} R/V \quad (K)$$

If $m_o = 0$, from Eq. (H), $m(h_i - u) = 0$. Since $m \neq 0$, for this initial condition

$$h_i = u. \quad (L)$$

It should be noted that Eq.(I) is valid whether the matter is ideal gas or not. Now assuming matter to be ideal gas, $dh = c_{po}dT$ and $du = c_{vo}dT$. Further with constant c_{po} and c_{vo}, $h_i = c_{po}T_i$ and $u = c_{vo}T$. Using these relations in Eq. (L) we obtain,

$$T = k_oT_i, \quad (M)$$

where $k_o = c_{po}/c_{vo}$. When the line pressure is large, gas dynamic considerations indicate that the flow is choked. Therefore, the mass flow rate depends only upon the line pressure and temperature. For fixed line conditions, $\dot{m}_i$ is a constant irrespective of the downstream cylinder pressure. Integrating Eq. (A) with the initial condition $m(t = 0) = m_o = 0$,

$$m = \dot{m}_i t + m_o = \dot{m}_i t. \quad (N)$$

From the ideal gas law $m(t) = P(t)V/RT$. Using Eqs. (M) and (N) we can solve for pressure

$$P(t) = \dot{m}_i t R k_oT_i/V = \alpha t. \quad (L)$$

Remarks

In the initial periods m_i is comparable to m_0, and as such T will be increasing as more mass is added (Eq. (J) . As $m_i >>> m_o$, then temperature reaches a constant value as indicated by Eq.(M). You can obtain (M) directly from (J) by setting $m_0 = 0$.

The temperature in the c.v is higher due to the conversion of flow work (i.e., the pumping work performed in pushing the mass into the cylinder) into thermal energy in the form of u. The mass entering the cylinder contains an enthalpy h which is converted into u, i.e., the Pv (or flow work) at the inlet is converted into internal energy.
Eq. (L) states that $P(t)/t = \alpha$. If $\dot{m}_i$, T_i, and R are known, α may be determined by examining the time gradient of P(t). Once α is known, the ratio k_o can be calculated, Since $R=c_{po}-c_{vo}$, than c_{vo} and c_{po} can be determined.
The kinetic energy of the fluid entering the tank has been neglected in this analysis. If the inlet line has a large diameter, the flow velocity is relatively low, and the enthalpy of the fluid in the line is at its stagnation enthalpy (since, $h_{stg} = h+V^2/2$ where h is called static enthalpy). At the throat of the tank, the enthalpy $h_{throat} < h$. but $V_{throat} >>> V$ and h_{stg} is same as in the line. Once the fluid enters the cylinder (of far larger diameter than the line), assuming the c.v to be situated several throat diameters downstream, $V^2/2 = 0$. Therefore, in this case the enthalpy of fluid entering the c.v is the same as the stagnation enthalpy that in a large diameter line.

If the cylinder is charged with a reciprocating compressor, the mass flow will depend upon the cylinder pressure, and hence mass flow may vary with time. Such an analysis has been performed before

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Example 5

Gas is discharged from a pressurized rigid cylinder. Determine the change in pressure in a rigid cylinder as the specific volume of the gas ($v = V/m$) contained in it changes.

Solution

For this problem, $\dot{m}_i = 0$, and $m_{c.v} = m$, and Eq. (32) simplifies to the form

$$(dm/dt) = -\dot{m}_e . \quad (A)$$

For a relatively short time period δt

$$dm = -dm_e. \quad (B)$$

For a rigid cylinder $\delta W_d = 0$. Since there is no shaft work, and the potential energy is negligible, Eq. (50) assumes the form

$$(dE_{c.v}/dt) = \dot{Q}_{cv} - \dot{m}_e\,(h_e + ke_e).$$

For the duration δt

$$dE_{c.v} = \delta Q_{c.v} - dm_e\,(h_e + ke_e). \quad (C)$$

Note that as matter is discharged, the cylinder temperature and pressure may vary so that h_e can change over time.

When gas leaves the tank its enthalpy h_e differs from that of the stagnant gas in the tank. For the mass near the exit, the specific energy e includes the energies of the stagnant and moving gas (i.e., u + ke). The energy balance between a unit mass of exiting gas and stagnant gas yields $h_e + ke_e = h$ (see Example 11). Omitting the subscript c.v, Eq. (C) may be written as

$$d(em) = \delta Q_{c.v} - dm_e\,h. \quad (D)$$

The specific energy is not uniform in the c.v. However, the mass containing kinetic energy adjacent to the exit is small as compared to the rest of the mass of stagnant gas. Therefore, the assumption $e \approx u$ or $E_{c.v} = U_{c.v} = m\times u$ is a good approximation. Expanding the LHS of Eq. (D) and using Eq. (B) to eliminate dm_e,

$$m\,du + u\,dm = \delta Q + dm\,h.$$

If the system is adiabatic, $\delta Q = 0$, and

$$dm\,(h - u) = m\,du. \quad (E)$$

Since $m = V/v$, $\ln(m) = \ln(V) - \ln(v)$ so that

$$dm/m = -\,dv/v. \quad (F)$$

Using Eqs. (E) and (F)

$$-(dv/v)\,(h - u) = du, \text{ or } -dv\,((Pv)/v) = du, \text{ i.e., } du + P\,dv = 0. \quad (G)$$

The relation in Eq. (G) is independent of the nature of the system. Assuming the gas to be ideal, $du = c_{vo}\,dT$ and $P = RT/v$, i.e., $R = c_{po} - c_{vo}$. Using these relations in Eq. (G),

$$(c_{vo}/T)\,dT = -((c_{po} - c_{vo})/v)\,dv. \quad (H)$$

Therefore,

$$dT/T = -(k_o - 1)\,(dv/v), \text{ that, upon integration, results in} \quad (I)$$

$$Tv^{k-1} = \text{Constant}. \quad (J)$$

Using the ideal gas law to replace T (with Pv/R) in Eq. (J), and simplifying

$$Pv^{k} = \text{Constant}. \quad (K)$$

As the specific volume increases due to a decrease in mass, the pressure also decreases. If the value of k is known, both temperature and pressure can be predicted.

Remarks

The above step by step procedure illustrates the process of simplification while using valid assumptions.

The Washburn experiment (discussed later in Chapter 7) involves gas discharge from a tank into the atmosphere with the pressurized tank and its valves immersed in an isothermal bath. The gas leaving the tank is always at the bath temperature. In that case, Pv = constant if gas is ideal, i.e., k = 1.

Example 6

Suppose pressurized air is admitted into a pneumatic piston–cylinder assembly that jacks up an automobile. As the cylinder of cross sectional area A is pressurized by the air, the force cA on the piston exceeds the weight of the car, thereby lifting it against gravity. Determine the volume and temperature, V(t) and T(t), for any given mass m(t).

Solution

For this problem, $\dot{m}_e = 0$, and $m_{c.v} = m$, and Eq. (32) simplifies to the form

$$(dm/dt) = -\dot{m}_i \,. \tag{A}$$

Assume the system to be adiabatic, and the kinetic and potential energies to be negligible. Therefore, h_i is time independent. Since the c.v is deformed, deformation work $\dot{W}_d$ is performed, and from Eq. (50)

$$(dUc.v/dt) = -\dot{W}_d + \dot{m}_i\, hi, \text{ where} \tag{B}$$

$$\dot{W}_d = P\, dV/dt. \tag{C}$$

If weight of automobile is W, then P= W/A. Initially the pressure is less than P and as such volume will not change until $P \geq P_0$. If the gas is admitted over a small duration δt, multiplying Eq. (B) by that period we have

$$dU = -P\, dV + d\, m_i h_i. \tag{D}$$

In the initial periods when PA is less than the weight, dV= 0; further If h_i is invariant, then integrating Eq.(D),

$$U - U_0 = \int_{V_0}^{V(t)} PdV + m_i\, h_i$$

Then

$$U - U_0 = m_i\, h_i,\ P < P, \tag{E}$$

$$\text{and}\quad (U - U_o) = -P_W\,(V - V_0) + m_i h_i\ ,\ P \geq P_0 \tag{F}$$

where subscript "0" represents the initial conditions. Eq. (E) presents results for charging problem (example 14). Assuming uniform properties within the c.v, U=mu and, hence, Eq.(F) becomes

$$(m\,u - m_0 u_o) = -P(V - V_0) + m_i h_i\,. \tag{G}$$

where V_o, m_o, and u_o are the initial volume, mass and specific internal energy. Note that if we set $V = V_0$, (i.e piston is not moving, the Eq. (G) converts back to charging problem solved in Example (14)). Employing the ideal gas law

$$m = P(t)V(t)/RT, \text{ and } m_o = P_0 V_o/RT_o. \tag{H}$$

Using Eqs. (G) and (H),

$$(PV/RT)\, c_{vo}\, R\, T(t) - (P_o V_o/RT_0)\, c_{vo}\, T_0 + P(V - V_o) = m_i h_i. \text{ i.e.,} \tag{I}$$

$$(PV - P_o V_o)(c_{vo}/R) + P_W(V - V_o)\,\Omega = m_i h_i. \tag{J}$$

With $R = c_{po} - c_{vo}$, solving for mass entered. One can determine m(t) at any time from inlet mass flow rate, i.e.,

$$m(t) = m_o + \dot{m}_i\ t = m_o + m_i\ (t) \qquad (K)$$

Once $\dot{m}_i$ is known, then m (t) is known from Eq. (K) and with known P, V(t) can be calculated using Eq. (J). Using ideal gas law ,

$$T(t)/T_0 = \{PV/(P_oV_o)\}/[1 + \{PV\ k/(P_oV_o) - (1+ (k-1)\ P/ P_o)\}\ (T_o/ kT_i)] \qquad (L)$$

Remarks

1.Charging Period

During the initial period, before the piston starts moving, $V = V_0$. Thus the air is charging the cylinder just like example 14 with increasing temperature. Letting $V \rightarrow V_o$ in Eq. (L) we recover the charging solution

$$T/T_o = \{P/P_o\}/[1 + \{P\ k/P_o - (1+ (k-1)\ P/ P_o)\}\ (T_o/ kT_i)] \qquad (M)$$

Which yields the expression for temperature of gas as a function of gas pressure in the cylinder. i.e if e place different cars, we will have different P's and hence different T. If initial pressure $P_o \approx 0$ in Eq. (M) reduces to $T = k\ Ti$ (as shown in Example 14).

2. Lifting Period

If the initial pressure $P_0 = P$ (i.e initial pressure is such that the weight can be lifted up with gas flow in) , then Eq.(L) shows that

$$T/T_o = \{V(t)/V_o\}/[1 + (V(t)/V_o -1)(T_o/ T_i)],,\ V(t) \geq V0\ , \qquad (N)$$

Which is independent of medium of gas pumped in! Then it follows that

$$m_i = P\ V(t) / [R\ T(t)] = P\ k/(k-1)\ [V-V_o\ ]/ h_i \qquad (O)$$

3.. For $V >>> V_0$, Eq. (N) simplifies to the form $T = T_i$ and Eq. (O) yields $m_i = m = PV/\{c_{p0}\ T_i\}$. In this example flow work is employed to lift the automobile so that $T = T_i$.
