
Example 1

Air is adiabatically and reversibly compressed from $P_1 = 1$ bar, and $T_1 = 300$ K to $P_2 = 10$ bar. Heat is then added at constant volume from a reservoir at 1000 K (T_R) until the air temperature reaches 900 K (T_3). During heat addition, about 10% of the added heat is lost to the ambient at 298 K. Determine:

The entropy generated σ_{12} in $\text{kJ kg}^{-1} \text{K}^{-1}$ for the first process 1–2;

The net heat added to the matter;

The heat supplied by the reservoir;

The entropy generated in isolated system during process from (2) to (3).

Solution

$$S_2 - S_1 - \int \delta Q/T_b = \sigma_{12}. \quad (\text{A})$$

Since the process is reversible,

$$\sigma_{12} = 0, \quad (\text{B})$$

which implies that no gradients exist within the system. Therefore,

$$T_b = T. \quad (\text{C})$$

Using Eqs. (A), (B), and (C)

$$S_2 - S_1 = \int \delta Q/T. \quad (\text{D})$$

Since the process is adiabatic $\delta Q = 0$, and $S_2 = S_1$ or $s_2 = s_1$. At state 1, from the air tables (Tables A-7), $p_{r1} = 1.386$, $u_1 = 214.07$, and $h_1 = 300.19$. Therefore, $s_1 = s^0(T_1) - R \ln P/1 = 1.702 - 0 = 1.702 \text{ kJ kg}^{-1} \text{K}^{-1}$.

For the isentropic process

$$p_{r2}(T_2)/p_{r1}(T_1) = p_2/p_1 = 10. \text{ Hence, } p_{r2} = p_{r1} 10 = 1.386 \times 10 = 13.86 \text{ so that } T_2 = 574 \text{ K, } u_2 = 415 \text{ kJ kg}^{-1}, h_2 = 580 \text{ kJ kg}^{-1}, \text{ and } s_2 = s_1 = 1.702 \text{ kJ kg}^{-1} \text{K}^{-1}.$$

Temperature gradients can develop inside a system during heat addition from a thermal reservoir or heat loss to the ambient, thereby making a process internally irreversible. In this example, the final states are assumed to be at equilibrium. Applying the First Law to the constant volume process, the heat added to the system can be evaluated as follows

$$q_{23} = u_3 - u_2 = 674.58 - 415 = 260 \text{ kJ kg}^{-1}.$$

If q_R denotes the heat supplied by reservoir, the heat added $q_{23} = 0.9 q_R$, i.e.

$$q_R = 288.88 \text{ kJ kg}^{-1}.$$

The heat loss to the ambient is $q_0 = 288.88 - 260 = 28.88 \text{ kJ kg}^{-1}$.

Since we must determine the entropy of an isolated system, assuming that there are no gradients outside that system, and selecting the system boundaries to include the reservoir at T_R and the ambient at T_0 , it follows that

$$s_3 - s_2 - q_R/T_R - q_0/T_0 = \sigma.$$

Now, $P_3/P_2 = T_3/T_2 = 900/574$, i.e., $P_3 = 15.68 \text{ atm}$, and $s_3 = 2.849 - 0.287 \ln(15.68/1) = 2.059 \text{ kJ kg}^{-1} \text{K}^{-1}$. Therefore,

$$2.059 - 1.702 - (289/1000) - (-29/298) = \sigma \text{ so that}$$

$$\sigma = 0.165 \text{ kJ kg}^{-1} \text{K}^{-1}.$$

Remarks

It is possible to tabulate p_r values for a particular gas using Eq. (50c), and the included software.

Example 2

A piston–cylinder assembly contains a 0.1 kmole mixture consisting of 40% CO_2 and 60% N_2 at 10 bars and 1000 K (state 1). The mixture is heated to 11 bars and 1200 K (state 2). The work output from the assembly is 65.3 kJ. Evaluate the entropy change $S_2 - S_1$ and σ_{12} for the following cases:

The boundary temperature T_b equals that of the gas mixture.

T_b is fixed and equals 1300 K during heat up.

Solution

$$S = \sum N_k \bar{s}_k(T, P_k) \quad (A)$$

For the mixture

$$S = N_{\text{CO}_2} \bar{s}_{\text{CO}_2}(T, P_{\text{CO}_2}) + N_{\text{N}_2} \bar{s}_{\text{N}_2}(T, P_{\text{N}_2}), \quad (B)$$

$$S_1 = [N_{\text{CO}_2} \bar{s}_{\text{CO}_2}(T, P_{\text{CO}_2}) + N_{\text{N}_2} \bar{s}_{\text{N}_2}(T, P_{\text{N}_2})]_1, \quad (C)$$

$$S_2 = [N_{\text{CO}_2} \bar{s}_{\text{CO}_2}(T, P_{\text{CO}_2}) + N_{\text{N}_2} \bar{s}_{\text{N}_2}(T, P_{\text{N}_2})]_2, \text{ where} \quad (D)$$

$$N_{\text{CO}_2} = 0.4 \times 0.1 = 0.04 \text{ kmole, and } N_{\text{N}_2} = 0.6 \times 0.1 = 0.06 \text{ kmole.}$$

Now, $\bar{s}_{\text{CO}_2}(T, P_k) = \bar{s}_{\text{CO}_2}^0(T) - \bar{R} \ln(P_{\text{CO}_2}/1)$, where

$$(P_{\text{CO}_2})_1 = 0.4 \times 10 = 4 \text{ bar, } (P_{\text{CO}_2})_2 = 0.4 \times 11 = 4.4 \text{ bar, and}$$

$$(P_{\text{N}_2})_1 = 0.6 \times 10 = 6 \text{ bar, } (P_{\text{N}_2})_2 = 0.6 \times 11 = 6.6 \text{ bar.}$$

Therefore, at conditions 1 and 2, respectively,

$$\begin{aligned} \bar{s}_{\text{CO}_2}(1200\text{K}, 4.4 \text{ bar}) &= \bar{s}_{\text{CO}_2}^0(1200 \text{ K}) - \bar{R} \ln(4.4 \div 1) \\ &= 234.1 - 8.314 \times \ln(4.4 \div 1) = 221.8 \text{ kJ kmole}^{-1} \text{ K}^{-1}, \text{ and,} \end{aligned}$$

$$\bar{s}_{\text{CO}_2}(1000\text{K}, 4 \text{ bar}) = \bar{s}_{\text{CO}_2}^0(1000 \text{ K}) - \bar{R} \ln(4 \div 1) = 216.6 \text{ kJ kmole}^{-1} \text{ K}^{-1}.$$

Likewise,

$$\begin{aligned} \bar{s}_{\text{N}_2}(1200\text{K}, 6.6 \text{ bar}) &= \bar{s}_{\text{N}_2}^0(1200 \text{ K}) - \bar{R} \ln(6.6 \div 1) \\ &= 279.3 - 8.314 \times \ln(6.6 \div 1) = 263.6 \text{ kJ kmole}^{-1} \text{ K}^{-1}, \text{ and} \end{aligned}$$

$$\bar{s}_{\text{N}_2}(1000\text{K}, 6 \text{ bar}) = 269.2 - 8.314 \times \ln 6 = 254.3 \text{ kJ kmole}^{-1} \text{ K}^{-1}.$$

Using Eqs. (C) and (D)

$$\begin{aligned} S_1 &= 0.04 \times 216.6 + 0.06 \times 254.3 = 23.92 \text{ kJ K}^{-1}, \\ S_2 &= 0.04 \times 221.8 + 0.06 \times 263.6 = 24.69 \text{ kJ K}^{-1}, \text{ and} \\ S_2 - S_1 &= 24.69 - 23.92 = 0.77 \text{ kJ K}^{-1}. \end{aligned}$$

$$S_2 - S_1 - Q_{12}/T_b = \sigma_{12}. \quad (E)$$

Applying the First Law, $Q_{12} = U_2 - U_1 + W_{12}$. Therefore,

$$U_2 = 0.04 \times 43871 + 0.06 \times 26799 = 3362.8 \text{ kJ,}$$

$$U_1 = 0.04 \times 34455 + 0.06 \times 21815 = 2687.1 \text{ kJ, and}$$

$$Q_{12} = 3362.8 - 2687.1 + 65.3 = 741 \text{ kJ.}$$

Using these results in Eq. (E)

$$\sigma_{12} = 0.77 - 741/1300 = 0.2 \text{ kJ K}^{-1}.$$

Example 3

0.5 kg of coffee is contained in a cup at a temperature of 370 K. The cup is kept in an insulated room containing air at a temperature of 300 K so that, after some time, it cools to 360 K. The air

mass is 100 kg. Assume the properties of coffee to be the same as those of water, and determine the following:

The change in internal energy $dU (= dU_{\text{coffee}} + dU_{\text{air}})$.

The initial entropy of the coffee and air if it is assumed that both subsystems exist at an equilibrium state.

The heat transfer across the cup boundary δQ .

The temperature change of the air.

The entropy change of coffee dS_{coffee} .

The entropy change of air dS_{air} .

The entropy generated.

Solution

Consider an isolated composite system consisting of two subsystems, i.e., coffee and air. In the absence of external interactions, isolated systems attain a stable equilibrium state. The internal energy change $dU = 0$ by applying the First Law for combined system, i.e.,

$$dU_{\text{coffee}} = -dU_{\text{air}}. \quad (\text{A})$$

$$S_{\text{coffee}} = m s = 0.5 \times s_{f,370 \text{ K}} = 0.5 \times 1.25 = 0.625 \text{ kJ K}^{-1}. \quad (\text{B})$$

$$S_{\text{air}} = 100 \times s_{300 \text{ K}} = 100 \times 1.7 = 170 \text{ kJ K}^{-1}. \quad (\text{C})$$

Therefore,

$$S = 170.625 \text{ kJ K}^{-1}. \quad (\text{D})$$

For the coffee $\delta Q - \delta W = dU_{\text{coffee}} = m c dT_{\text{coffee}} = 0.5 \times 4.184 \times (-10) = -20.92 \text{ kJ}$.

Since there is no volume change, $\delta W = 0$, and

$$\delta Q = -20.92 \text{ kJ}. \quad (\text{E})$$

Applying the First Law $dU_{\text{coffee}} = -dU_{\text{air}}$ (see Eq. (A)), since $dU_{\text{coffee}} = -20.92 = -dU_{\text{air}} = -100 \times 1.0 \times dT_{\text{air}}$,

$$dT_{\text{air}} = 0.21 \text{ K}. \quad (\text{F})$$

The air temperature does not rise significantly.

Assuming the coffee temperature to be uniform within the cup, we will select the system so as to exclude boundaries where temperature gradients exist. Using the entropy balance equation for closed systems, and for internally reversible processes,

$$dS_{\text{coffee}} = \delta Q/T = -20.92 / ((360 + 370) \times 0.5) = -0.0573 \text{ kJ K}^{-1}. \quad (\text{G})$$

Employing Eq. (B),

$$S_{\text{coffee}} = 0.625 - 0.0573 = 0.5677 \text{ kJ K}^{-1}.$$

$$dS_{\text{air}} = 20.92 / ((300 + 300.21) \times 0.5) = +0.0697 \text{ kJ K}^{-1}, \text{ and}$$

$$S_{\text{air}} = 170.0697 \text{ kJ K}^{-1}.$$

Forming a combined system that includes both coffee and air, $\delta Q = 0$. Applying the entropy balance equation $dS - 0 = \delta\sigma$,

$$dS = dS_{\text{coffee}} + dS_{\text{air}} = 0.0573 + 0.0697 = 0.0124 \text{ kJ K}^{-1} \text{ so that}$$

$$\delta\sigma = 0.0124 \text{ kJ K}^{-1}.$$

You will also find that $\delta\sigma \rightarrow 0$ when the coffee temperature almost equals that of the air.

Remarks

In this case, $\delta\sigma > 0$ during the irreversible process in the isolated system.

$\delta\sigma \rightarrow 0$ when the coffee temperature almost equals the air temperature (or as reversibility is approached).

Vaporization of water into the air has been neglected.

a. Example 4

A gas undergoes an expansion in a constant diameter horizontal adiabatic duct. As the pressure decreases, the temperature can change and the velocity increases, since the gas density decreases. What is the maximum possible velocity?

Solution

From mass conservation

$$d(V/v) = 0, \quad (\text{A})$$

where V denotes velocity. Therefore, $dV/v + V d(1/v) = 0$. Applying energy conservation

$$d(h + V^2/2) = 0, \text{ and} \quad (\text{B})$$

utilizing the entropy equation

$$dh = T ds + v dP, \text{ or } ds = dh/T - v dP/T. \quad (\text{C})$$

Using Eqs. (A) and (B),

$$dh = -V dV = -V^2 dv/v. \quad (\text{D})$$

From Eqs. (D) and (C),

$$ds = -V^2 dv/(T v) - v dP/T. \quad (\text{E})$$

For an adiabatic duct, $ds = \delta\sigma$. Since $\delta\sigma \geq 0$ for an irreversible process,

$$ds \geq 0. \quad (\text{F})$$

Using Eqs. (F) and (E), $V^2 \leq -v^2 (\partial P/\partial v)_s$. Typically $(\partial P/\partial v)_s < 0$. Hence $V^2 > 0$. For a reversible (i.e., isentropic) process,

$$V^2 = -v^2 (\partial P/\partial v)_s \quad (\text{G})$$

which is the velocity of sound in the gas.

b. Example 5

Nitrogen is pumped into a 0.1 m³ rigid tank. The initial state of the gas is at 300 K and 1 atm. Determine the chemical work done to isentropically pump 0.016 kmole of the gas into the tank to a 10 bar pressure.

Solution

Applying the ideal gas law,

$$N_1 = 1 \times 0.1 \div (0.08314 \times 300) = 0.004 \text{ kmole.}$$

Therefore, the entropy change $dS = d(\bar{s} N) = 0$, i.e., $d\bar{s}/\bar{s} = dN/N$, or

$$\bar{s}_2/\bar{s}_1 = N_1/N_2 = 0.004 \div 0.020 = 0.2.$$

Now, $\bar{v}_2 = 0.1 \div 0.02 = 5 \text{ m}^3 \text{ kmole}^{-1}$, and $\bar{v}_1 = 0.1 \div 0.004 = 25 \text{ m}^3 \text{ kmole}^{-1}$ so that

$$\bar{s}_2/\bar{s}_1 = (\bar{c}_{v0} \ln(T_2/T_{\text{ref}}) + \bar{R} \ln(\bar{v}_2/\bar{v}_{\text{ref}})) / (\bar{c}_{v0} \ln(T_1/T_{\text{ref}}) + \bar{R} \ln(\bar{v}_1/\bar{v}_{\text{ref}}))$$

Using the values $T_{\text{ref}} = 273 \text{ K}$, $\bar{v}_{\text{ref}} = 1 \text{ m}^3 \text{ kmole}^{-1}$, $\bar{c}_{v0} = 20 \text{ kJ kmole}^{-1} \text{ K}^{-1}$, and $\bar{R} = 8.314 \text{ kJ kmole}^{-1} \text{ K}^{-1}$,

$$T_2 = 186 \text{ K.}$$

The chemical work, $W_{\text{chem,rev}} = -\int \mu dN = -(U_2 - U_1)$. Now,

$$U_2 - U_1 = 0.02 \times 20 \times (186 - 273) - 0.004 \times 20 \times (300 - 273) = -36.96 \text{ kJ, i.e.,}$$

$$W_{\text{chem,rev}} = +36.96 \text{ kJ.}$$

c. Example 6

Determine the chemical potential of pure O₂ at $T = 2000 \text{ K}$ and $P = 6 \text{ bar}$, and O₂ present in a gaseous mixture at $T = 2000 \text{ K}$ and $P = 6 \text{ bar}$, and $X_{\text{O}_2} = 0.3$, assuming the mixture to behave as an ideal gas.

Solution

$\mu = \bar{g} = (\bar{h} - T\bar{s})$, where for ideal gases $\bar{s} = \bar{s}^0 - R \ln(P/P_{\text{ref}})$. Using values from tables (tables A-19),

$$\mu_{\text{O}_2} = 67881 \text{ kJ kmole}^{-1} - 2000 \text{ K} \times (268.655 \text{ kJ kmole}^{-1} \text{ K}^{-1} - 8.314 \text{ kJ kmole}^{-1} \text{ K}^{-1}$$

$$\times \ln(6 \text{ bar} \div 1 \text{ bar})) = -439,636 \text{ kJ kmole}^{-1}.$$

For an ideal gas mixture according to the Gibbs–Dalton Law,

$$\bar{s}_k(T, p_k) = \bar{s}_k^0(T) - \ln(p_k/P_{ref}) = (268.655 \text{ kJ kmole}^{-1} \text{ K}^{-1} - 8.314 \text{ kJ kmole}^{-1} \text{ K}^{-1} \times \ln[(6 \text{ bar} \times 0.3) \div 1 \text{ bar}]) = 272.747 \text{ kJ kmole}^{-1} \text{ K}^{-1}.$$

Since $\mu_k = (\bar{h}_k - T\bar{s}_k)$, $\mu_{O_2} = -477,613 \text{ kJ (kmole O}_2 \text{ in the mixture)}^{-1}$.

Remarks

The chemical potential of O₂ decreases as its concentration is reduced from 100% (pure gas) to 30%.

You will later see that the chemical potential plays a major role in determining the direction of chemical reactions (in Chapter 10) and of mass transfer (in this chapter), just as temperature determines the direction of heat transfer.

a. Example 7

Pressurized gas tanks are employed in space power applications. As the gas contained in the tanks is used, the tank pressure falls (say, from P_{t,1} to P_{t,2}) so that the work done per unit mole can vary. Determine the work that can be done if a 2 m³ turbine and the tank are kept in an isothermal bath, P_e = 1 bar, P_{t,1} = 50 bars, and P_{t,2} = 1 bar.

Solution

For an open system,

$$\delta \bar{w}_{\text{shaft, rev}} = -\bar{v} dP = -(\bar{R} T/P) dP, \text{ i.e.,} \quad (\text{A})$$

$$\bar{w}_{\text{shaft, rev}} = -\bar{R} T \ln(P_e/P_i). \quad (\text{B})$$

The pressure P_i varies as the gas is progressively withdrawn from the tank. From Eq. (B)

$$\delta \bar{W}_{\text{shaft, rev}} = \bar{w}_{\text{shaft, rev}} dN_i = -\bar{R} T \ln(P_e/P_i) dN_{i, \text{turbine}}, \text{ where} \quad (\text{C})$$

$$dN_{\text{tank}} = -dN_{i, \text{turbine}}. \quad (\text{D})$$

Since P_{tank}V = N_{tank} $\bar{R} T$, dP_{tank} = dN_{tank} $\bar{R} T/V$, and

$$dN_{\text{tank}} = dP_{\text{tank}} V / \bar{R} T. \quad (\text{E})$$

Therefore, using Eqs. (C) and (E)

$$\delta \bar{W}_{\text{shaft, rev}} = -\bar{R} T \ln(P_e/P_i) dP_{\text{tank}} V / \bar{R} T = -V \ln(P_e/P_i) dP, \quad (\text{F})$$

where P_i = P_{tank}. Hence,

$$\delta \bar{W}_{\text{shaft, rev}} = -V P_e \ln(P_e/P_{\text{tank}}) P_{\text{tank}} / P_e, \text{ and}$$

$$\bar{W}_{\text{shaft, rev}} = V P_e ((P_{\text{tank},2}/P_e) \ln(P_{\text{tank},2}/P_e) - (P_{\text{tank},1}/P_e) \ln(P_{\text{tank},1}/P_e)). \quad (\text{G})$$

Using the values V = 2 m³, P_e = 1 bar, P_{tank,1} = 50 bars, P_{tank,2} = 1 bar,

$$\bar{W}_{\text{shaft, rev}} = 2 \times 1 \times 100 (1 \div 1 \ln(1 \div 1) - (50 \div 1) \ln(50 \div 1)) = 39120 \text{ kJ}$$

Remarks

If we select the control volume to include both the turbine and the tank, assuming that there is no accumulation of energy or entropy in the turbine,

$$dU_{\text{tank}} = \delta Q - PdV - \delta \bar{W}_{\text{shaft, rev}} - \bar{h}_e dN_e. \quad (\text{H})$$

Applying the entropy balance equation to the tank,

$$dS_{\text{tank}} = \delta Q/T - dN_e \bar{s}_e. \quad (\text{I})$$

From Eqs. (H) and (I),

$$\delta \bar{W}_{\text{shaft, rev}} = T dS_{\text{tank}} - dU_{\text{tank}} - PdV + T dN_e \bar{s}_e - \bar{h}_e dN_e. \quad (\text{J})$$

Since dN_e = -dN_{tank}, dV = 0, dS_{tank} = $\bar{s}_{\text{tank}} dN_{\text{tank}} + N_{\text{tank}} d\bar{s}_{\text{tank}}$, $\bar{h}_e(T) = \bar{h}_{\text{tank}}(T) = \bar{u}_{\text{tank}} + \bar{R}T$, dU_{tank} = $\bar{u}_{\text{tank}} dN_{\text{tank}} + N_{\text{tank}} d\bar{u}_{\text{tank}}$, and d $\bar{u}_{\text{tank}} = 0$,

$$\delta W_{\text{shaft,rev}} = T (\bar{s}_{\text{tank}} dN_{\text{tank}} + N_{\text{tank}} d\bar{s}_{\text{tank}}) - d(dN_{\text{tank}} \bar{u}_{\text{tank}} + N_{\text{tank}} d\bar{u}_{\text{tank}}) - 0 - T dN_{\text{tank}} \bar{s}_e + (\bar{u}_{\text{tank}} + RT) dN_{\text{tank}}$$

Simplifying this relation

$$dW_{\text{shaft,rev}} = dN_{\text{tank}} T (s_{\text{tank}} - s_e) = -dN_{\text{tank}} TR \ln(P_{\text{tank}}/P_e) = -V dP_{\text{tank}} \ln(P_{\text{tank}}/P_e). \quad (K)$$

If the process within the control volume is adiabatic and reversible,

$$dU_{\text{tank}} = 0 - 0 - \delta \bar{W}_{\text{shaft,rev}} - \bar{h}_e (T) dN_e, \text{ or} \quad (H)$$

$$\begin{aligned} \delta \bar{W}_{\text{shaft,rev}} &= -\bar{u}_{\text{tank}} dN_{\text{tank}} - N_{\text{tank}} d\bar{u}_{\text{tank}} + (\bar{u}_{\text{tank}} + \bar{R}T(t))dN_{\text{tank}} \\ &= -N_{\text{tank}} \bar{c}_{v0} dT_{\text{tank}} + \bar{R}T(t)dN_{\text{tank}} \\ &= -N_{\text{tank}} \bar{c}_{v0} dT_{\text{tank}} + VdP - \bar{R}N_{\text{tank}} dT_{\text{tank}} \\ &= -dT_{\text{tank}} N_{\text{tank}} (\bar{c}_{v0} + \bar{R}) + VdP = -dT_{\text{tank}} N_{\text{tank}} \bar{c}_{p0} + VdP. \end{aligned}$$

Therefore the relationship between the temperature and N_{tank} is of the form $T_{\text{tank}}/T_{\text{tank},1} = (P_{\text{tank}}/P_{\text{tank},1})^{(k-1)/k} = (N_{\text{tank}} T_{\text{tank}}/N_{\text{tank},1} T_{\text{tank},1})^{(k-1)/k}$, i.e.,

$$(T_{\text{tank}}/T_{\text{tank},1})^{(1/k)} = (N_{\text{tank}}/N_{\text{tank},1})^{(k-1)/k}, \text{ and}$$

$$\delta \bar{W}_{\text{shaft,rev}} = -dT_{\text{tank}} N_{\text{tank},1} (T_{\text{tank}}/T_{\text{tank},1})^{(1/(k-1))} \bar{c}_{p0} + VdP.$$

d. Example 8

Consider a 1 meter long turbine that operates steadily and produces a net power output of 1000 kW. Gases enter the turbine at 1300 K (T_i) and 10 bar (P_i), and leave at 900 K (T_e) and 1 bar (P_e). The turbine walls are insulated, but its blades are cooled. The cooling rate per unit area of the blade is given by the relation $h(T_{\text{avg}} - T_{\text{blade}})$ where $T_{\text{avg}} = (T_i + T_e)/2$. The Nusselt number ($Nu = hC/\lambda$) on the gas side of the blade is 1000, where h denotes the convective heat transfer coefficient ($\text{kW m}^{-2} \text{K}^{-1}$), C the chord length (which is 15 c.m. along axial direction), and λ the thermal conductivity of the hot gases ($= 70 \times 10^{-6} \text{ kW m}^{-2} \text{K}^{-1}$). The blade $A = C \times$ blade height which is assumed to be the same as the chord length. There are approximately 40 blades for each rotor and 3 rotors for every meter of length. Assume that $c_p = 1.2 \text{ kJ kg}^{-1} \text{K}^{-1}$ and $R = 0.287 \text{ kJ kg}^{-1} \text{K}^{-1}$.

Write the generalized overall energy conservation equation.

What is the heat loss rate if the blade temperature $T_{\text{blade}} = 900 \text{ K}$.

Determine the gas mass flow rate.

Write the entropy balance equation and simplify it for this problem.

Determine the entropy generation rate.

Solution

The energy equation can be written in the form

$$dE/dt = \dot{Q} - \dot{W} + \dot{m}_i (h + ke + pe)_i - \dot{m}_e (h + ke + pe)_e.$$

At steady state, $dE_{c.v.}/dt = 0$, $ke = 0$, $pe = 0$, $dm_{c.v.}/dt = 0$, hence $\dot{m}_i = \dot{m}_e = \dot{m}$, i.e.,

$$0 = \dot{Q} - \dot{W} + \dot{m} (h_i - h_e). \quad (A)$$

$$\dot{Q} = h A (T_{\text{avg}} - T_{\text{blade}}), \text{ and} \quad (B)$$

$$A = 0.15 \times 0.15 \times 40 \times 3 = 2.7 \text{ m}^2. \quad (C)$$

Since, $h C/\lambda = 1000$,

$$h = 1000 \times 70 \times 10^{-6} \div 0.15 = 0.467 \text{ kW m}^{-2} \text{K}^{-1}. \quad (D)$$

Using Eqs. (B), (C), and (D),

$$\dot{Q} = 0.462 \times 2.3 \times (1100 - 900) = 252 \text{ kW}. \quad (E)$$

$$h_e - h_i = c_{p0}(T_i - T_e) = 1.2 \times (1300 - 900), \text{ and} \quad (F)$$

$$\dot{W} = 1000 \text{ kW}. \quad (G)$$

Using Eqs. (A), (E), (F), and (G),

$$0 = 252 - 1000 - \dot{m} \times 1.2 \times (1300 - 900), \text{ i.e.,}$$

$$\dot{m} = 2.6 \text{ kg s}^{-1}. \quad (\text{H})$$

The entropy balance equation at steady state is

$$\begin{aligned} dS_{c.v.}/dt &= 0 \\ &= \dot{Q}/T_{b,j} + \dot{m}(s_i - s_e) + \dot{\sigma}_{c.v.} \end{aligned} \quad (\text{I})$$

$$\begin{aligned} (s_e - s_i) &= c_{p0} \ln(T_e/T_i) - R \ln(P_e/P_i) = 1.2 \\ \ln(900/1300) - 0.287 \ln(1/10) \end{aligned} \quad (\text{J})$$

$$\begin{aligned} &= -0.441 + 0.66 = 0.220 \text{ kJ kg}^{-1} \text{ K}^{-1}. \quad (\text{K}) \end{aligned}$$

Using Eqs. (H)–(K),

$$\begin{aligned} \dot{\sigma}_{c.v.} &= -(-252/900) - 2.6 \times (-0.22) = 0.28 + 0.57 = 0.85 \text{ kW K}^{-1}. \end{aligned}$$

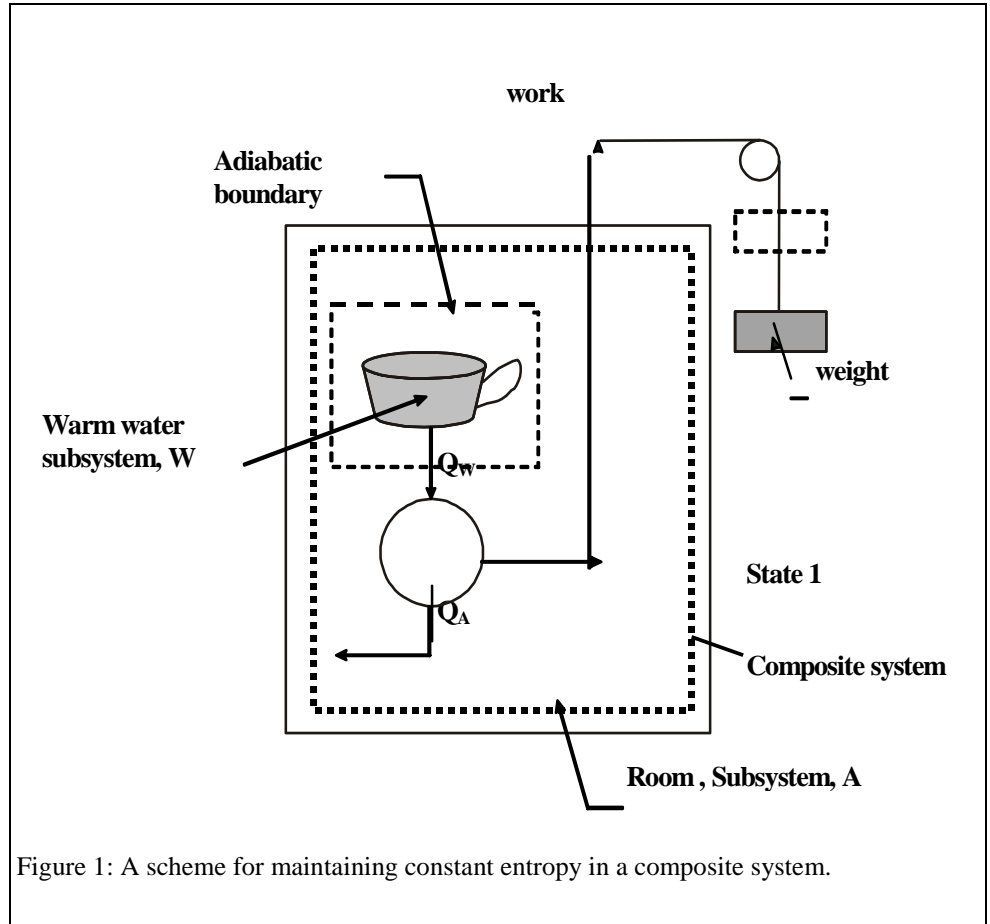


Figure 1: A scheme for maintaining constant entropy in a composite system.

Remark

The entropy generation due to blade cooling results in a work loss of $T_o \dot{\sigma}_{c.v.} = 298 \times 0.85 = 250$ kW (this is discussed further in Chapter 4). However, the increased gas temperature at the turbine inlet results in a larger work output.

e. Example 9

One kg of hot coffee at a temperature of 350 K ($T_{W,0}$) is kept in an adiabatic room that contains 0.4 kg of air at a temperature of 290 K ($T_{A,0}$). The cup is initially insulated. A Carnot engine is used to cool the coffee, lift a weight, and reject heat to the room air until the coffee and air temperatures equilibrate (cf. Figure 1). During the equilibration process:

- How does the internal energy of the composite system change?
- How does the entropy of the coffee change?
- How does the entropy of the room air change?
- How does the entropy of the composite system change?

Assuming coffee temperatures to be 340 K, 339.5 K, ... K, calculate T_A such that $dS_W + dS_A = 0$. Determine the initial internal energy of the composite system and plot the internal energy as a function of air temperature as the coffee cools.

Solution

Using the First Law $\delta Q - \delta W = dU$. Since, $\delta Q = 0$ and $\delta W > 0$, $dU < 0$. Therefore, the internal energy will decrease, since energy is converted into work.
 The entropy of the coffee decreases, since heat is transferred from it.
 The entropy of the air increases, since heat is transferred to it.

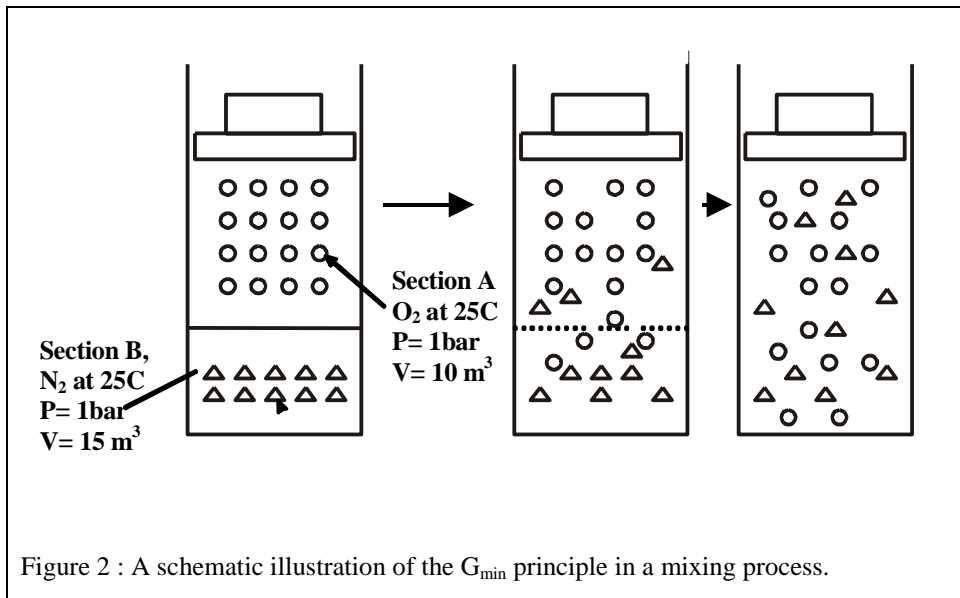


Figure 2 : A schematic illustration of the G_{\min} principle in a mixing process.

There is no entropy change in the composite system, since a Carnot engine is used for which the entropy changes in its source (coffee) and sink (air) are equal.

The total entropy change $dS_A + dS_W = dS = 0$. Therefore, $dS_A = -dS_W$ so that $dS_W = m_W c_W dT_W/T_W$. If the coffee cools by 0.5 K, $dT_W = -0.5$ K, and

$$dS_W = 1 \times 4.184 \times (-0.5 \div 350) = -0.00598 \text{ kJ K}^{-1}.$$

$$dS_A = m_A c_{vA} dT_A/T_A = -dS_W = 0.00598 \text{ kJ K}^{-1}.$$

$$dT_A = 0.00598 \times 290 \div (0.4 \times 0.713) = 6.078 \text{ K}.$$

$$U = U_W + U_A = m_W c_W (T_W - 273) + m_A c_{vA} (T_A - 273) = 327.02 \text{ kJ}.$$

$$\text{After cooling by 0.5 K } U = U_W + U_A = m_W c_W (T_W - 273) + m_A c_{vA} (T_A - 273) = 326.66 \text{ kJ}.$$

The internal energy decreases as work is delivered. These calculations may be repeated for the other temperatures, i.e., $T_w = 389, 388.5$ K.

f. Example 10

Consider section (A) in a constant–pressure device to consist of a 10 m^3 volume that contains molecular oxygen at 25°C and 100 kPa . Section (B) in the same device consists of the remaining volume of 15 m^3 which contains molecular nitrogen at the same temperature and pressure. When the partition is removed, molecules of both species diffuse into one another. The molecules are instantaneously distributed throughout the section they diffuse into. Plot the relationship of G_{A+B} with respect to $Y_{N_2,A}$. What is the value of G for the combined system at equilibrium? Assume the two specific heats $c_{pN_2} = 1.04 \text{ kJ kg}^{-1} \text{ K}^{-1}$, and $c_{pO_2} = 0.92 \text{ kJ kg}^{-1} \text{ K}^{-1}$ (cf Figure 2).

Solution

$$N_A = N_{O_2} PV / \bar{R} T = 100 \times 10 / (298 \times 8.314) = 0.404 \text{ kmole.}$$

Similarly, $N_B = N_{N_2} = 0.605 \text{ kmole.}$

$$\bar{h}_{N_2} = 1.04 \times 28 \times 298 = 8678 \text{ kJ kmole}^{-1}, \text{ and, } \bar{h}_{O_2} = 0.92 \times 32 \times 298 = 8773 \text{ kJ kmole}^{-1}.$$

Now,

$$G = N_{O_2,A} \bar{g}_{O_2} + N_{N_2,B} \bar{g}_{N_2}, \text{ and} \quad (A)$$

$$\bar{g}_{O_2,A} = \bar{g}_{O_2} - T \bar{s}_{O_2,A}(T, p_{O_2,A}), \text{ where}$$

$$\bar{s}_{O_2,A}(T, p_{O_2,A}) = \bar{c}_{p,O_2} \ln(T/T_{ref}) - \bar{R} \ln(p_{O_2,A}/p_{ref}).$$

The reference state is selected to be 298 K and 1 bar. Since $p_{O_2,A} = Y_{O_2,A} P$,

$$\bar{g}_{O_2,A} = \bar{g}_{O_2}(T, P) + \bar{R} T \ln Y_{O_2,A}. \quad (B)$$

Similarly,

$$\bar{g}_{N_2,B} = \bar{g}_{N_2}(T, P) + \bar{R} T$$

$$\ln Y_{N_2,B}, \text{ i.e., (C)}$$

$$\bar{g}_{O_2,A} = 8773 + 8.314$$

$$\times 298 \ln 1 = 8773 \text{ kJ}$$

$$\text{kmole}^{-1}, \text{ and } \bar{g}_{N_2,B} =$$

$$8678 \text{ kJ kmole}^{-1}.$$

Initially,

$$G_1 = 0.404 \times 8773 +$$

$$0.605 \times 8678 = 8794$$

$$\text{kJ.}$$

Assume that 10% of the oxygen molecules (i.e., 0.0605 kmole) cross into section B after the partition is removed and 10% of the N_2 molecules (i.e., 0.0404 kmole) likewise cross into section A (at the same temperature and

pressure, i.e., 298 K and 1 bar). Following the molecular crossover, the number of moles contained in Section A are

$$N_{N_2,A} = 0.10 \times 0.605 = 0.0605 \text{ kmole, and}$$

$$N_{O_2,A} = 0.404 - 0.1 \times 0.404 = 0.364 \text{ kmole.}$$

Similarly the number of moles contained in section B are

$$N_{N_2,B} = 0.605 - 0.1 \times 0.605 = 0.545 \text{ kmole, and}$$

$$N_{O_2,B} = 0.1 \times 0.4 = 0.0404 \text{ kmole.}$$

Therefore,

$$Y_{N_2,A} = 0.0605 \div (0.0605 + 0.364) = 0.143, \text{ and}$$

$$Y_{O_2,A} = 0.364 \div (0.0605 + 0.364) = 0.857.$$

Similarly in Section B,

$$Y_{N_2,B} = (0.545) \div (0.545 + 0.0404) = 0.931, \text{ and}$$

$$Y_{O_2,B} = 0.0404 \div (0.0404 + 0.545) = 0.069.$$

Now,

$$G = N_{O_2,A} \bar{g}_{O_2,A} + N_{N_2,A} \bar{g}_{N_2,A} + N_{O_2,B} \bar{g}_{O_2,B} + N_{N_2,B} \bar{g}_{N_2,B}, \text{ where}$$

$$\bar{g}_{N_2,A} = 8678 + 8.314 \times 298 \times \ln 0.143 = 3859 \text{ kJ kmole}^{-1}.$$

Similarly,

$$\bar{g}_{N_2,B} = 8678 + 8.314 \times 298 \times \ln 0.931 = 8501 \text{ kJ kmole}^{-1},$$

$$\bar{g}_{O_2,A} = 8773 + 8.314 \times 298 \times \ln 0.857 = 8391 \text{ kJ kmole}^{-1}, \text{ and}$$

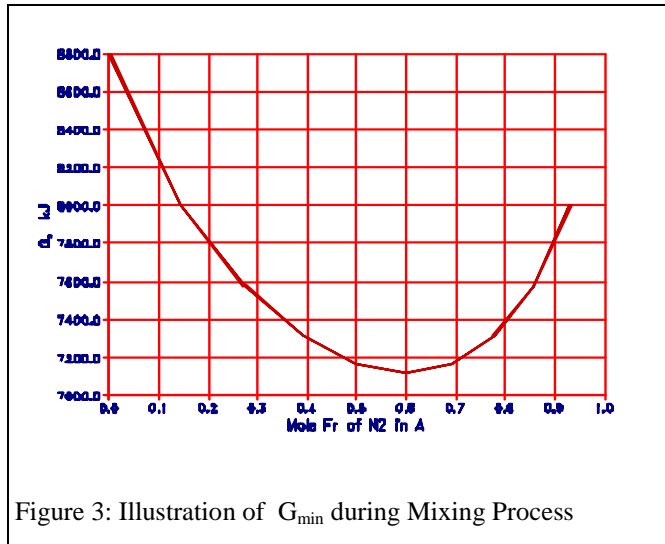


Figure 3: Illustration of G_{min} during Mixing Process

$$\bar{g}_{O_2,B} = 8773 + 8.314 \times 298 \times \ln 0.069 = 2149 \text{ kJ kmole}^{-1}, \text{ so that}$$

$$G = 0.364 \times 8391 + 0.0605 \times 3859 + 0.0404 \times 2149 + 0.545 \times 8501 = 8007 \text{ kJ.}$$

These calculations can be repeated for 20%, 30%, etc. of each species diffusing into each other. Figure 3 contains a plot of G with respect to $Y_{N_2,A}$. Note that if all of the molecular nitrogen diffuses into section A and all of the O_2 diffuses into section B the value of G reverts to G_1 . The minimum value is reached when $Y_{N_2,A} = 0.6$.

At equilibrium

$$Y_{O_2} = 0.404 + (0.404 + 0.605) = 0.4, Y_{N_2} = 0.6,$$

$$\bar{g}_{O_2} = 8773 + 8.314 \times 298 \times \ln 0.4 = 6503 \text{ kJ, and}$$

$$\bar{g}_{N_2} = 8678 + 8.314 \times 298 \times \ln 0.6 = 7412 \text{ kJ.}$$

Hence,

$$G_2 = 0.404 \times 6503 + 0.605 \times 7412 = 7111 \text{ kJ.}$$

$dG_{T,P} < 0$ when mixing (which is irreversible) occurs. We will discuss the G_{\min} principle further in Chapters 7 and 8.

Remarks

Explicit Solution for Equilibrium Concentration With LaGrange Method

Applying Eq.(107) at constant, T,P,m and zero other work

$$dG_{T,P,m} = - T_b \delta\sigma \quad (1)$$

we need to evaluate either σ or G. The G can not be evaluated for sections A and B together since an irreversible process occurs between them. However G can be evaluated for each section since we assume process within each section occurs reversibly. Then $dG_{T,P,m} = \{dG_A + dG_B\} < 0$; each section act as open system. In Chapters 4 , 11 and 12 , we will show that for the open system, $dG_A = dN_{O_2,A} \bar{g}_{O_2,A} + dN_{N_2,A} \bar{g}_{N_2,A}$ where $dN_{O_2,A}$ is the change in Section A for O_2 moles due to transfer from Section B ; $dG_B = dN_{O_2,B} \bar{g}_{O_2,B} + dN_{N_2,B} \bar{g}_{N_2,B}$. The concentration at $G = G_{\min}$ can be obtained from Eqs. (A), (B), and (C) by minimizing $G(T, P, N_{N_2}, N_{O_2})$ subject to $N_{O_2} = N_{O_2,0}$ (the initial value of O_2) and $N_{N_2} = N_{N_2,0}$ (the initial value of N_2). Using the LaGrange multiplier method,

$$F = G_A(T,P,N_{N_2,A},N_{O_2,A}) + G_B(T,P,N_{N_2,B},N_{O_2,B}) + \lambda_1(N_{O_2,A} + N_{O_2,B} - N_{O_2,0}) + \lambda_2(N_{N_2,A} + N_{N_2,B} - N_{N_2,0}). \quad (D)$$

where λ is the LaGrange multiplier. Therefore,

$$G(T,P,N_{N_2},N_{O_2}) = N_{O_2}(\bar{g}_{O_2}(T,P) + \bar{R} T \ln(N_{O_2}/(N_{N_2} + N_{O_2}))) + N_{N_2}(\bar{g}_{N_2}(T,P) + \bar{R} T \ln(N_{O_2}/(N_{N_2} + N_{O_2}))). \quad (E)$$

Using the relations $\partial F/\partial N_{O_2,A} = 0$, $\partial F/\partial N_{O_2,B} = 0$, $\partial F/\partial N_{N_2,A} = 0$, $\partial F/\partial N_{N_2,B} = 0$, and differentiating Eq. (E),

$$N_{O_2,A}(\bar{R} T/(N_{O_2,A} - (N_{N_2,A} + N_{O_2,A}))^{-1}) + \bar{g}_{O_2}(T,P) + \bar{R} \ln(N_{O_2,A}/(N_{N_2,A} + N_{O_2,A})) + \lambda_1 = 0.$$

Simplifying the equation

$$\bar{R} T N_{N_2,A}/(N_{N_2,A} + N_{O_2,A}) + \bar{g}_{O_2}(T, p_{O_2,A}) + \lambda_1 = 0, \text{ or}$$

$$Y_{N_2,A} + \bar{g}_{O_2}(T, p_{O_2,A}) = -\lambda_1' \quad (G)$$

where $\lambda_1' = \lambda/RT$. Similarly,

$$Y_{N_2,B} + \bar{g}_{O_2}(T, p_{O_2,A}) = -\lambda_1' \quad (H)$$

It is seen from Eqs. (G) and (H) that mole fractions in both sections A and B must be the same at equilibrium (i.e., at G_{\min}), i.e.,

$$Y_{N_2,A} = Y_{N_2,B} \text{ or } p_{O_2,A} = p_{O_2,B}.$$
