
Example 1

Air is expanded to perform work in a piston–cylinder assembly. The air is initially at $P_1 = 35$ bar and $T_1 = 2000$ K, and the expansion ratio $r_v (= v_2 / v_1)$ is 7. The ambient temperature $T_0 = 298$ K and pressure $P_0 = 1$ bar.

Determine the useful work that is delivered for an isentropic process.

If process is nonadiabatic and $P_2 = 2.5$ bar, what are the absolute closed system availabilities at the initial and final states? Determine the optimum and useful optimum work.

If a dynamometer measures the useful work to be 0.8 kJ for non-adiabatic process and the initial volume $V_1 = 0.000205$ m³, determine the heat loss, irreversibility (or lost work), and the entropy generated during the process.

Solution

From the tables for air (Table A-7) at 2000 K, $u_1 = 1679$ kJ kg⁻¹, $s_1^0 = 3.799$ kJ kg⁻¹ K⁻¹, $v_{r1} = 2.776$, and $P_{r1} = 2068$.

For the isentropic processes,

$v_{2s}/v_1 = v_{r2s}/v_{r1} = 7$. $\therefore v_{r2s} = 19.43$, and from the tables $P_{r2} = 161$, $T_2 = 1090$ K, and $u_{2s} = 835$ kJ kg⁻¹. Hence,

$P_2/P_1 = P_{r2}/P_{r1} = 161/2068 = 0.0779$, and $P_2 = 2.725$ bar,

and the isentropic work

$$w_s = u_1 - u_{2s} = (1679 - 835) = 844 \text{ kJ kg}^{-1}.$$

The specific volumes

$v_1 = RT_1/P_1 = 0.164$ m³ kg⁻¹, and $v_2 = 1.148$ m³ kg⁻¹ (using the expansion ratio).

Hence, the useful work

$$w_{u,s} = 844 - 100 \times (1.148 - 0.164) = 745.6 \text{ kJ kg}^{-1}.$$

Since the cylinder mass is constant, applying the ideal gas law, $T_2 = T_1 P_2 v_2 / (P_1 v_1)$, i.e.,

$T_2 = 2.5 \times 7 \times 2000 \div 35 = 1000$ K. The initial entropy

$s_1 = s_1^0 - R \ln (P_1/P_0) = 3.799 - 0.287 \ln (35/1) = (3.799 - 1.020) = 2.779$ kJ kg⁻¹ K⁻¹, and the final entropy

$s_2 = s_2^0 - R \ln (P_2/P_0) = 2.97 - 0.287 \ln (2.5/1) = 2.71$ kJ kg⁻¹ K⁻¹.

The absolute availabilities

$\phi_1 = u_1 - T_0 s_1 + P_0 v_1 = 1679 - 298 \times 2.779 + 100 \times 0.164 = 867.23$ kJ kg⁻¹, and

$\phi_2 = u_2 - T_0 s_2 + P_0 v_2 = 758.9 - 298 \times 2.71 + 100 \times 1.148 = 66.1$ kJ kg⁻¹.

Therefore,

$w_{u,opt} = \phi_1 - \phi_2 = 867.2 - 66.1 = 801.1$ kJ kg⁻¹. Now,

$w_{opt} = w_{u,opt} + P_0 (v_2 - v_1) = 801.1 + 100 \times (1.148 - 0.164) = 899.5$ kJ kg⁻¹.

Hence, the optimal heat transfer

$q_{opt} = (u_2 - u_1) + w_{opt} = 758.9 - 1679 + 899.5 = -20.6$ kJ kg⁻¹.

Applying the relation $V_2/V_1 = v_2/v_1 = 7$, since $V_1 = 0.000205$ m³,

$V_2 = 7 \times 0.000205 = 0.00144$ m³

The mass $m = P_1 V_1 / RT_1 = 35 \times 10^2 \times 0.000205 / (0.287 \times 2000) = 0.00125$ kg.

Since the total useful work $W_u = 0.8$ kJ, on a mass basis $w_u = 0.8 / 0.00125 = 640$ kJ kg⁻¹.

Therefore, the work $w = 640 + 100 \times (1.148 - 0.164) = 738.4 \text{ kJ kg}^{-1}$.
 The heat transfer $q = (u_2 - u_1) + w = (758.9 - 1679) + 738.4 = -181.7 \text{ kJ kg}^{-1}$.
 The irreversibility $i = w_{u,\text{opt}} - w_u = 801.2 - 640 = 161 \text{ kJ kg}^{-1}$ (or 0.201 kJ).
 The entropy generation $\sigma = I/T_0 = 161/298 = 0.0007 \text{ kJ K}^{-1}$ (or $0.54 \text{ kJ kg}^{-1} \text{ K}^{-1}$).

Remarks

For the last part of the problem $\sigma = 161.2/298 = 0.54 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The entropy change in the system $s_2 - s_1 = -0.07 \text{ kJ kg}^{-1} \text{ K}^{-1}$, the atmospheric entropy change $s_0 = -q/T_0 = 182/298 = 0.61 \text{ kJ kg}^{-1} \text{ K}^{-1}$ so that $\sigma = -0.07 + 0.61 = 0.54 \text{ kJ kg}^{-1} \text{ K}^{-1}$ (as before).

The entropy change can be negative for nonadiabatic processes, since the entropy is lowered when a system is cooled.

We note that $|q_{\text{opt}}| < |q|$.

Now we will illustrate the optimum process. The work $w_{u,\text{opt}}$ could have been obtained for same initial and final system states through a totally reversible process which generated no entropy in the isolated system (for which S , V , and m are fixed).

At the initial states for the system M and the ambient A , $U_{1,M} = 1679 \times 0.00125 = 2.1 \text{ kJ}$, (cf. Point 1 on Figure 1) and we assume that $U_{1,A} = 0$. Therefore, the combined isolated system energy is $U_{1,M+A} = 2.1 \text{ kJ}$.

At their final states

$U_{2,M} = 758.9 \times 0.00125 = 0.95 \text{ kJ}$, i.e., the energy change $\Delta U_M = -1.15 \text{ kJ}$.

Applying the First law

$Q_A - W_A = U_{2,A} - U_{1,A}$, where

$W_A = P_0 \Delta V_A = -0.062 \text{ kJ}$. Now,

$Q_A = 0.00125 \times 20.6 = 0.026 \text{ kJ}$. Therefore,

$U_{2,A} - U_{1,A} = 0.026 - (-0.062) = 0.088 \text{ kJ}$, i.e., $U_{2,A} = 0.088 \text{ kJ}$, and

$U_{2,M+A} = 0.95 + 0.088 = 1.038 \text{ kJ}$.

Assume that the optimum work is used to raise a weight so that the ambient potential energy increases by an amount

$\Delta \text{PE} = 801.2 \times 0.00125 = 1 \text{ kJ}$ (based on the useful work).

The internal energy change in the isolated system

$\Delta U_{M+A} = U_{2,M+A} - U_{1,M+A} = 1.038 - 2.1 = -1.06 \text{ kJ}$.

The internal energy of the combined isolated system is lowered (cf. Path 1–2_{opt} on Figure 1), although S , V , and m are held constant.

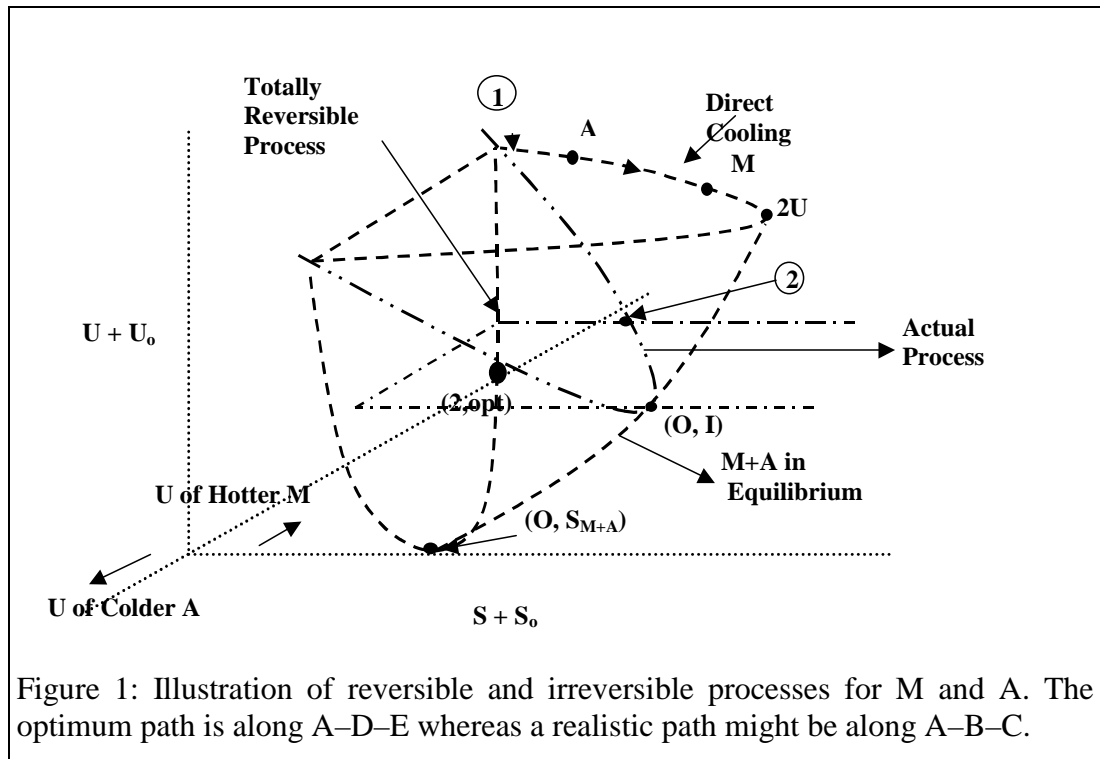
For the actual process considered in part five of the problem $U_{2,A} - U_{1,A} = Q_A$.

With $Q_A = 0.227 \text{ kJ}$,

$U_{2,A} = 0.227 + 0.062 = 0.289 \text{ kJ}$, and

$U_{2,M+A} = 0.95 + 0.289 = 1.239 \text{ kJ}$ (cf. Point 2 on Figure 1).

The actual process increases the isolated system entropy (cf. Path 1–2 on Figure 1). The combined system energy $U_{2,M+A}$ is higher in this case than for the corresponding reversible process, since a larger amount of heat is rejected to environment. While the increased heat transfer raises the ambient internal energy, in case a weight is lifted, the ambient gains a lower amount of potential energy ΔPE due to the smaller amount of work that is done on it.



In case the expansion is continued to the dead state, $T_0 = 25^\circ\text{C}$, $P_0 = 1$ bar, using the properties of air at that state, it is possible to determine that

$$\phi_0 = u_0 - T_0 s_0 + P_0 v_0 = 213 - 298 \times 1.677 + 100 \times 0.855 = -201.2 \text{ kJ kg}^{-1},$$

$$w_{u,\text{opt},0} = \phi_1 - \phi_0 = 867.3 - (-201.2) = 1068.5 \text{ kJ kg}^{-1},$$

$$w_{\text{opt},0} = w_{u,\text{opt},0} + P_0 (v_0 - v_1) = 1068.5 + 100 \times (0.855 - 0.164) = 1137.6 \text{ kJ kg}^{-1},$$

and

$$q_{\text{opt}} = (u_0 - u_1) + w_{\text{opt},0} = 213 - 1679 + 1137.6 = -328.5 \text{ kJ kg}^{-1}, \text{ i.e., the heat entering the ambient under this condition } Q_{\text{opt},0} = 0.374 \text{ kJ. Furthermore,}$$

$$U_{0,A} = 0.374 + 0.062 = 0.436 \text{ kJ and } U_{0,M} = 213 \times 0.00125 = 0.266 \text{ kJ so that}$$

$$U_{0,M+A} = 0.436 + 0.266 = 0.702 \text{ kJ (cf. Point 0 on Figure 1). Therefore,}$$

$$U_{0,M+A} - U_{1,M+A} = U_{0,M} - U_{1,M} + U_{0,A} - U_{1,A} = 0.266 - 2.1 + 0.436 - 0 = -1.398 \text{ kJ.}$$

This is the lowest possible energy value that can be contained in the combined isolated system keeping the combined S , V and m constant. In terms of the potential energy change, at this point a lifted weight will reach its highest elevation.

For the actual process (cf. to Point 2 on Figure 1) a lifted weight will rise to a lower elevation and the internal energy and entropy of the combined isolated system will have higher values.

Removing all restraints between M and at the dead state A will result in no change in state of the isolated system (if both M and A consist of air). However, if the gas composition is different in M and A, removing the restraints will result in mixing. This will be discussed later in the context of chemical availability.

a. Example 2

Two kg of water are to be heated from a temperature $T_1 = 25^\circ\text{C}$ to $T_2 = 100^\circ\text{C}$. Natural gas heaters with an 85% efficiency are used for the purpose. How much natural gas is required, assuming that it can release $18,400 \text{ kJ m}^3$ of heat.

How much electrical work is required by an electrical range to do so?(Figure 2a)
 Determine the minimum work required using an availability analysis (Figure 2b)
 If the cost of natural gas is \$4 per MJ and the electricity cost is 4¢ per kW–hr,
 determine the costs associated with the problem.

Solution

Assume that the ambient temperature $T_0 = 298 \text{ K}$ and that the water specific heat $c = 4.184 \text{ kJ kg}^{-1} \text{ K}^{-1}$.

Applying the First law

$$q - 0 = u_2 - u_1 = c (T_2 - T_1) = 4.184 \times (100 - 25) = 313.8 \text{ kJ kg}^{-1}.$$

The actual heat input that is required

$$q_{\text{in}} = 313.8/0.85 = 369.2 \text{ kJ kg}^{-1} \text{ or } Q_{\text{in}} = 2 \times 369.2 = 738.4 \text{ kJ}.$$

The volumetric amount of natural gas required = $738.4 \div 18400 = 0.04 \text{ m}^3$.

Assuming that the electrical work is 100% efficient

$$W_{\text{elec}} = -313.8 \times 2 = -627.6 \text{ kJ}.$$

The optimum work

$$W_{\text{opt}} = u_1 - u_2 - T_0 (s_1 - s_2) + P_0 (v_1 - v_2).$$

Since $v_1 \approx v_2$ for liquids due to negligible expansion,

$$W_{\text{opt}} = c (T_2 - T_1) - T_0 (c \ln (T_2/T_1)) = -4.184 \times 75 + 298 \times 4.184 \times \ln (373/298) = -313.8 + 279.9 = -33.9 \text{ kJ kg}^{-1}. \text{ Therefore,}$$

$$W_{\text{opt}} = -33.9 \times 2 = -67.8 \text{ kJ, and}$$

$W_{\text{opt}} = W_{\text{min}}$, the minimum work required in order to heat the water.

The cost comparisons to heat the 2 kg of water are as follows:

Natural Gas Heating: $\$4 \times 10^{-6} \times 738.4 = 0.296\text{¢}$.

Electrical Heating: $4\text{¢ kWh}^{-1} \times 313.8 = 0.7\text{¢}$.

Heat pump: $4\text{¢ kWh}^{-1} \times 33.9 = 0.076\text{¢}$.

Remarks

Using a heat pump, the electrical bill for heating the water can be reduced by 89%. Availability analyses provide the information on how best to achieve

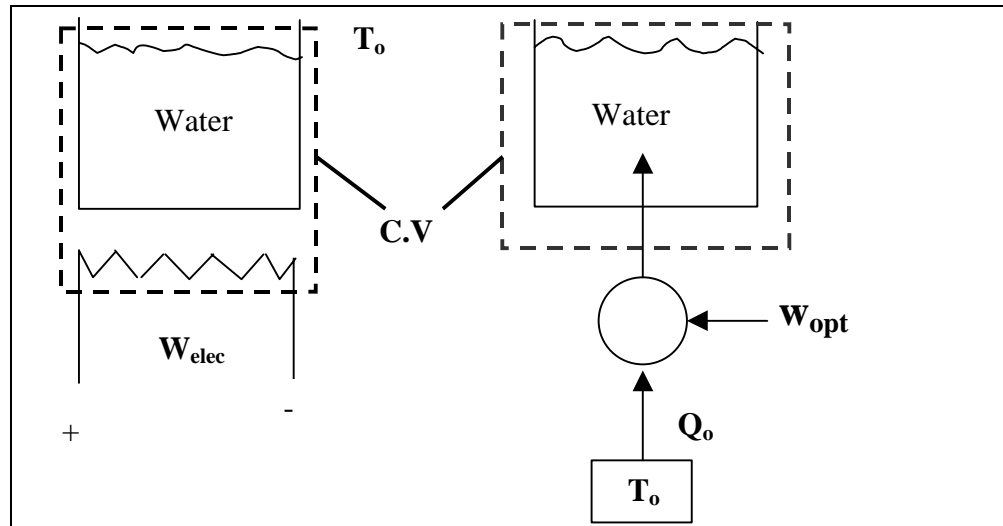


Figure 2: a. Direct heating of water with an electrical range; b. heating of water with a heat pump where we use ambient Q_0 to supply a part of the heat

desired end states with minimum work input or by obtaining maximum work output. The only allowed interactions are the ones with the environment.

The heat pump can be run by using 33.9 kJ kg^{-1} of electrical work to run it. In turn the heat pump will accept 279 kJ kg^{-1} from the ambient air at 25 C and deliver $313.8 (= 279.9 + 33.9) \text{ kJ kg}^{-1}$ of heat to the water. The heat pump must be operated between a fixed temperature thermal reservoir at a temperature $T_0 = 25^\circ\text{C}$ and the variable-temperature hot water reservoir.

This example is pertinent also to domestic heating applications.

b. Example 3

This example illustrates the interaction between a thermal energy reservoir, its ambient, a steady state steady flow process, and a cyclical process. Consider the inflow of water in the form of a saturated liquid at 60 bar into a nuclear reactor (state 1). The reactor temperature is 2000 K and it produces steam which subsequently expands in a turbine to saturated vapor at a 0.1 bar pressure (state 2). The ambient temperature is 25°C . The reactor heat transfer is 4526 kJ per kg of water. Assume that the pipes and turbines are rigid.

What is the maximum possible work between the two states 1 and 2?

If the steam that is discharged from turbine is passed through a condenser (cf. Figure 3) and then pumped back to the nuclear reactor at 60 bar , what is the maximum possible work under steady state cyclical conditions? Assume that the inlet condition of the water into the pump is saturated liquid.

Solution

If the boundary is selected through the reactor, for optimum work $I = \sigma = 0$. Under steady state conditions time derivatives are zero, and, since the body does not deform $\dot{W}_u = 0$, so that $\dot{W}_{cv} = \dot{W}_{\text{shaft}}$ and $\dot{m}_i = \dot{m}_e = \dot{m}$. Therefore,

$$\dot{m}(\psi_i - \psi_e) + \dot{Q}_{R,1} (1 - T_0 / T_{R,1}) - \dot{W}_{cv,opt} = 0. \quad (\text{A})$$

Dividing Eq. (A) throughout by the mass flow rate,

$$W_{cv,opt} = (\psi_i - \psi_e) + q_{R,1} (1 - T_0 / T_{R,1}). \quad (\text{B})$$

Using the steam tables (A-4A)

$$\psi_i = 1213.4 - 298 \times 3.03 = 310.5 \text{ kJ kg}^{-1}, \text{ and} \quad (\text{C})$$

$$\psi_e = 2584.7 - 298 \times 8.15 = 156.0 \text{ kJ kg}^{-1}. \quad (\text{D})$$

Therefore,

$$q_{R,1} = 4526 \text{ kJ kg}^{-1}, \text{ and} \quad (\text{E})$$

$$W_{cv,opt} = 4526(1 - 298 / 2000) + (310.5 - 156) = 4006 \text{ kJ kg}^{-1}. \quad (\text{F})$$

For the cycle, $\psi_i = \psi_e$. Therefore,

$$W_{cv,opt,cycle} = q_{R,1} (1 - T_0 / T_{R,1}) = 3852 \text{ kJ kg}^{-1}.$$

Note that this work is identical to that of a Carnot cycle with an efficiency of $(1 - T_0 / T_{R,1})$.

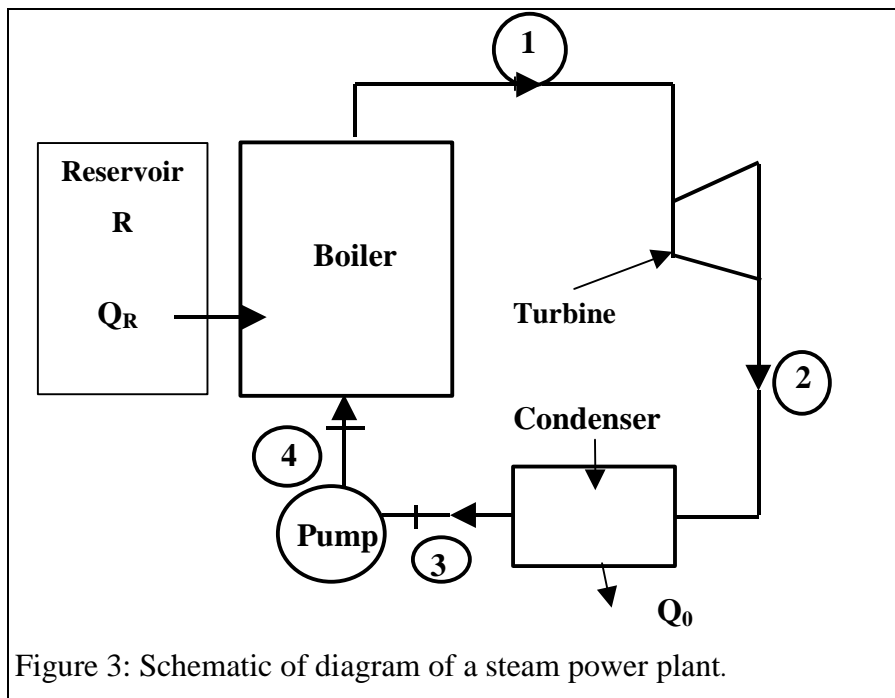


Figure 3: Schematic of diagram of a steam power plant.

Remarks

A realistic cyclical process contains inherent irreversibilities due to irreversible heat transfer and internal irreversibilities so that

$$W_{cv,cycle} < W_{cv,opt,cycle}$$

The work $w_{cv,cycle}$ usually deteriorates over time, since internal irreversibilities in the cycle increase. Once the state is known, it is possible to ascertain ψ at various points during a process to determine $w_{cv,opt}$ and w_{cv} , and to calculate $\sigma_{cv} = (w_{cv,opt} - w_{cv}) / T_0$.

c. Example 5

Hot air at a temperature of 400°C flows into an insulated heat exchanger (the fire tube boiler shown in Figure 4) at a rate of 10 kg s⁻¹. It is used to heat water from a saturated liquid state to a saturated vapor condition at 100°C. If the air exits the heat exchanger at 200°C, determine the water flow in kg s⁻¹ and the irreversibility. Assume that $c_p = 1 \text{ kJ kg}^{-1} \text{ K}^{-1}$, $h_{fg} = 2257 \text{ kJ kg}^{-1}$, and $T_0 = 298 \text{ K}$.

Solution

The energy required to heat the water is obtained from the first law:

$$dE_{c.v.}/dt = \dot{Q}_0 + \dot{Q}_{R,1} + \dot{Q}_{R,2} + \dot{Q}_{R,3..} - \dot{W}_{c.v.} + \sum \dot{m}_i e_{T,i} - \sum \dot{m}_e e_{T,e}$$

Since the fire tube boiler is assumed to be adiabatic, steady and non-work producing device

$$0 = + \dot{m}_a \{ h_{a,i} - h_{a,e} \} + \dot{m}_w \{ h_f - h_g \}$$

or

$$\dot{m}_a c_p (T_2 - T_1) = \dot{m}_w h_{fg},$$

where the subscripts a and w, respectively, refer to the air and water. Therefore,

$$\dot{m}_w = 10 \times 1 \times 200 \div 2257 = 0.886 \text{ kg s}^{-1}.$$

The optimum work

$$\dot{W}_{cv,opt} = (\dot{m}_a \psi_{a,i} + \dot{m}_w \psi_{w,i}) - (\dot{m}_a \psi_{a,e} + \dot{m}_w \psi_{w,e}), \text{ i.e.,} \quad (\text{A})$$

$$\psi_{a,i} = h_{a,i} - T_0 s_i = 1 \times 673 - 298 \times (1 \times \ln(673/298)) = 430.2 \text{ kJ kg}^{-1},$$

$$\psi_{a,e} = 473 - 298 \times 1 \times \ln(473/298) = 335.3 \text{ kJ kg}^{-1},$$

$$\psi_{w,i} = 419 - 298 \times 1.31 = 28.6 \text{ kJ kg}^{-1}, \text{ and}$$

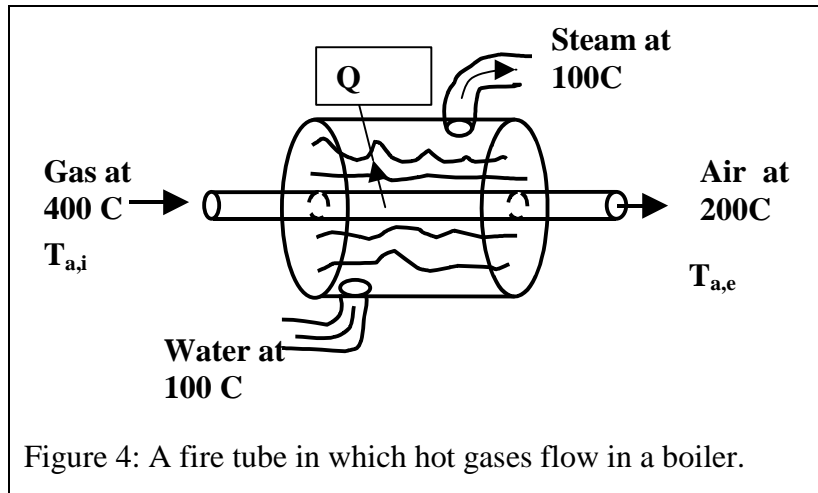


Figure 4: A fire tube in which hot gases flow in a boiler.

$$\psi_{w,e} = 2676.1 - 298 \times 7.35 = 485.8 \text{ kJ kg}^{-1}.$$

Therefore,

$$\dot{W}_{cv,opt} = 10 \times 430.2 + 0.89 \times 28.6 - (10 \times 335.3 + 0.89 \times 485.8) = 544 \text{ kW}, \text{ and}$$

$$I = \dot{W}_{cv,opt} - \dot{W} = 544 - 0 = 544 \text{ kW}.$$

Remarks

Hot combustion products enter the fire tubes of fire tube boilers at high temperatures and transfer heat to the water contained in the boiler drum. The water thereby evaporates, producing steam. This example reveals the degree of irreversibility in such a system.

The irreversibility exists due to the temperature difference between the hot gases and the water. An alternative method to heat the water would be to extract work by running a Carnot engine that would operate between the variable-temperature hot gases and the uniform-temperature ambient. A portion of the Carnot work can be used to run a heat pump in order to transfer heat from the ambient to the water and generate steam. The remainder of the work would be the maximum possible work output from the system. However, such a work output is unavailable from conventional heat exchangers in which the entire work capability is essentially lost.

We now discuss this scenario quantitatively. Assume that the air temperature changes from $T_{a,i}$ to $T_{a,e}$ as it transfers heat to the Carnot engine. For an elemental amount of heat $\delta \dot{Q}$ extracted from the air, the Carnot work

$$\delta \dot{W}_{CE} = \delta \dot{Q} (1 - T_0/T). \quad (B)$$

Since

$$\delta \dot{Q} = \dot{m}_a c_p dT, \quad (C)$$

$$\delta \dot{W}_{CE} = -\dot{m}_a c_p dT (1 - T_0/T).$$

Upon integration,

$$\begin{aligned} \dot{W}_{CE} &= \dot{m}_a c_p \{(T_e - T_i) - T_0 \ln(T_e/T_i)\}. \quad (D) \\ &= 10 \times 1 \times (200 - 298 \times \ln(473/673)) = 949 \text{ kW}. \end{aligned}$$

This is the Carnot work obtained from the transfer of heat from the air. Now, a portion of this work will be used to run a heat pump operating between constant temperatures T_0 and T_w (100°C) in order to supply heat to the water. The heat pump COP is given by the expression

$$\text{COP} = \dot{Q}_H / \dot{W}_{\text{in,heat pump}} = T_w / (T_w - T_0) = 4.97.$$

Since the heat transfer $\dot{Q}_H = 2257 \times 0.89 = 2009 \text{ kW}$, The work input $\dot{W}_{\text{in,heat pump}} = 2009 \div 4.97 = 404 \text{ kW}$. Therefore, the net work that is obtained

$$\dot{W}_{\text{cv,opt}} = 949 - 404 = 545 \text{ kW}.$$

This is identical to the answer obtained for the irreversibility flux using the availability analysis. Due to the high cost of fabricating such a system, conventional heat exchangers are instead routinely used.

We now examine the feasibility of installing a Carnot engine between the hot gases and the water that exists at 100°C so that heat could be directly pumped into water. You will find that it is impossible to achieve the same end states as in the heat exchanger while keeping $\sigma_{\text{cv}} = 0$ without any interaction with the environment.

d. Example 5

A nuclear reactor transfers heat to water in a boiler that is at a 900 K temperature, thereby producing steam at 60 bar, and 500°C. The steam exits an adiabatic turbine in the form of saturated vapor at 0.1 bar. The vapor enters a condenser where it is condensed into saturated liquid at 0.1 bar and then pumped to the boiler using an isentropic pump.

Determine:

The optimum work.

The availability efficiency.

The overall irreversibility of the cycle.

The irreversibility in the boiler, turbine, and condenser.

Solution

Analyzing the Rankine cycle:

The *turbine* work is

$$\begin{aligned}q_{12} - w_{12} &= h_2 - h_1, \text{ i.e.,} \\w_{12} &= 2585 - 3422 = 837 \text{ kJ kg}^{-1}.\end{aligned}\tag{A}$$

The heat rejected in the *condenser*

$$\begin{aligned}q_{23} - w_{23} &= h_3 - h_2, \text{ i.e.,} \\q_{23} = q_{\text{out}} &= 192 - 2585 = -2393 \text{ kJ kg}^{-1}.\end{aligned}$$

Likewise, in the *pump*

$$q_{34} - w_{34} = h_4 - h_3 \approx v_3 (P_4 - P_3), \text{ or}\tag{B}$$

Since properties for the liquid state at 4 may be unavailable, they can be otherwise determined. The work

$$w_{34} = -0.001 \times (60 - 0.1) \times 100 = -6 \text{ kJ kg}^{-1}.$$

From Eq. (B) $h_3 = 192 \text{ kJ kg}^{-1}$ (sat liquid at 0.1), and

$$h_4 = 192 + 6 = 198 \text{ kJ kg}^{-1}.$$

In the *boiler*

$$q_{\text{in}} = q_{41} - w_{41} = h_1 - h_4 = 3422 - 198 = 3224 \text{ kJ kg}^{-1}.$$

Therefore, the cyclical work

$$w_{\text{cyc}} = w_t - w_p = 837 - 6 = q_{\text{in}} - q_{\text{out}} = 3224 - 2393 = 831 \text{ kJ kg}^{-1}.$$

The efficiency

$$\eta = w_{\text{cyc}}/q_{\text{in}} = 831/3224 = 0.26.$$

Integrating the general availability balance equation over the cycle

$$w_{\text{cyc,opt}} = q_{\text{in}} (1 - T_0/T_b) = 3224 (1 - 298/900) = 2156 \text{ kJ kg}^{-1}.$$

The actual Rankine cycle work = 831 kJ kg^{-1} and the actual cycle efficiency $\eta = 0.26$. The Carnot work is 2156 kJ kg^{-1} and the Carnot efficiency $\eta_{\text{Carnot}} = 0.67$. The relative efficiency is

$$\eta_{\text{Avail}} = w_{\text{cyc}}/w_{\text{cyc,opt}} = \eta/\eta_{\text{Carnot}} = 831/2156 = 0.39.$$

The overall irreversibility of the cyclical process is

$$I = w_{\text{cyc,opt}} - w = 2156 - 831 = 1325 \text{ kJ kg}^{-1}.$$

An availability analysis can be performed on the various system components as follows:

For the turbine,

$s_1 = 6.88 \text{ kJ kg}^{-1} \text{ K}^{-1}$, and $s_2 = 8.15 \text{ kJ kg}^{-1} \text{ K}^{-1}$ so that $s_2 > s_1$. Furthermore,

$\psi_1 = h_1 - T_0 s_1 = 3422 - 298 \times 6.88 = 1372 \text{ kJ kg}^{-1}$, and, likewise,

$\psi_2 = 2585 - 298 \times 8.15 = 156.3 \text{ kJ kg}^{-1}$. Therefore,

$w_{t,\text{opt}} = \psi_1 - \psi_2 = 1372 - 156.3 = 1216 \text{ kJ kg}^{-1}$, and

$I_t = 1216 - 837 = 379 \text{ kJ kg}^{-1}$.

For the condenser,

$$\psi_3 = 191.8 - 298 \times 0.649 = -1.6 \text{ kJ kg}^{-1}, \text{ and}$$

$$W_{\text{cond,opt}} = \psi_2 - \psi_3 = 156.3 - (-1.6) = 157.9 \text{ kJ kg}^{-1}. \text{ Therefore,}$$

$$I_{\text{cond}} = 157.9 \text{ kJ kg}^{-1}.$$

For the pump,

$$s_4 = s_3 = 0.649 \text{ kJ kg}^{-1} \text{ K}^{-1}, \text{ and}$$

$$\psi_4 = h_4 - T_0 s_4 = 198 - 298 \times 0.649 = 4.6 \text{ kJ kg}^{-1}. \text{ Consequently,}$$

$$W_{\text{p,opt}} = \psi_3 - \psi_4 = -1.6 - 4.6 = -6.2 \text{ kJ kg}^{-1}.$$

$$\text{Since } w_p = -6.2 \text{ kJ kg}^{-1}, I_p = 0 \text{ kJ kg}^{-1}.$$

For the boiler,

$$W_{\text{b,opt}} = q_b (1 - T_0/T_b) + \psi_4 - \psi_1 = 2156 + 4.6 - 1372 = 789 \text{ kJ kg}^{-1}, \text{ and}$$

$$I_b = W_{\text{b,opt}} - W_b = 789 - 0 = 789 \text{ kJ kg}^{-1}.$$

The total irreversibility = 379 + 158 + 0 + 789 = 1326 kJ kg⁻¹ is the same as that calculated before.

The availability input at the boiler inlet $\psi_4 = 4.6 \text{ kJ kg}^{-1}$.

Results are summarized in tabular form below.

$$\psi_0 = h_0 - T_0 s_0 \approx h_f^{\text{sat}}(25 \text{ C}) - 298 * s_f^{\text{sat}}(25 \text{ C}) = 104.89 - 298 * 0.3674 = -4.6 \text{ kJ/kg}.$$

State	T, C	p, bar	x	H kJ/kg	s kJ/kg	q kJ/kg	w kJ/kg	ψ kJ/kg	i kJ/kg	$\psi' = \psi - \psi_0$
1	500	60	-	3422	6.8	0		1372		1376.6
2		0.1	1.0	2585	8.15	0	837	156.3	379	160.9
3		0.1	0.0	191.8	0.65	-2393	0	-1.6	158	3.0
4		60	-	198	0.65	-	-6	4.6	0	9.2
1		60	-	3422	6.8	3224	-	1372	789	1376.6

The availability input through heat transfer in the boiler $\psi_{41} = q_b (1 - T_0/T_b) = 3224 \times (1 - 298/2000) = 2156 \text{ kJ kg}^{-1}$. **Error! Reference source not found.** illustrates the exergy band diagram for the cyclic process.

Remarks

In this example, the processes comprising the Rankine cycle are all reversible. The irreversibility arises due to the temperature difference between the thermal energy reservoir and the boiler. In this case the maximum work output $W_{\text{cyc,opt}}$ can be obtained by placing two Carnot heat engines, one between the reactor and the boiler (to supply heat to the boiler), and the second between the condenser and its ambient (to reject heat to the ambient).

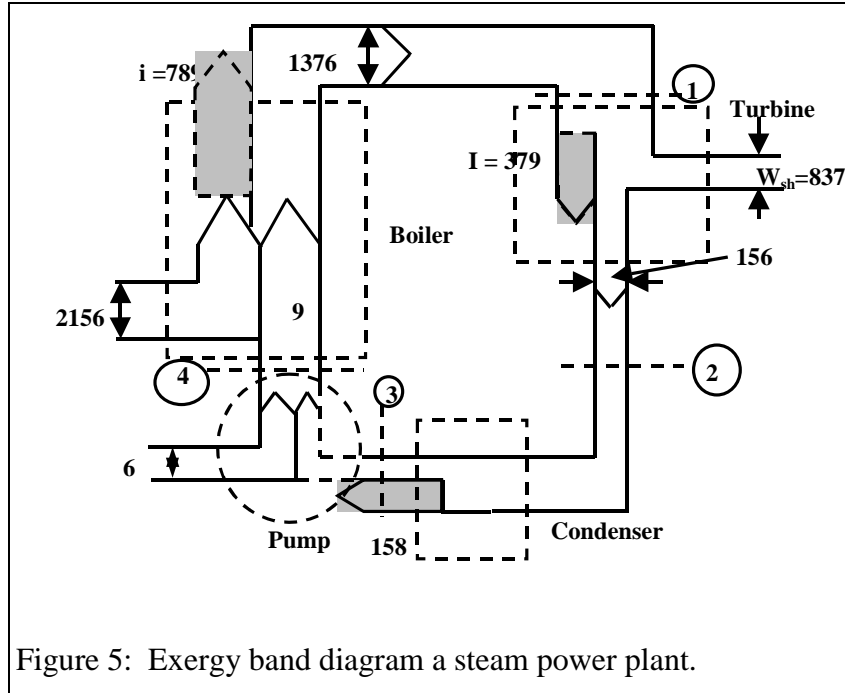


Figure 5: Exergy band diagram a steam power plant.

The boiler accounts for 24.5% of the total irreversibility.

e. Example 6

Determine the maximum work that can be performed if a gas mixture consisting of 40% O₂ and 60% N₂ is expanded from 2000 K and 60 bars to the dead state at which it is at thermo-mechanical-chemical equilibrium.

Solution

The maximum work

$$\bar{w}_{\max, \infty} = \bar{w}_{\max, 0} + \bar{w}_{\text{ch}}, \text{ where} \quad (\text{A})$$

$$\bar{w}_{\max, 0} = \bar{\phi} - \bar{\phi}_0.$$

Now, $\bar{\phi} = \bar{u} - T_0 \bar{s} + P_0 \bar{v}$, where $\bar{v} = 0.08314 \times 2000 \div 60 = 2.77 \text{ m}^3 \text{ kmole}^{-1}$. The specific entropy, $\bar{s}_{\text{N}_2} = \bar{s}_{\text{N}_2}^0 - \bar{R} \ln(p_{\text{N}_2}/1) = 251.6 - 8.314 (\ln(0.4 \times 60)/1) = 225.2 \text{ kJ kmole}^{-1} \text{ K}^{-1}$, and $\bar{s}_{\text{O}_2} = \bar{s}_{\text{O}_2}^0 - \bar{R} \ln(p_{\text{O}_2}/1) = 268.7 - 8.314 (\ln(0.6 \times 60)/1) = 238.9 \text{ kJ kmole}^{-1} \text{ K}^{-1}$. The mixture initial specific entropy and internal energy are $\bar{s} = (0.4 \bar{s}_{\text{N}_2} + 0.6 \bar{s}_{\text{O}_2}) = 233.49 \text{ kJ kmole}^{-1} \text{ K}^{-1}$, and $\bar{u} = (0.4 \times 48,181 + 0.6 \times 51,253) = 50,024 \text{ kJ kmole}^{-1}$.

Therefore,

$$\bar{\phi} = 50,024 - 298 \times 233.4 + 100 \times 2.77 = -19252.2 \text{ kJ kmole}^{-1},$$

$$\bar{u}_0 = 0.4 \times 6190 + 0.6 \times 6203 = 6197.8 \text{ kJ kmole}^{-1},$$

$$\bar{s}_0 = 0.4(191.52 - 8.314 \times \ln(0.4/1)) + 0.6(205.0 - 8.314 \times \ln(0.6/1)) = 205.2 \text{ kJ kmole}^{-1} \text{ K}^{-1},$$

$$\bar{v}_0 = 0.08314 \times 298/1 = 24.78 \text{ m}^3 \text{ kmole}^{-1},$$

$$\bar{\phi}_0 = 6197.8 - 298 \times 205.2 + 100 \times 24.78 = -52473.8 \text{ kJ kmole}^{-1}.$$

Therefore,

$w_{\max,0} = -19252.2 - (-52473.8) = 33,221.6 \text{ kJ kmole}^{-1}$. Since

$w_{\text{ch}} = \bar{\phi}_0 - \bar{\phi}_\infty$ and

$$\bar{\phi}_\infty = \sum X_k \bar{\phi}_{k,\infty}$$

$$\bar{\phi}_{\text{O}_2,\infty}(T_0, P_{\text{O}_2,\infty}) = \bar{u}_{\text{O}_2}(T_0, p_{\text{O}_2,\infty}) - T_0 \bar{s}_{\text{O}_2}(T_0, p_{\text{O}_2,\infty}) + P_0 \bar{v}_{\text{O}_2}(T_0, p_{\text{O}_2,\infty})$$

$$= 6203 - 298 \times (205.0 - 8.314 \times \ln(0.21/1)) + 100 \times (0.08314 \times 298/0.21)$$

$$= -46981 \text{ kJ kmole}^{-1}.$$

Likewise,

$$\bar{\phi}_{\text{N}_2,\infty}(T_0, p_{\text{N}_2,\infty})$$

$$= 6190 - 298 \times (191.5 - 8.314 \times \ln(0.79/1)) + 100 \times (0.8314 \times 298/0.79)$$

$$= 6190 - 298 \times 193.5 + 100 \times 31.4 = -48,333 \text{ kJ kmole}^{-1}.$$

Therefore,

$$\bar{\phi}_\infty = 0.6 \times (-46981) + 0.4 \times (-48333) = -47521.8 \text{ kJ kmole}^{-1}, \text{ and}$$

$$w_{\text{ch}} = -52473.8 - (-47521.8) = -4952 \text{ kJ kmole}^{-1}.$$

$$w_{u, \max, \infty} = 33,221.6 + (-4952) = 28,269.6 \text{ kJ/mole}$$

Remarks:

The chemical work per kmole of mixture is negative since more work input is necessary to compress 0.4 kmoles of N_2 from 0.4 bars to 0.79 bars compared to work output obtained from the expansion of 0.6 kmoles of O_2 from 0.6 bars to 0.21 bars

f. Example 7

A wet cooling tower is used to cool the water discharged from the condenser of a power plant. (**Error! Reference source not found.**) Water enters the tower at 45°C (state 1) and leaves at 25°C (state 2). Additional makeup water enters the tower at 25°C. Air enters the tower at 20°C and 35% relative humidity, and leaves it at 35°C and 70% relative humidity. Assume $c_{p,v} = 0.603 \text{ kJ kg}^{-1} \text{ K}^{-1}$. Determine:

The mass flow rate of dry air; and

The optimum work.

Solution

With $p_1^{\text{sat}}(20\text{C}) = 0.02339 \text{ bars}$ and $p_1^{\text{sat}}(35\text{C}) = 0.05628 \text{ bars}$. Using Eqs. (86') and known values of RH, $p_{v1} = 0.0082 \text{ bars}$, $p_{v2} = 0.0394 \text{ bars}$. Hence $p_{a,1} = 1 - 0.008 = 0.992 \text{ bar}$, and $p_{a,2} = 0.961 \text{ bar}$. Then from Eq. (100), $w_1 = 0.0051$, $w_2 = 0.025$ (Alternately use psychrometric charts in Appendix Fig B-1). Using constant specific heats $h_{a,1} = c_{pa} T$ (in C) = 20 kJ kg^{-1} , $h_{a,2} = 35 \text{ kJ kg}^{-1}$, and from Tables A-4, $h_{v1} = 2547.2$, $h_{v2} = 2565.3$, $s_g(25\text{C}) = 8.558$, with $R = 8.314/18.02 = 0.461 \text{ kJ/kgK}$, $s_{g1} = s_g^{\text{sat}}(25\text{C}) - R \ln\{p_{v1}/P_v^{\text{sat}}(T_1)\} = 8.6672 \text{ kJ/kg K}$, $s_{g2} = 8.3531 \text{ kJ/kg K}$, $h_{f,3} = 188.45 \text{ kJ kg}^{-1}$, $s_{f,3} = 0.6387 \text{ kJ kg}^{-1} \text{ K}^{-1}$, $h_{f,4} = h_{f,5} = 104.89 \text{ kJ kg}^{-1} \text{ K}^{-1}$, $s_{f,4} = 0.3674 \text{ kJ kg}^{-1} \text{ K}^{-1}$.

From mass balance for dry air

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$

For water species

$$\frac{dm_{H_2O}}{dt} = \dot{m}_{v1} + \dot{m}_{f3} + \dot{m}_{f5} - \dot{m}_{v2} - \dot{m}_{f4}$$

$$\dot{m}_{f5} = \dot{m}_{v1} - \dot{m}_{v2}$$

Dividing by \dot{m}_a

$$\dot{m}_{f5} / \dot{m}_a = \dot{m}_{v1} / \dot{m}_a - \dot{m}_{v2} / \dot{m}_a = w_2 - w_1$$

Then

$$\dot{m}_{f5} / \dot{m}_a = 0.025 - 0.005 = 0.020 \text{ kg of water per kg dry air}$$

Through an energy balance

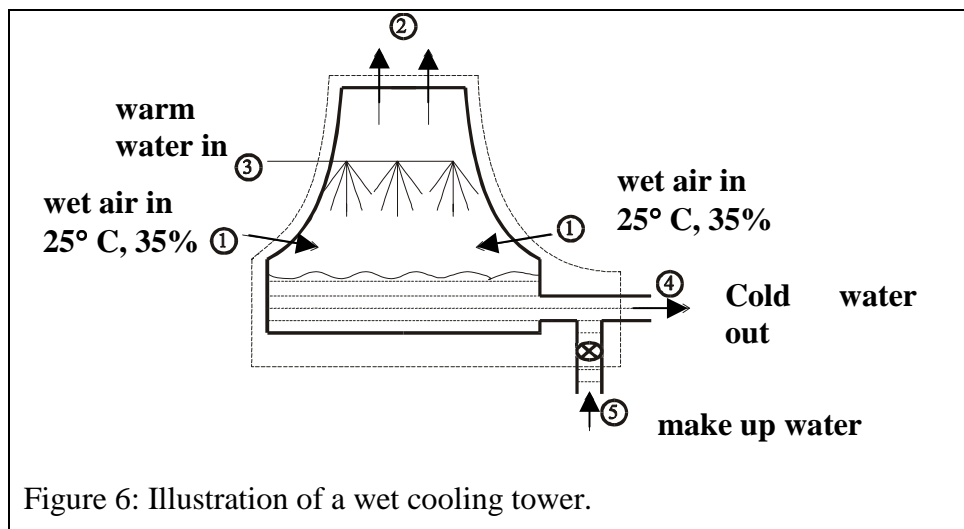
$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \dot{m}_{v1} h_{v1} + \dot{m}_{a1} h_{a1} + \dot{m}_{f3} h_{f3} + \dot{m}_{f5} h_{f5} - \dot{m}_{v2} h_{v2} - \dot{m}_{a2} h_{a2} - \dot{m}_{f4} h_{f4}$$

Dividing by \dot{m}_a and assuming steady state

$$0 = \frac{\dot{Q}}{\dot{m}_a} - \frac{\dot{W}}{\dot{m}_a} + \dot{m}_{v1} h_{v1} / \dot{m}_a + \dot{m}_{f3} h_{f3} / \dot{m}_a + \dot{m}_{f5} h_{f5} / \dot{m}_a - \dot{m}_{v2} h_{v2} / \dot{m}_a - \dot{m}_{f4} h_{f4} / \dot{m}_a$$

Since $\dot{Q} = 0$, $\dot{W} = 0$, $\dot{m}_{f4} = \dot{m}_{f3}$ and using mass balance equation,

$$0 = 0 - 0 + w_1 h_{v1} + h_{a1} + \dot{m}_{f3} / \dot{m}_a \{ h_{f3} - h_{f4} \} + \{ w_2 - w_1 \} h_{f5} - w_2 h_{v2} - h_{a2}$$



Solving

$$\begin{aligned} \dot{m}_{f3} / \dot{m}_a &= [\{ w_2 h_{v2} + h_{a2} \} - \{ w_1 h_{v1} + h_{a1} \} - \{ w_2 - w_1 \} h_{f5}] / \{ h_{f3} - h_{f4} \} \\ &= [\{ 0.025 * 2565.3 + 35 \} - \{ 0.005 * 2538.1 + 20 \} - \{ 0.02 \} * 104.9] / \\ &\{ 188.45 - 104.9 \} = 0.794 \text{ kg of water per kg dry air} \end{aligned}$$

The optimum work

$$\dot{W}_{\text{opt}} = \dot{m}_a \Psi_{a,1} + \dot{m}_{v,1} \Psi_{v,1} + \dot{m}_{f,3} \Psi_{f,3} + \dot{m}_{f,5} \Psi_{f,5} - \dot{m}_a \Psi_{a,2} - \dot{m}_{v,2} \Psi_{v,2} - \dot{m}_{f,4} \Psi_{f,4}, \text{ i.e.,}$$

Dividing by \dot{m}_a

$$w_{\text{opt}} = \{ (h_{a,1} - h_{a,2} - T_0(s_{a,1} - s_{a,2})) \} + w_1 \{ h_{v,1} - T_0 s_{v,1} \} - w_2 \{ h_{v,2} - T_0 s_{v,2} \} + \dot{m}_{f,3}/\dot{m}_a (h_{f,3} - h_{f,4} - T_0(s_{f,3} - s_{f,4})) + \{w_2 - w_1\} \Psi_{f,5}$$

$$\Psi_{f,5} = h_{f,5} - T_0 s_{f,5} = 104.89 - 298 * 0.3674 = -4.595 \text{ kJ/kg}$$

$$w_{\text{opt}} = [-15 - 298 \times \{1 \ln(293/308) - 0.287 \ln(0.992/0.961)\}] + 0.005 [2538.1 - 298 * \{8.6672 - 0.461 \ln(0.0082/0.02339)\}] - 0.025 \times [2565.3 - 298 * \{8.3531 - 0.461 \ln(0.0394/0.05628)\}] + 0.794 [188.45 - 104.89 - 298 \times (0.6387 - 0.3674)] + 0.02 * (-5.595) = [2.594 + 0.005 * (-188.722) - 0.025 * 27.0909 + 0.794 * 2.713 - 0.112] = 3.015 \text{ kJ kg}^{-1} \text{ (of dry air).}$$

Expressing in kJ per kg of water pumped,

$$w_{\text{opt}} = 3.015 / 0.791 = 3.811 \text{ kJ kg}^{-1} \text{ of water pumped to cooling tower.}$$

For air we use reference temperature as 273.15 K, and reference pressure as 1 bar.

$$\Psi_2 = 35 - 298 * \{1 \ln(308/273.15) - 0.287 \ln(0.961/1)\} + 0.025 [2565.3 - 298 * \{8.3531 - 0.461 \ln(0.0394/0.05628)\}] = -3.79 + 0.025 * (27.09) = -3.11 \text{ kJ of mix at 2 per kg of dry air.}$$

$$\Psi_2' = \Psi_2 - \Psi_0$$

$$\Psi_0 = 25 - 298 * \{1 \ln(298/273.15) - 0.287 \ln(0.961/1)\} + 0.025 [2547.2 - 298 * \{8.558 - 0.461 \ln(0.05628/0.03169)\}] = -4.60 + 0.025 * 75.82 = -2.71 \text{ kJ of mix at 2 per kg of dry air.}$$

$$\Psi_2' = \Psi_2 - \Psi_0 = -3.11 + 2.71 = -0.40 \text{ kJ/kg, dry air}$$

$$\Psi_\infty = 25 - 298 * \{1 \ln(298/273.15) - 0.287 \ln(0.992/1)\} + 0.025 [2547.2 - 298 * \{8.558 - 0.461 \ln(0.0082/0.03169)\}] = -1.8 + 0.025 * (-188.9) = -6.523 \text{ kJ/kg of dry air}$$

$$w_{\text{ch}} = \Psi_0 - \Psi_\infty = -2.71 + 6.523 = 3.81 \text{ kJ/kg, dry air}$$

Remarks

The results are given per kg of dry air since dry air mass flow remains constant during cooling or heating of wet mixtures.

The optimum work is lost in the power plant. The design could have used (1) a Carnot engine to obtain work from the warm water, (2) run a heat pump to heat the air, (3) vaporize some water, and (4) add vapor to the exiting air stream.

The same method can be used to determine the work required to separate water vapor from air so that the vapor does not condense on the evaporator coils in air conditioning devices (and, consequently, does not reduce the heat transfer rate).