Solution to Home Assignment 1

Solution to Problem#1

1. a)

$$\oint (xy^3dx + 3x^2y^2dy) = \left[3x^2\int_5^7 y^2dy\right]_{x=2} + \left[y^3\int_2^4 x\ dx\right]_{y=7} + \left[3x^2\int_7^5 y^2dy\right]_{x=4} + \left[y^3\int_4^2 x\ dx\right]_{y=5}$$

$$= \frac{3.2^2(7^3-5^3)}{3} + \frac{7^3(4^2-2^2)}{2} + \frac{3.4^2(5^3-7^3)}{3} + \frac{5^3(2^2-4^2)}{2} = 872 + 2058 - 3488 - 750 = -1308$$

b)

$$\oint (2e^x ydy + e^x y^2 dx) = \oint d(e^x y^2) = 0$$ (exact differential and hence path independent)

First function is path dependent and second function is path independent

Solution to Problem#2

$$dS = \frac{dU}{T} + \frac{pdV}{T} = MdU + NdV \text{ where } M = \frac{1}{T} \quad and \quad N = \frac{p}{T}$$

Condition for exact differential:

$$\left[\frac{\partial M}{\partial V}\right]_U = \left[\frac{\partial N}{\partial U}\right]_V \quad i.e. \quad \left[\frac{\partial}{\partial V}\left(\frac{1}{T}\right)\right]_U = \left[\frac{\partial}{\partial U}\left(\frac{p}{T}\right)\right]_V$$

Also exact differential for dS is $\left[\frac{\partial}{\partial p}\left(\frac{1}{T}\right)\right]_H = \left[\frac{\partial}{\partial H}\left(-\frac{V}{T}\right)\right]_p$

Similarly for the exact differential dT are $\left[\frac{\partial}{\partial V}\left(-\frac{1}{S}\right)\right]_A = \left[\frac{\partial}{\partial A}\left(-\frac{p}{S}\right)\right]_V$ and

$\left[\frac{\partial}{\partial p}\left(-\frac{1}{S}\right)\right]_G = \left[\frac{\partial}{\partial G}\left(\frac{V}{S}\right)\right]_p$;

the condition for exact differential for dp is $\left[\frac{\partial}{\partial T}\left(\frac{1}{V}\right)\right]_G = \left[\frac{\partial}{\partial G}\left(\frac{S}{V}\right)\right]_T$ and

that for dV is $\left[\frac{\partial}{\partial A}\left(-\frac{S}{p}\right)\right]_T = \left[\frac{\partial}{\partial T}\left(-\frac{1}{p}\right)\right]_A$

Solution to Problem#3

False
True
False
False

False
True
False
False

<u>Solution to Problem#4</u>

General - for all sections of the problem:
Subscript A and B will indicate properties for sections A and B and added subscripts 1 and 2 will indicate the initial and the final conditions.

Given data: P_{A1}=1.48 bar
T_{A1}=320 K
m_A=1 kg
and also, P_{B1}=0.42 bar
T_{B1}=290 K

Find: P_{A2}, V_{A2}, T_{A2}, P_{B2}, V_{B2}, T_{B2}

Assumptions: 1. No energy added/removed by container.
2. QS process means internally reversible process.
3. Air behaves as ideal gas for the operating conditions mentioned.

We know, R_u = 0.08314 bar.m^3/kgmole.K and molar mass of air = 28.97 kg/kgmole, hence, R_{air}= R_u/molar mass of air = 2.8699X10^{-3} bar.m^3/kg.K. From now on, whenever R is mentioned in this problem, it will mean R_{air} unless otherwise specified.

<u>Calculate V_{A1}, V_{B1}, m_B</u>

Ideal gas relation gives, for section A, $P_{A1}V_{A1} = m_A R T_{A1}$
or, $\mathbf{V_{A1}} = m_A R T_{A1} / P_{A1}$ = 1X2.8699X10^{-3}X320 / 1.48
= 0.6205 m^3

As the container is rigid, $V_{A1}+V_{B1}$ = 2.0 m^3
hence, $\mathbf{V_{B1}}$ = 2.0 - V_{A1} = 1.3795 m^3.

Again, from ideal gas relation for section B,
$\mathbf{m_B} = P_{B1}V_{B1} / RT_{B1}$ = 0.42X1.3795 /(2.8699X10^{3}X290)
= 0.6962 kg.

Part a) i)

If only the pin is removed and expansion in A is QS, process in B is unknown

we have six unknowns: P_{A2}, V_{A2}, T_{A2}, P_{B2}, V_{B2}, T_{B2}

We have six equations for the six unknowns:

$P_{A2} = P_{B2}$ - From mechanical equilibrium

$T_{A2} / T_{A1} = (P_{A2} / P_{A1})^{(k-1)/k}$ - Adiabatic reversible (isentropic) process inA

$\Delta U_A = -\Delta U_B$ - 1st law applied to the whole container δQ and δW both being zero

$P_{A2}V_{A2} = m_A RT_{A2}$ - Ideal gas relation for section A
$P_{B2}V_{B2} = m_B RT_{B2}$ - Ideal gas relation for section A
$V_{A2} + V_{B2} = 2.0\ m^3$ - As the whole container is rigid

So it will not be difficult to find the six unknowns.

Solution process:

Let $P_{A2} = P_{B2} = P_2$ (1)

From the second relation above,

$T_{A2} / 320 = (P_{A2} / 1.48)^{(1.4-1)/1.4}$

or, $T_{A2} = 286.09 \text{ X } P_2^{0.2857}$ (2)

From the third relation above,

$m_A C_v (T_{A2} - T_{A1}) = - m_B C_v (T_{B2} - T_{B1})$

Assuming C_v does not change at low temps and pressures,

$T_{A2} - 320 = 0.6962 (290 - T_{B2})$ (3)

From the fourth and the fifth relations above, we get respectively,

$P_2V_{A2} = 1X2.8699X10^{-3}\ T_{A2}$ (4)

and, $P_2V_{B2} = 0.6962X2.8699X10^{-3}\ T_{B2}$ (5)

The sixth relation gives,

$V_{A2} + V_{B2} = 2.0$ (6)

Solving equations (1) through (6), we obtain,

$P_2 = 0.749$ bar

$T_{A2} = 263.41$ K **$T_{B2} = 371.1$ K**

$V_{A2} = 1.01\ m^3$ **$V_{B2} = 0.99\ m^3$**

Part a) ii)

If only the pin is removed and compression in B is QS, process in A is unknown

we have six unknowns: P_{A2}, V_{A2}, T_{A2}, P_{B2}, V_{B2}, T_{B2}

We have six equations for the six unknowns:

$P_{A2} = P_{B2}$ - From mechanical equilibrium
$T_{B2} / T_{B1} = (P_{B2} / P_{B1})^{(k-1)/k}$ - Adiabatic reversible (isentropic) process inA
$\Delta U_A = -\Delta U_B$ - 1st law applied to the whole container δQ and δW both being zero
$P_{A2}V_{A2} = m_A RT_{A2}$ - Ideal gas relation for section A
$P_{B2}V_{B2} = m_B RT_{B2}$ - Ideal gas relation for section A
$V_{A2} + V_{B2} = 2.0\ m^3$ - As the whole container is rigid

So it will not be difficult to find the six unknowns.

Equations (1) and (3) through (6) are identical to part i) however, only equation (2) will change as follows: $T_{B2} / 290 = (P_2 / 0.42)^{(1.4-1)/1.4}$ (7)

Solving equations (1), (3) - (6) and (7), we obtain,

$P_2 = 0.749$ bar

$T_{A2} = 283.65$ K **$T_{B2} = 342.1$ K**

$V_{A2} = 1.087\ m^3$ **$V_{B2} = 0.913\ m^3$**

Part b)

If both the pin and the insulation are removed

we have six unknowns: P_{A2}, V_{A2}, T_{A2}, P_{B2}, V_{B2}, T_{B2}

We have six equations for the six unknowns:

$P_{A2} = P_{B2}$ - From mechanical equilibrium

$T_{A2} = T_{B2} = T_2$ - From thermal equilibrium

$\Delta U_A = -\Delta U_B$ - 1st law applied to the whole container δQ and δW both being zero

$P_{A2}V_{A2} = m_A R T_{A2}$ - Ideal gas relation for section A

$P_{B2}V_{B2} = m_B R T_{B2}$ - Ideal gas relation for section A

$V_{A2} + V_{B2} = 2.0\ m^3$ - As the whole container is rigid

So it will not be difficult to find the six unknowns.

The third relation gives,

$$m_A\, C_v\, (T_{A2} - T_{A1}) = -\, m_B\, C_v\, (T_{B2} - T_{B1})$$

or, $1.0 X C_v\, (T_2 - 320) = -\, 0.6962 X C_v\, (290 - T_2)$ (since, $T_{A2} = T_{B2} = T_2$)

solving for T_2, we obtain,

$$\mathbf{T_2 = 307.69\ K}$$

Subsequently, using the rest of the relations, we will obtain,

$$\mathbf{V_{B2} = 0.821\ m^3}$$

$$\mathbf{V_{A2} = 1.179\ m^3}$$

and $\mathbf{P_{A2} = P_{B2} = P_2 = 0.749\ bar}$

Discussions:

One interesting point to note is that in all cases (parts a) i), ii) and b)) the final pressure is the same, i.e., P_2 in all cases is same and is 0.749 bar. However, what varies case to case is the temperature.

Also to note is that when in case a) (i), $P_{B1}V_{B1}^k = 0.65896$, the value of $P_{B2}V_{B2}^k = 0.739521$; which shows that the process in chamber B is not isentropic. Anisentropic condition may result due to a process not being adiabatic and/or it not being internally reversible (QS). From the conditions given in the problem, the process in section B has to be adiabatic since it is insulated from all sides. This clearly indicates that the process in B is not isentropic because it is not internally reversible, i.e. it is not a QS process even though that in section A is QS.

The next point to check is whether the process in section A is QS for part a) ii). It is seen that when $P_{A1}V_{A1}^k = 0.7588$, the value of $P_{A2}V_{A2}^k = 0.8418$; which shows that in a fashion similar to that demonstrated in part a) i) when the process in section B is QS, the process going on in section A is not QS.

<u>Solution to Problem#5</u>

General balance equation for quantity *A*

$$\frac{\partial A}{\partial t} = \dot{A}_{in} - \dot{A}_{out} + \dot{A}_g$$

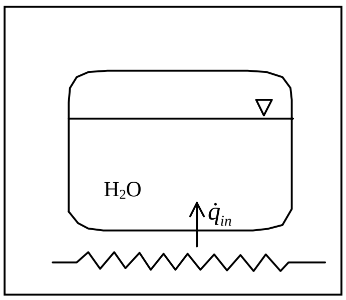
Amount of *A* generated (or consumed) in the system

Amount of *A* exiting the system

Amount of *A* entering the system

Variation of *A* with the time

a) $t < t_L$



The system is closed
The gas volume is assumed to be constant Vg
The gas is assumed to be ideal

Balance equations

Air $$\frac{\partial m_A}{\partial t} = 0 - 0 + 0$$

Vapor $$\frac{\partial m_V}{\partial t} = 0 - 0 + \dot{m}_{V,g}$$

Steam generated by evaporation

Water $$\frac{\partial m_W}{\partial t} = 0 - 0 - \dot{m}_{V,g}$$

Energy $$\frac{\partial U}{\partial t} = \dot{q}_{in} - 0 + 0$$

The following considerations may apply

$$\dot{q}_{in} = \mathrm{W}_{\mathrm{elec}}$$

$$\frac{\partial U}{\partial t} = \left(m_C c_S + m_A c_A + m_V c_V + m_W c_W\right)\frac{\partial T}{\partial t} + m_{V,g}\Delta H_{ev}\left(T\right)$$

$$m_V\left(T\right) = \frac{P_{sat}\left(T\right)\cdot MW_{H_2O}\cdot \mathrm{V_g}}{RT}$$

Partial pressure of steam is supposed to be $P_{sat}(T)$. $P_{sat}(T)$ can be found by Clausius-Clapeyron equation (or Antoine equation).

System of equations

$$\begin{cases} m_V(T) = \dfrac{P_{sat}(T)\cdot MW_{H_2O}\cdot \mathrm{V_g}}{RT} \\ \dfrac{\partial m_V}{\partial t} = -\dfrac{\partial m_W}{\partial t} \\ (m_C c_S + m_A c_A + m_V c_V + m_W c_W)\dfrac{\partial T}{\partial t} - \dfrac{\partial m_V}{\partial t}\Delta H_{ev}(T) = \mathrm{W_{elec}} \end{cases}$$

I.C. $\quad t = 0 \quad T = T_{amb}$

$$m_V = m_V(T_{amb})$$

$$m_W = m_{W,0} - m_V(T_{amb})$$

Variables: $T(t),\ m_V(t),\ m_W(t)$

Global mass balance

$$\frac{\partial m_{TOT}}{\partial t} = 0$$

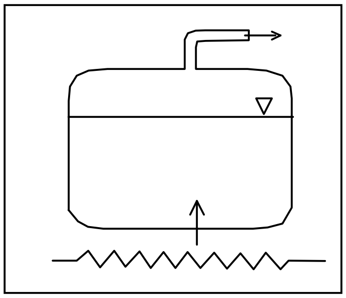
The system is closed m_{TOT} =constant

We can find time t_L from the condition

$$P_{sat}(T) + P_A(T) = P_L \qquad P_A(T) = \frac{m_A RT}{MW_{air}\cdot \mathrm{V_g}}$$

P_L being the pressure required to lift the weight

b) $t > t_L$



The system is open
The control volume is the presscooker
The gas volume is assumed to be constant $\mathrm{V_g}$
The gas is supposed to be ideal

Balance equations

Air $$\frac{\partial m_A}{\partial t} = 0 - \dot{m}_{A,out} + 0$$

Vapor $$\frac{\partial m_V}{\partial t} = 0 - \dot{m}_{V,out} + \dot{m}_{V,g}$$

Water $$\frac{\partial m_W}{\partial t} = 0 - 0 - \dot{m}_{V,g}$$

Energy $$\frac{\partial U}{\partial t} = \dot{q}_{in} - \dot{h}_{out} + 0$$

The following considerations may apply

$$\dot{m}_{A,out} = \frac{m_A}{m_A + m_V}\dot{m}_{out} \quad , \quad \dot{m}_{V,out} = \frac{m_V}{m_A + m_V}\dot{m}_{out}$$

$$\dot{q}_{in} = \mathrm{W_{elec}}$$

$$P_{sat}(T) + P_A(T) = P_L$$

Gas pressure is constant

$$m_V(T) = \frac{P_{sat}(T) \cdot MW_{H_2O} \cdot \mathrm{V_g}}{RT}$$

Partial pressure of steam is $P_{sat}(T)$

$$\dot{h}_{out} = c_A T \frac{\partial m_A}{\partial t} + c_V T \frac{\partial m_V}{\partial t}$$

System of equations

$$\begin{cases} P_{sat}(T) + \dfrac{m_A RT}{MW_{air} \cdot \mathrm{V_g}} = P_L \\ m_V(T) = \dfrac{P_{sat}(T) \cdot MW_{H_2O} \cdot \mathrm{V_g}}{RT} \\ \dfrac{\partial m_A}{\partial t} = -\left(\dfrac{m_A}{m_A + m_V}\right) \cdot \left(\dfrac{\partial m_A}{\partial t} + \dfrac{\partial m_V}{\partial t} + \dfrac{\partial m_W}{\partial t}\right) \\ (m_C c_S + m_A c_A + m_V c_V + m_W c_W)\dfrac{\partial T}{\partial t} - \dfrac{\partial m_V}{\partial t}\Delta H_{ev}(T) = \mathrm{W_{elec}} - \left(c_A T \dfrac{\partial m_A}{\partial t} + c_V T \dfrac{\partial m_V}{\partial t}\right) \end{cases}$$

I.C. $t = 0 \quad T = T(t_L)$

$$m_A = m_{A,0}$$
$$m_V = m_V T(t_L)$$
$$m_W = m_{W,0} - m_V T(t_L)$$

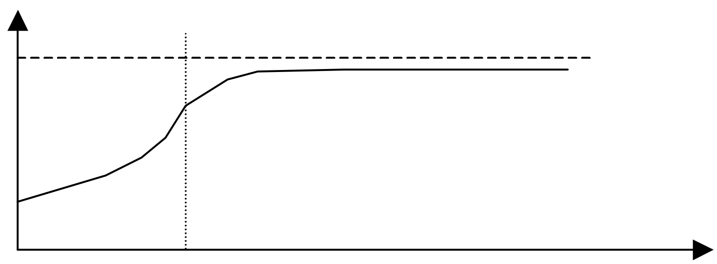
Initial conditions are evaluated from the previous problem

Variables: $T(t),\ m_A(t),\ m_V(t),\ m_W(t)$

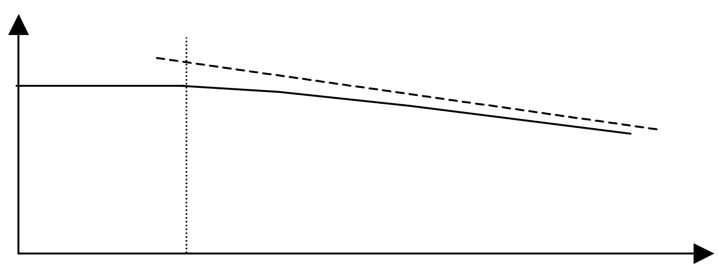
Global mass balance

$$\frac{\partial m_{TOT}}{\partial t} = -\dot{m}_{out} = -\left(\frac{\partial m_A}{\partial t} + \frac{\partial m_V}{\partial t} + \frac{\partial m_W}{\partial t}\right)$$

Behavior of the function *T(t)* (qualitative)



Behavior of the function $m_{TOT}(t)$ (qualitative)



Equations don't hold if the evaporated water is too much.