SECTION 2.4

Constructing proofs in Fitch

Writing out a long formal proof in complete detail, let alone reading or checking it, can be a pretty tedious business. The system *F* makes this less painful than many formal systems, but it's still not easy. This book comes with a second program, Fitch, that makes constructing formal proofs much less painful. Fitch can also check your proof, telling you whether it is correct, and if it isn't, which step or steps are mistaken. This means you will never be in any doubt about whether your formal proofs meet the standard of rigor demanded of them. And, as a practical matter, you can make sure they are correct before submitting them.

There are other ways in which Fitch makes life simpler, as well. One is that Fitch is more flexible than the system *F*. It lets you take certain shortcuts that are logically correct but do not, strictly speaking, fall under the rules of *F*. You can always go back and expand a proof in Fitch to a formally correct proof, but we won't often insist on this.

Let us now use Fitch to construct a simple formal proof. Before going on, you will want to read the first few sections of the chapter on how to use Fitch in the manual.

You try it

1. We are going to use Fitch to construct the formal proof of SameCol(a,d) from premises SameCol(a,b), b = c and c = d. Launch Fitch and open the file Exercise 2.17 (note that this file may appear *after* Exercise 13.x in the Fitch Exercise Files folder). Here we have the beginnings of the formal proof. The premises appear above the Fitch bar. It may look slightly different from the proofs we have in the book, since in Fitch the steps don't have to be numbered, for reasons we will soon find out. (If you would like to have numbered steps,you can choose **Show Step Numbers** from the **Proof** menu. But don't try this yet.)

2. Before we start to construct the proof, notice that at the bottom of the proof window there is a separate pane called the "goal strip," containing the goal of the proof. In this case the goal is to prove the sentence SameCol(a,d). If we successfully satisfy this goal, we will be able to get Fitch to put a check mark to the right of the goal.

3. Let's construct the proof. What we need to do is fill in the steps needed to complete the proof. Add a new step to the proof by choosing **Add Step After** from the **Proof** menu. In the new step, enter the sentence SameCol(a,c) either by typing it in or by using the toolbar at the top of the proof window.

4. Once you have entered SameCol(a,c), add another step below this and enter the goal sentence SameCol(a,d). Use the mouse to click on the word **Rule?** that appears to the right of SameCol(a,d). In the menu that pops up, go to the **Elimination Rules** and select =. If you did this right, the rule name should now say = **Elim**. If not, try again.

5. Next cite the third premise and the intermediate sentence you first entered. You do this in Fitch by clicking on the two sentences, in either order. If you click on the wrong one, just click again and it will be uncited. Once you have the right sentences cited, choose **Verify Proof** from

the **Proof** menu (or toolbar). The last step should check out, as it is a valid instance of =**Elim**. The step containing SameCol(a,c) will not check out, since we haven't yet indicated what it follows from. Nor will the goal check out, since we don't yet have a complete proof of SameCol(a,d).

6. Move the focus slider (the triangle in the left margin) to the problematic step, SameCol(a,c). You will see below the goal strip that it says A rule must be specified. Fitch always provides such helpful advice if it rejects your proof, telling you here what is wrong with the selected step. Justify the step using the = **Elim rule**. Cite the first and second premises. Check the step using **Check Step**; it will check out if you have justified it correctly.

7. Now the whole proof, including the goal, should check out. To find out if it does, use **Verify Proof** again. The proof should look like this, except for the absence of numbers on the steps. (Try out **Show Step Numbers** from the **Proof** menu now. The highlighting on support steps will go away and numbers will appear.)

| 1. SameCol(a,b)
| 2. b = c
| 3. c = d
|-
| 4. SameCol(a,c) = **Elim:** 1,2
| 5. SameCol(a,d) = **Elim:** 3,4

8. Save your proof as **Proof 2.17**, and use Submit to check it.

9. Now add a step before the first introduced step (the one containing SameCol(a,c)), and enter the sentence b = b. Do this by moving the focus slider to the step containing SameCol(a,c) and choosing **Add Step Before** from the **Proof** menu. (If the new step appears in the wrong place, choose **Delete Step** from the **Proof** menu.) Enter the sentence b = b and justify it by using the rule =**Intro**. Check that the step is allowed.

10. Now check the proof. You will see that it is still correct, even though it contains the unnecessary step b = b. We included it for practice and to show that a proof will still work if it contains unnecessary (though legitimate) steps; anesthetically, of course, it is best to avoid them.

11. Finally, here's a trick you will find handy: Reload **Exercise 2.17**. Add a new proof line. Click on the goal sentence at the very bottom of the window. This puts the focus on the goal sentence. Choose **Copy** from the **Edit** menu, and then click back on the empty step at the end of your proof. Choose **Paste** from the **Edit** menu and the goal sentence will be entered into this step. Add a step before it, and do the proof over, just the same as before. Use **Verify Proof** to check it.

Congratulations

Now read the rest of §2.4 (pp 60-62) in the book.