Phys 450

Homework set 1 due February 11, 2002

Problem 1

A group of three particles share a total energy of 6ε. The allowed energy levels for each of the particles are 0, ε, 2ε, 3ε, 4ε, ………….

(a) Enumerate the number of different ways in which the total energy 6ε can be shared among the three particles.
(b) If each of these ways is equally likely, calculate the relative populations of the different energy levels.

Problem 2

(a) For a one-dimensional random walk, write down a formula for the probability that after N steps, the particle is displaced by a distance $x = n\delta - (N-n)\delta = (2n-N)\delta$, i.e. the probability that the particle takes n steps to the right and $(n-N)$ steps to the left, with δ the distance covered in each step.

(b) Plot the distribution for $N = 14$ steps.

(c) Plot a gaussian distribution on the same graph to overlay your discrete probability distribution. What standard deviation (in units of δ) did you pick for the gaussian distribution?

Problem 3

(a) Verify that the "spreading" Gaussian function

$$c(x,t) = \frac{C_0}{\sqrt{4\pi Dt}} \exp[-x^2/(4Dt)]$$

satisfies the one-dimensional diffusion equation $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$

(b) Verify that $c(x,t)$ of part (a) describes the spreading of a constant number of particles, i.e. $\int_{-\infty}^{\infty} \mathrm{dx}\, c(x,t)$ is a time-independent constant.

Problem 4

(a) For a 100 kDa protein of radius 3 nm, estimate the mass, density, drag coefficient, diffusion coefficient, and root-mean-square velocity. Assume that the protein is in water at 300 K.

(b) How long will it take to diffuse a distance of 40 nm?

(c) If there is an applied force of 2 pN, how long will it take to move 40 nm in the direction of the force.

Problem 5

The motor protein kinesin generates a force of 6 pN. Given that the viscosity of the cytoplasm is ~ 1000 times that of water (for large objects like organelles), how fast could a single kinesin molecule move a bacterium (radius ~ 1 μm) though a cell?

Problem 6

The density of molecules in a gravitational field falls exponentially with height.

(a) At what height above sea level is the oxygen concentration 37% (or 1/e) of the concentration at sea level?

(b) At what height is the concentration reduced to 10%

Problem 7

A solution of gold spheres, at equilibrium, has a concentration gradient such that the density of spheres decreases e-fold every 20 mm from bottom to top. Given that the density of gold is 19.3 times that of water, what is the diameter of the particles?

Problem 8

Consider a bacterium (radius ~1 μm) swimming through water at a speed of 25 μm/s.

(a) How much force do the flagella of the bacteria have to generate to maintain that speed?

(b) The bacterium stops swimming and slows down due to the drag force. How much time before the bacterium comes to a stop?

(c) How much distance does the bacterium cover in that time?

This is also known as the correlation time. Note that the bacterium is still subject to Brownian motion, so it does not actually stop, but undergoes a random walk instead.