

# Estimating Data Envelopment Analysis Frontiers for Nonsubstitutable Inputs and Outputs: The Case of Urban Mass Transit

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## Abstract

Conventional data envelopment analysis (DEA) models assume that inputs are substitutable for each other, and that outputs are substitutable for each other. However, recent DEA articles frequently include outputs that cannot be substituted for each other and inputs that cannot be substituted for each other. In this paper, we demonstrate that conventional DEA models report invalid efficiency scores when outputs and/or inputs are nonsubstitutable. We use artificial data to illustrate the differences between the efficient frontiers of substitutable and nonsubstitutable variables. Assuming that the inputs and outputs are nonsubstitutable, we compare the DEA scores from a conventional DEA model with those from a new model, the Fixed Proportion Additive (FPA) model, which we developed to deal with nonsubstitutable variables. Then, we apply the conventional and FPA models to real-world data involving urban mass transit systems, where the outputs are nonsubstitutable, and where the inputs are nonsubstitutable. Finally, we make recommendations for model use when inputs or outputs are nonsubstitutable, one involving the development of new models and the others involving adaptations that can be made if one wishes to use conventional models.













































