9 Basics of options, including trading strategies

- Option: The option of buying (call) or selling (put) an asset.
- European option: Can only be exercised at expiration.
- American option: Can be exercised anytime until the expiration.
- In-the-money: Either (a) a call option where the asset price is greater than the strike price or (b) a put option where the asset price is less than the strike price.
- Out-of-the-money: Opposite of in-the-money.

9.1 Factors Affecting Options Prices

- The list of factors:
  1. Current stock price S_0
  2. Strike price: K
  3. Time to expiration: T
  4. Volatility of stock price: σ
  5. Risk-free interest rate: r
  6. Dividends expected during life of option: D

- Signs of partial derivatives

	S_0	K	T	σ	r	D
European call (c)	+	-	?	+	+	-
European put (p)	-	+	?	+	-	+
American call (C)	+	-	+	+	+	-
American put (P)	-	+	+	+	-	+

- S_0: the higher the spot price today, the higher the spot price at the expiration (or before). Payoff on the call would be higher, payoff on the put would be lower.
- K: a higher strike price lowers the payoff for the call and increases the payoff for the put.
- T: harder to predict beforehand for European options. For American options, the longer the time to expiration, the longer the time you have for the asset price to move until you're in the money.
- σ: the higher the volatility of the asset price, the higher the probability of having a big increase or decrease of the asset price of the life of the option.

- r: the interest rate is a gain for the holder of calls options (earn interest on their cash while the wait to exercise the option) and a loss for the holders of put options.
- D: The holders of put options are receiving dividends as they wait to exercise the option and the holders of call options are foregoing the dividends.

9.2 Upper and Lower Bound

1. Upper bound – call options

   - For both American and European options, the option can never be worth more than the stock. Therefore, in particular

$$\begin{aligned} C &\leq S_0 \\ c &\leq S_0 \end{aligned}$$

   - Otherwise, arbitrageurs can make a riskless profit by selling the call and using the proceeds to buy the stock.

2. Upper bound – put options

   (a) For both American and European put options, the option can never be worth more than $K - S_T \leq K$ (since S_T is unobservable, we take K as the upper bound):

$$\begin{aligned} P &\leq K \\ p &\leq K \end{aligned}$$

   (b) We can say more for European options because we know the exercise date. At that date, the option cannot be worth more than K, so therefore the value today satisfies

$$p \leq Ke^{-rT}$$

   (c) Otherwise, arbitrageurs can make a riskless profit by selling the option and investing the proceeds at the riskless rate.

3. Lower bound – call on non-dividend paying stock

   (a) European call

   - It must satisfy

$$c \geq S_0 - Ke^{-rT}$$

   - To see this, consider two portfolios:
     - **A**: One European call plus cash ($= Ke^{-rT}$).
     - **B**: One share of stock.
   - For portfolio **A**, the cash grows to K by time T.
     - If $S_T > K$, the call is exercised and portfolio is worth S_T.
     - If $S_T < K$, the call is worthless at time T and the portfolio is worth K.
     - Conclusion: At time T, portfolio **A** is worth $\max(S_T, K)$.
   - Portfolio **B** is worth S_T at time T.

- **A** is always worth at least as much as **B** and can be worth more at time T.
- For there to be no arbitrage, we must have the same relation today:

$$c + Ke^{-rT} \geq S_0$$

otherwise we could make a profit by selling the stock now and buying the call. The equation above can be written

$$c \geq S_0 - Ke^{-rT}$$

- In fact we can say more. The worst that can happen to a call is that it's worthless at expiration.
  - c cannot be negative
  - $c \geq \max(S_0 - Ke^{-rT}, 0)$

(b) These lower bounds also apply to American calls because American calls have more flexibility than European calls and so must be worth more for given values of S_0, K, r, and T.

4. Lower bound – put on non-dividend paying stock

   (a) European put

   - Has a lower bound of $Ke^{-rT} - S_0$
   - Consider two portfolios:
     - **C**: One European put plus one share
     - **D**: Cash $(= Ke^{-rT})$
   - If $S_T < K$, the option is exercised and portfolio **C** is worth K.
   - If $S_T > K$, the option is worthless, so the portfolio is worth S_T.
   - Conclusion: portfolio **C** is worth $\max(S_T, K)$ at time T.
   - The cash grows to K at time T.

- Conclusion: portfolio **C** is always worth as much as **D** and possibly more at time T.
- Conclusion: For absence of arbitrage, we must have the same relation today:

$$p + S_0 \geq Ke^{-rT}$$

otherwise we could make a profit by borrowing money and buying the put and the asset. The equation above can be written

$$p \geq Ke^{-rT} - S_0$$

- As with a call, the worst that can happen to the put is that it is worthless at time T, so that

$$p \geq \max(Ke^{-rT} - S_0, 0)$$

(b) American put – also satisfies these bounds because of its extra flexibility.

9.3 Put-Call Parity

1. European option
   - Consider the following two portfolios
     - **A**: one European call with strike price K and exercise date T + cash ($= Ke^{-rT}$).
     - **B**: one European put with the **same** strike price K and exercise date T + one share of stock.
   - At time T, both portfolios are worth $\max(S_T, K)$.
   - Because the options are European, they cannot be exercised before T.
     - Conclusion: They must also have the same relative value today.
     - Conclusion: $c + Ke^{-rT} = p + S_0$
   - This equality is the **put-call parity**.

- If put-call parity does not hold, arbitrageurs can make a sure profit by shorting the securities in the expensive portfolio and buying the securities in the cheaper one.
  - Example 1: If $c + K^{-rT} < p + S_0$, then short both the put and the stock and use the proceeds ($= p + S_0$) to buy the call and invest the difference ($= p + S_0 - c$) at the risk free rate.

    At expiration, one of the two options is in the money and one is out. In either case, the arbitrageur ends up buying one share at a price of K, leaving him with a profit of $(p + S_0 - c)e^{rT} - K$.
  - Example 2: If $c + K^{-rT} > p + S_0$, then arbitrage by shorting the call, borrowing $p + S_0 - c$ (which is less than Ke^{-rT}), and buying the put and the stock. The investor will end up selling one share for K, earning a profit of $K - (p + S_0 - c)e^{rT}$.

2. American options

   - American puts and calls do not satisfy put-call parity but do satisfy the following weaker relation

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$$

   - Proof: Question 9.18 in Hull.

9.4 Early Exercise of American Options

1. **Calls on a non-dividend stock**

   (a) Early exercise is never optimal if the investor plan to hold the stock past the expiration of the option.

   - Consider an option (deep) in the money ($S_t > K$)
   - If you wait until the expiration of the option to exercise it, you can earn interest on your cash.
   - If the price continue to rise, you can always exercise later at no loss.
   - If the price falls (below the strike price), you may have preferred never to exercise.

(b) What if you don't plan to hold the stock past the expiration and you think the stock is overpriced?

- You're better off selling the option.
- The option will be bought by another investor who does want to hold the stock.
- Such investor must exist otherwise the current stock price would not be as high.

(c) Call price vs. intrinsic value

- The intrinsic value (value of option if exercised immediately) is $\max(S_0 - K, 0)$.
- The above argument show that the call is always worth more than its intrinsic value (never optimal to exercise early): $C > S_0 - K$

(d) More formally:

- Since the American call is worth at least as much as the European call because of the extra flexibility:

$$C \geq c$$

- From the lower bound on European call:

$$c \geq S_0 - Ke^{-rT}$$

- Combining the two:

$$C \geq S_0 - Ke^{-rT}$$

- Assuming that the interest rate is positive ($r > 0$), we have $e^{-rT} < 1$, which imply that

$$C > S_0 - K$$

- This therefore shows that there is a positive value associated with the option to wait.

(e) Increasing r, T, or σ raises C for given S_0.

- Higher r: you earn more interest on your cash as you wait to exercise the option.
- Longer T: you earn interest for longer on your cash as you wait to exercise the option.
- Higher σ: increases the probability that the stock will do very well (high price). It also increases the probability that the stock will do very bad but we don't care since we have an option and not the stock.

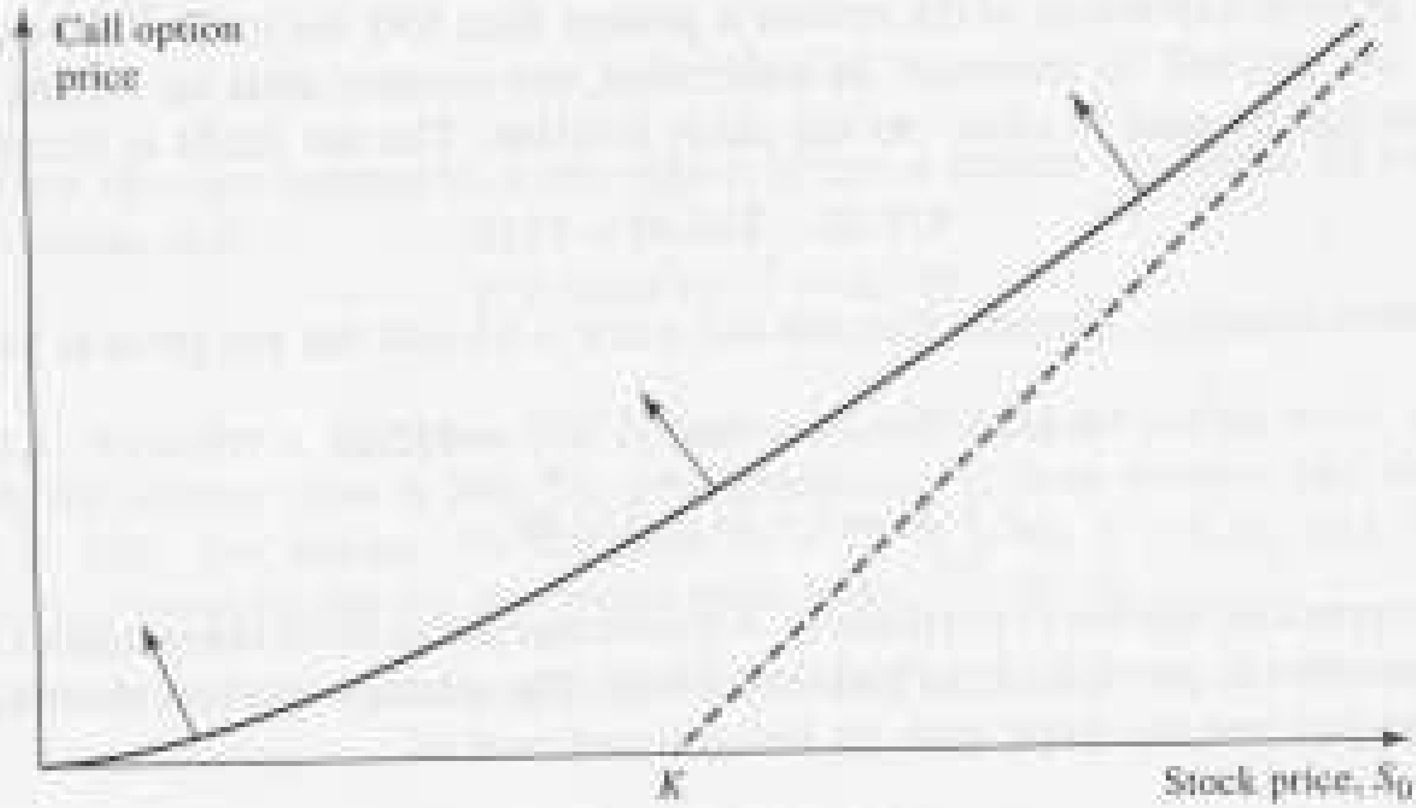


Figure 8.3 Variation of price of an American or European call option on a non-dividend-paying stock with the stock price, S_0

2. **Puts on a non-dividend stock**

   (a) Early exercise is optimal if option is sufficiently deep in the money.

      - There is a lower bound of zero on the price of the stock but no upper bound.
      - A price near its lower bound can fall only a little but can rise arbitrarily high.
      - Waiting to exercise a put deep in the money can bring only small gains but large losses.
      - Exercise early when stock price hits a trigger price.
      - Also, we can earn interest on the profit from early exercise.

(b) Put price vs. intrinsic value

- An American put always can be exercised immediately

$$P \geq K - S_0$$

- For sufficiently low S_0, exercise is optimal: $P = K - S_0$ for $S_0 < A$.

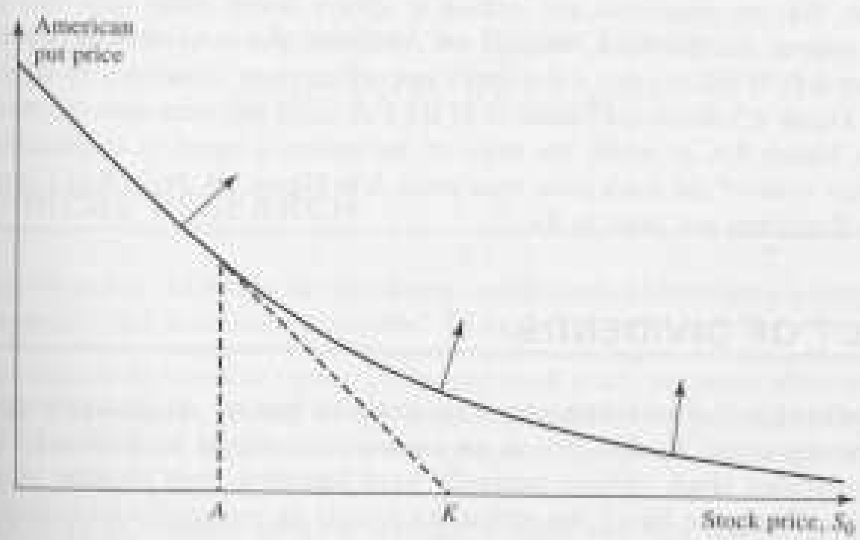


Figure 8.4 Variation of price of an American put option with stock price, S_0

(c) Contrast with European put

- For a European put: $p \geq Ke^{-rT} - S_0$
- And note that $Ke^{-rT} - S_0 < K - S_0$ because $r > 0$.
- Also, $P \geq p$ because there are values of S_0 for which exercise is optimal.
- Conclusion: there exist S_0 for which $p < K - S_0$. because
  i. $P \geq p$
  ii. Exist S_0 for which $P = K - S_0$.
- Conclusion: An European put can be worth less than its intrinsic value, whereas an American put cannot.

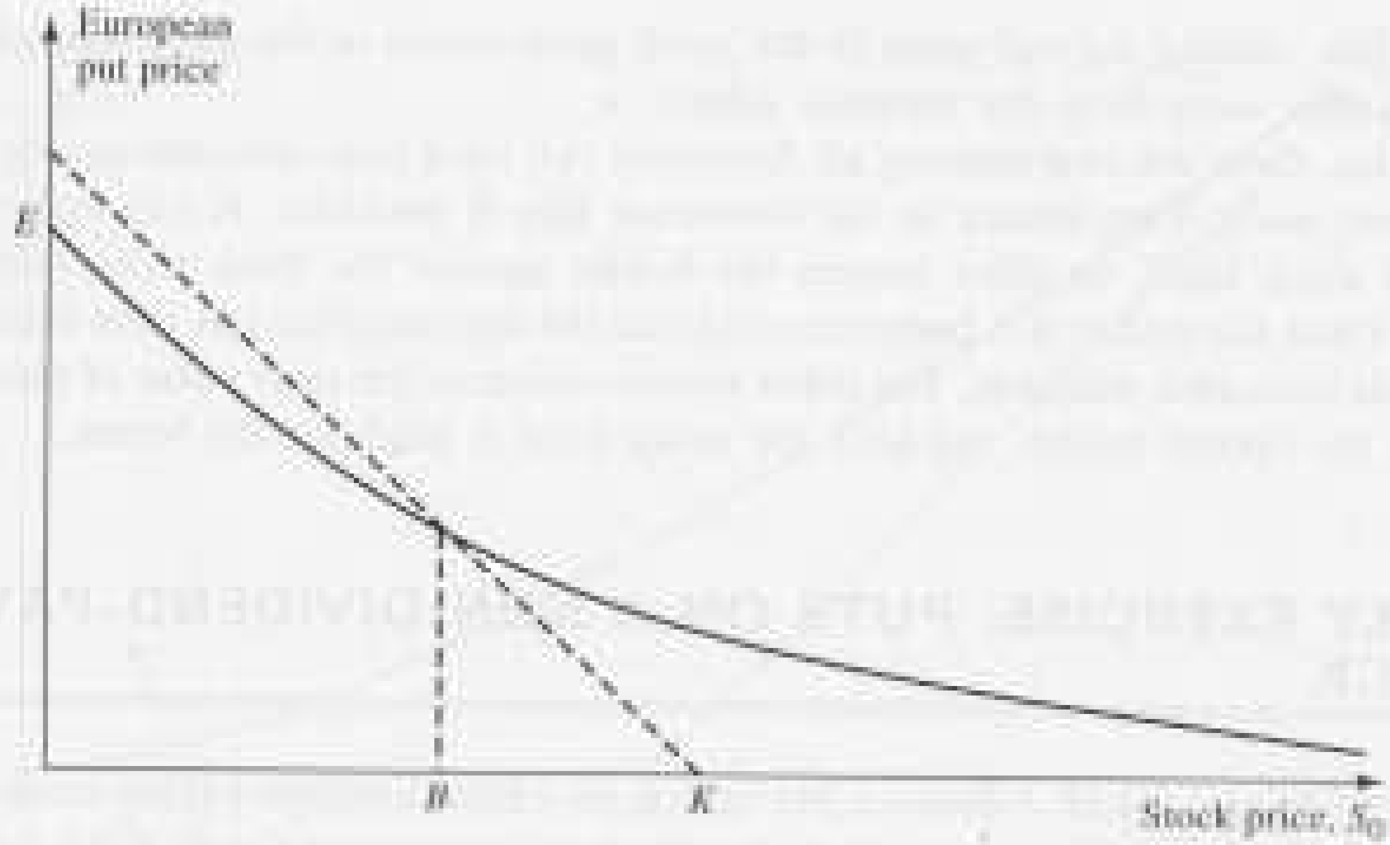


Figure 8.5 Variation of price of a European put option with the stock price, S_0

- Note that $B \geq A$ because at A we have $P = K - S_0$, at B we have $p = K - S_0$, and we always have $P \geq p$. So at B we must have $P \geq p = K - S_0$.

9.5 Dividends

1. Lower bound for calls

   - Define the portfolios
     - **E**: one European call plus cash ($= D + Ke^{-rT}$) where D is the present value of dividends during the life of the option.
     - **F**: one share
   - By the same arguments as before: $c \geq S_0 - D - Ke^{-rT}$
   - D is the present value of the dividends foregone by not having the stock. So they are an asset given up by holding the option.

2. Lower bound for puts

   - Define the portfolios

     – **G**: one European put plus one share

     – **H**: cash ($= D + Ke^{-rT}$)

   - The same argument as before gives

$$p \geq D + Ke^{-rT} - S_0$$

3. Upper bounds: same as before

4. Early exercise

   American calls may be exercised early when there are dividends because not exercising forgoes the dividend. If early exercise occurs, it will be immediately before a dividend date.

5. Put-call parity

   - The relation for European options is

$$c + D + Ke^{-rT} = p + S_0$$

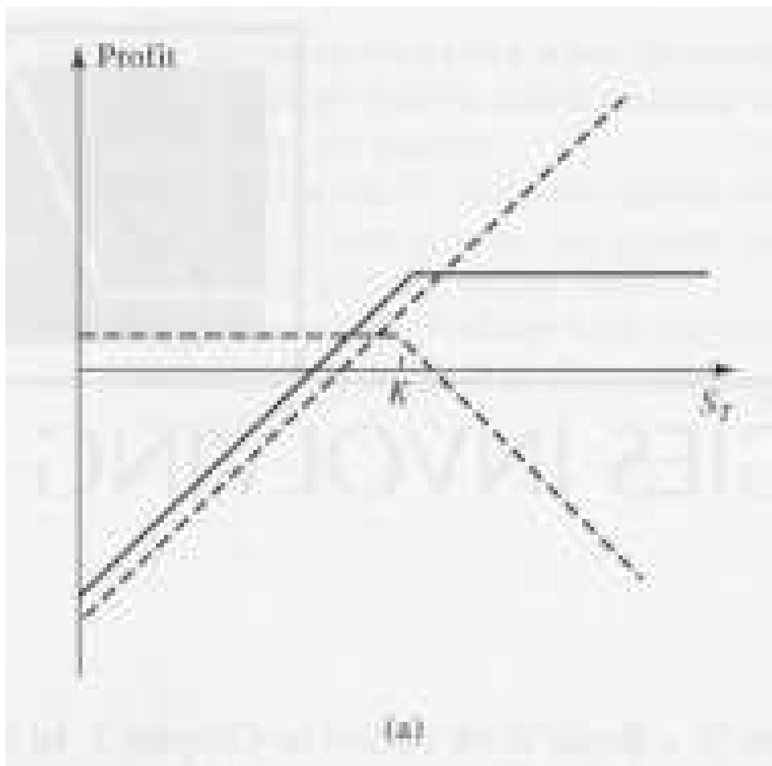
   - For American options:

$$S_0 - D - K \leq C - P \leq S_0 - Ke^{-rT}$$

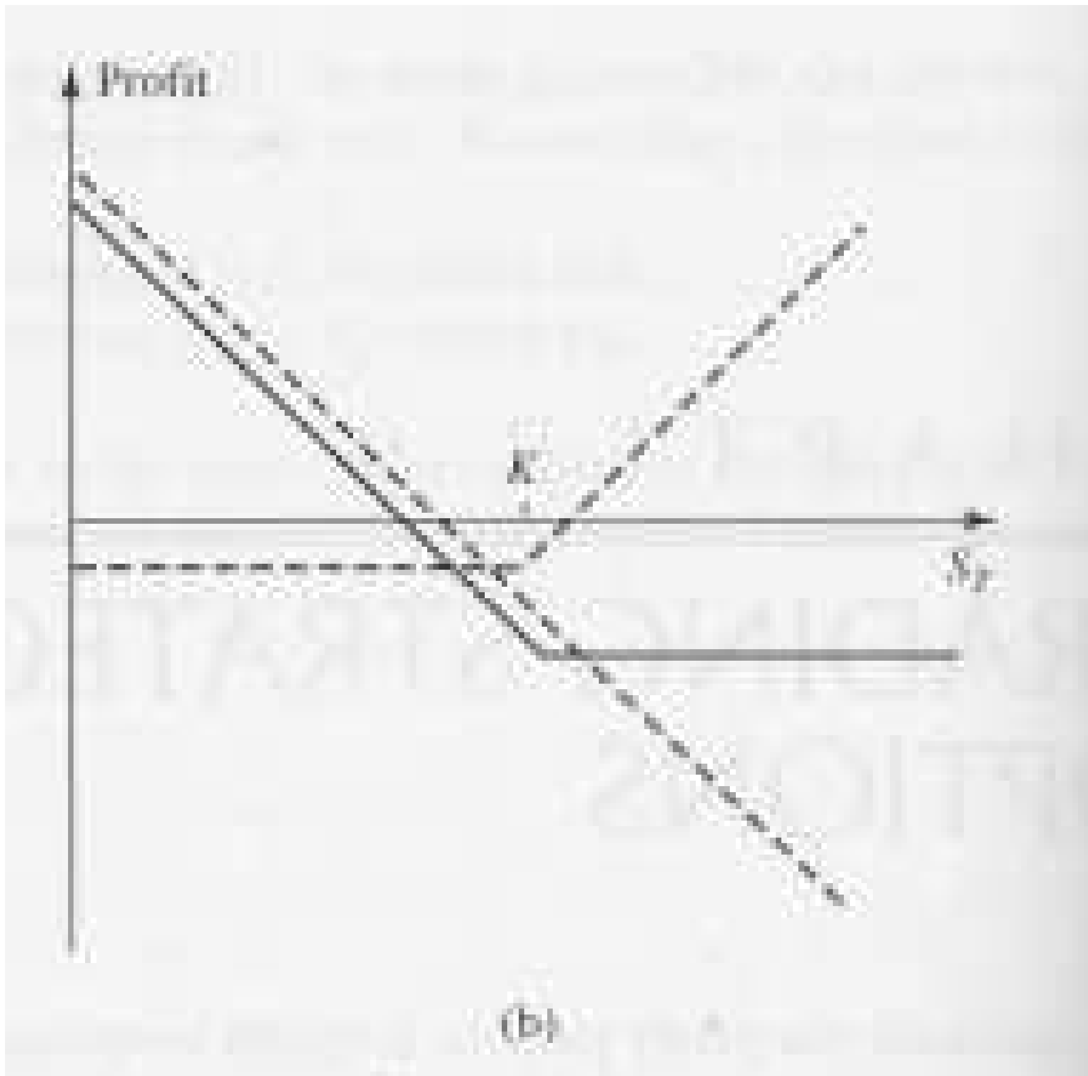
9.6 Trading Strategies Involving Options

9.6.1 A single option and a stock

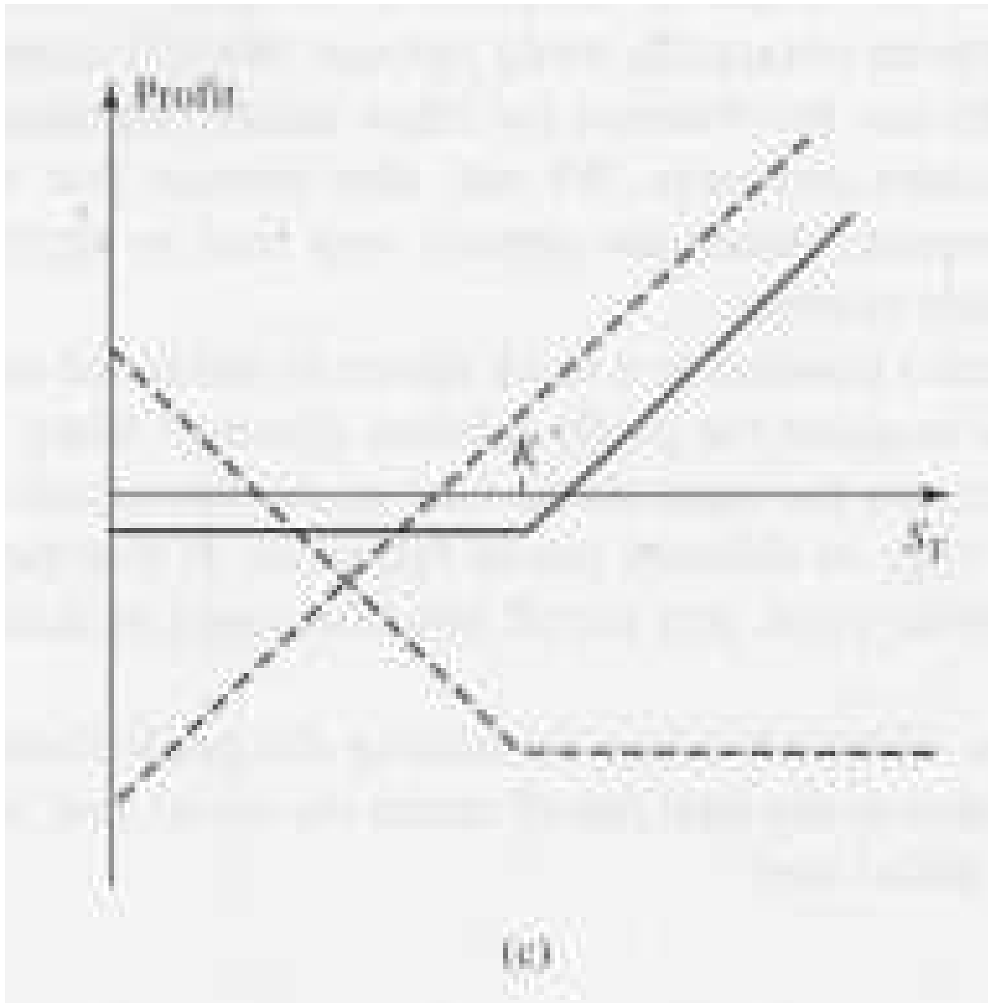
1. **Covered call:** long stock plus short call (the stock covers the risk from the call is S_t rises)



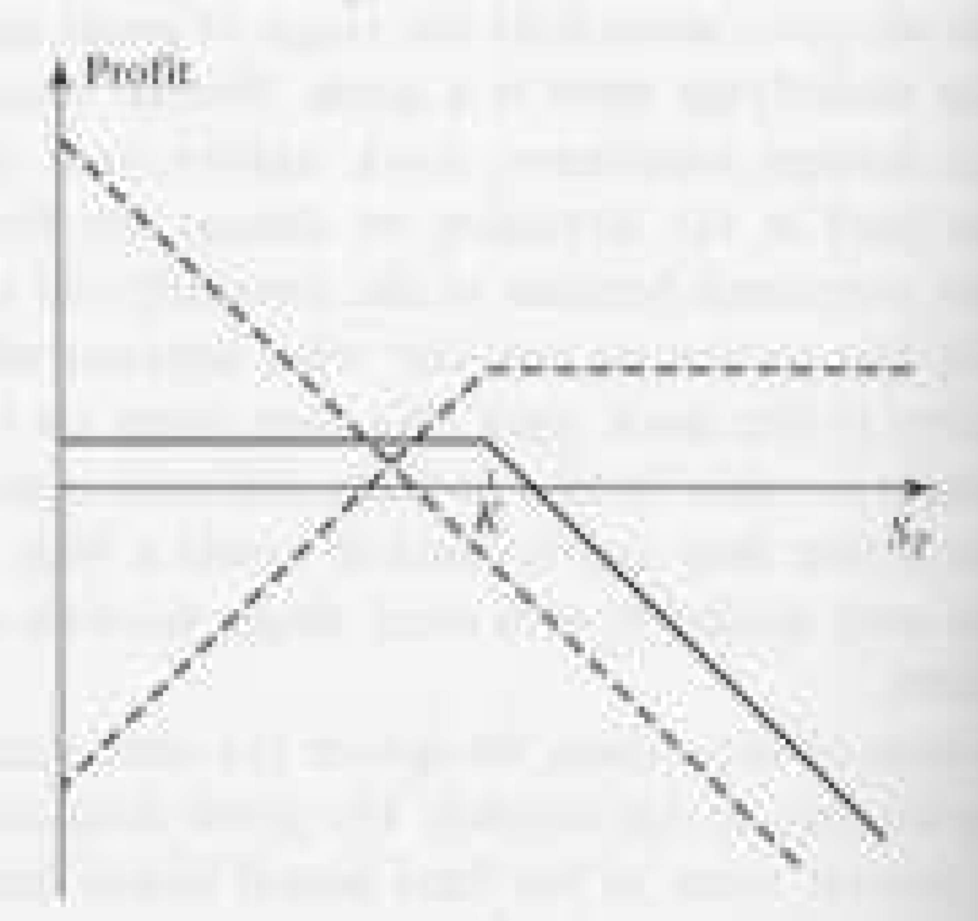
2. **Inverse covered call:** short stock plus long call (the call covers the risk from the stock is S_t rises)



3. **Protective put:** long stock plus long put (the put covers the loss from the stock is S_t falls)



4. **Inverse protective put:** short stock plus short put (the stock covers the loss from the put is S_t falls)



(d)

5. **Relations.** The result of each of the foregoing looks like a put or call. We can see this formally from the put-call parity

$$p + S_0 = c + Ke^{-rT} + D$$

- The LHS is a long put and long stock (like a protective put). If we rearrange the expression as

$$p + S_0 - Ke^{-rT} - D = c$$

  we get that the call equals the put plus the stock less cash equal to $Ke^{-rT} + D$ (which is what you would have to pay for the call).

- Similarly,

$$\begin{aligned} & -p - S_0 = -c - Ke^{-rT} - D \\ \Leftrightarrow \quad & -p - S_0 + Ke^{-rT} + D = -c \end{aligned}$$

  which corresponds to the inverse protective put.

- Rearrange the put-call parity equation as

$$S_0 - c = Ke^{-rT} + D - p$$

or a long stock and short call equals cash plus a short put, which is the covered call.

- Finally

$$-S_0 + c = -Ke^{-rT} - D + p$$

which is the inverse covered call.

9.6.2 Spreads

A spread is a trading strategy involving two or more options of the same type (i.e., two or more calls, two or more puts).

1. **Bull spreads.** Used by people who expect (or hope) S_t will rise but who do not want to pay the amount required for a long call.
   - Buy a call with one strike price (K_1) and sell a call with a higher strike price ($K_2 > K_1$).

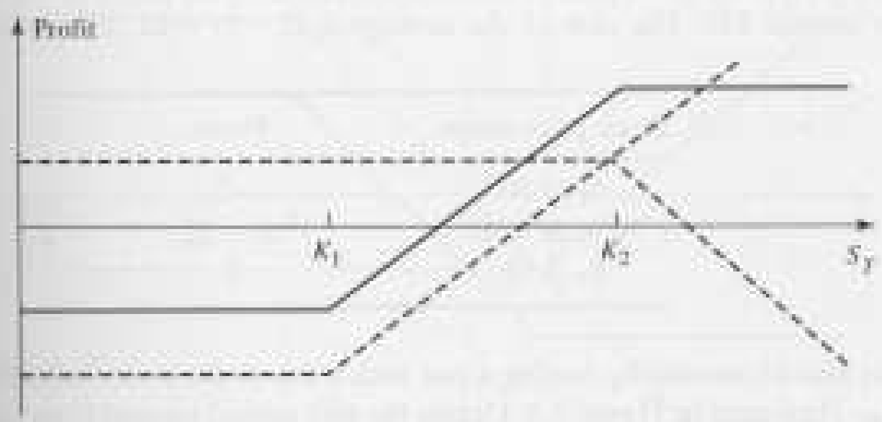


Figure 9.2 Bull spread created using call options

- Bull spreads created with call options require an initial investment to pay the difference between the short and long calls. The payoff is non-negative (the profit can be negative).
- A bull spread can also be created with puts. You buy a put with a low strike price and sell a put with a higher strike price.

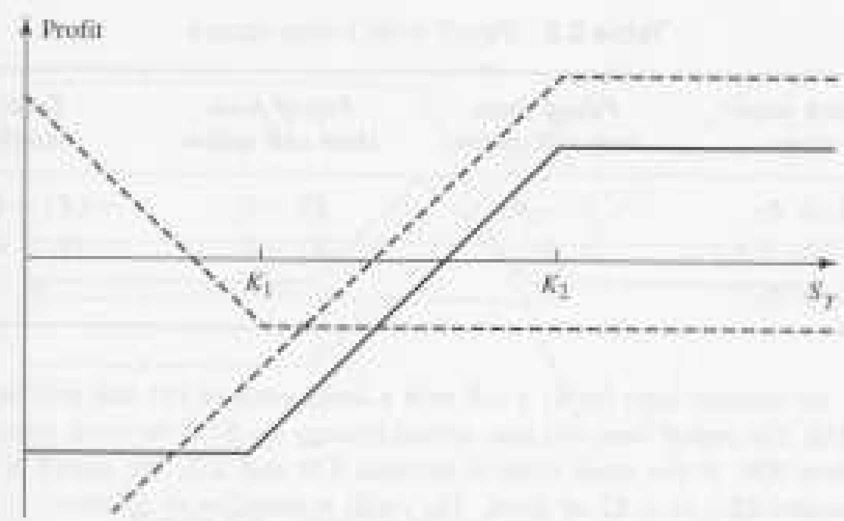


Figure 9.3 Bull spread created using put options

- Bull spreads created with puts yield an initial cash inflow and have payoffs that are non-positive. They also put a floor on the possible loss.
- Both kinds of bull spreads are profitable when the stock price rises.

2. **Bear spreads.** These are the mirror image of bull spreads. They are profitable when stock prices fall.

   - Buy a call with a **high** strike price and sell a call with a **low** strike price.

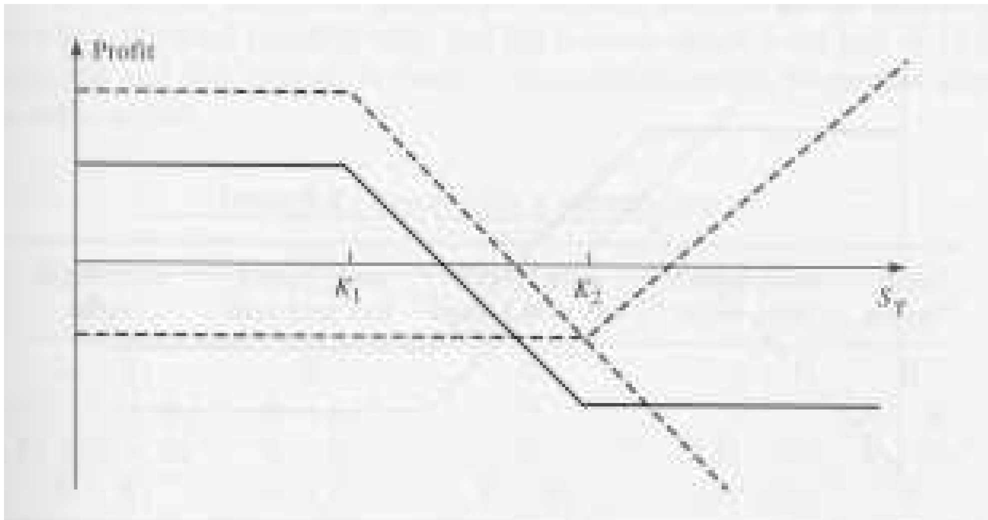


Figure 9.4 Bear spread created using call options

- Buy a put with a high strike price and sell a put with a low strike price.

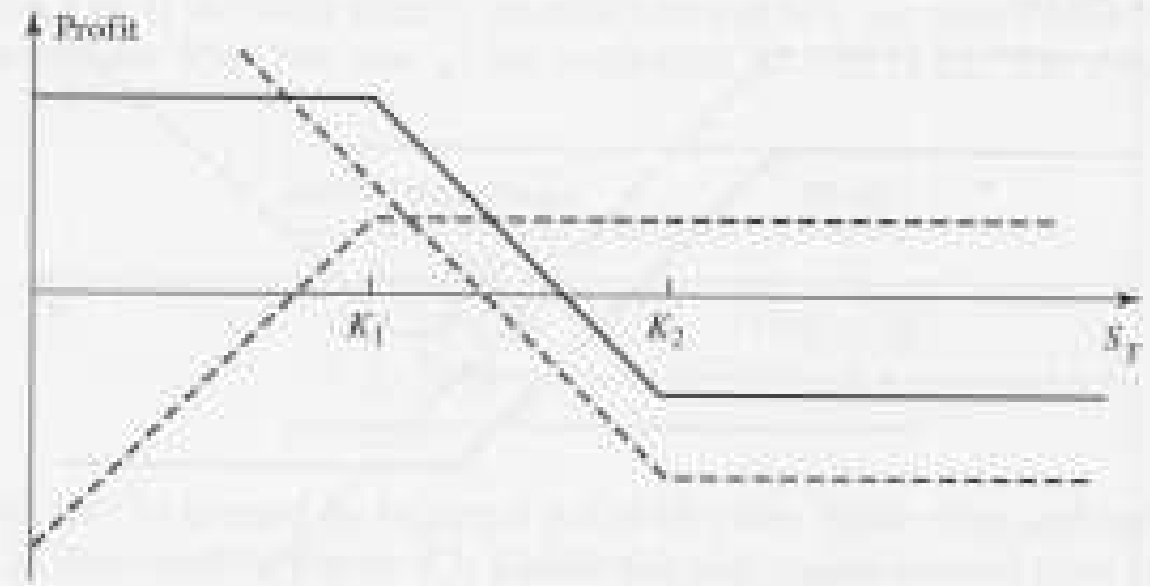


Figure 9.5 Bear spread created using put options

9.6.3 Butterfly spread

- Involves three options of the same type with different strike prices
- For example
  1. Buy a call with a low strike price K_1
  2. Buy a call with a high strike price K_3
  3. Sell two calls with a strike price K_2 halfway between K_1 and K_3
- The butterfly spread is profitable if S_T is near K_2. It's a good investment if you think large movements in S_t are unlikely. It's also an insurance against big movements in either directions.

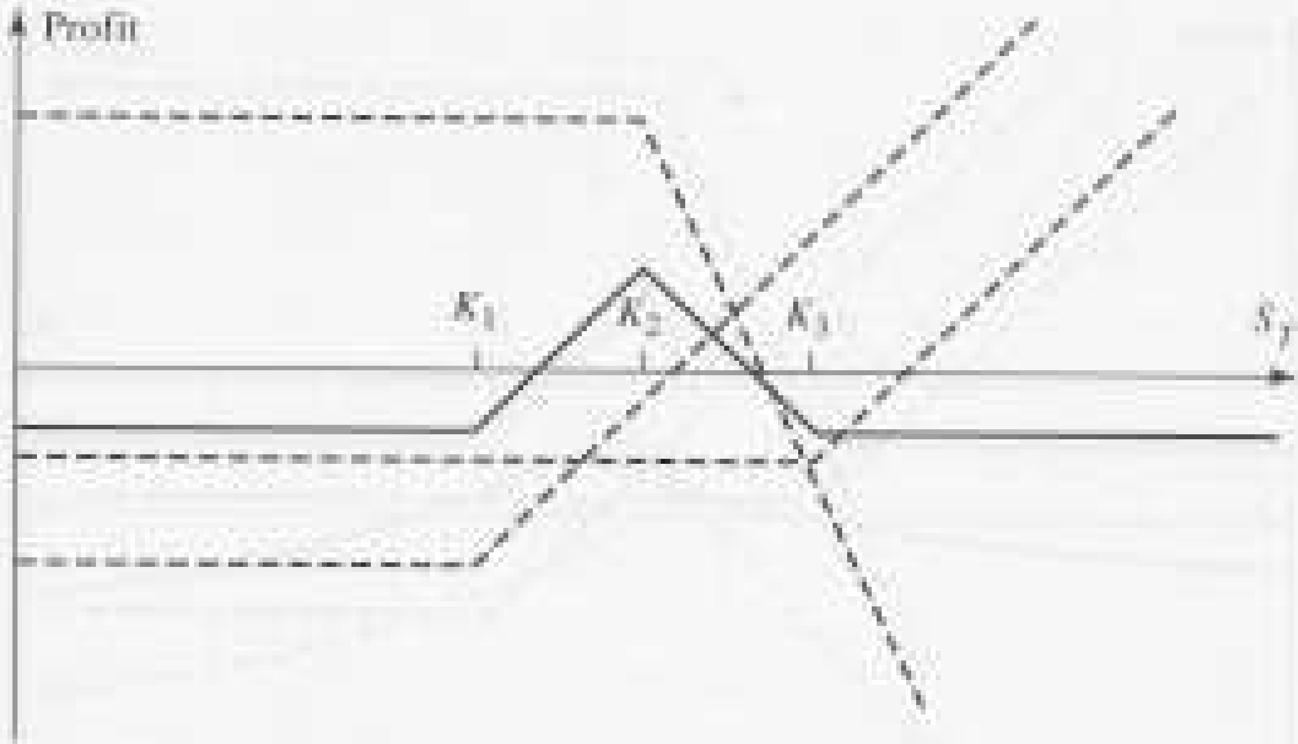


Figure 9.6 Butterfly spread using call options

9.6.4 Others

1. **Calendar spreads.** A combination of options with the same strike price but different expiration dates.
2. **Combinations.** A trading strategy that takes a position in both puts and calls on the same stock.

(a) **Straddle:** buy a put and a call with the same strike and expiration date

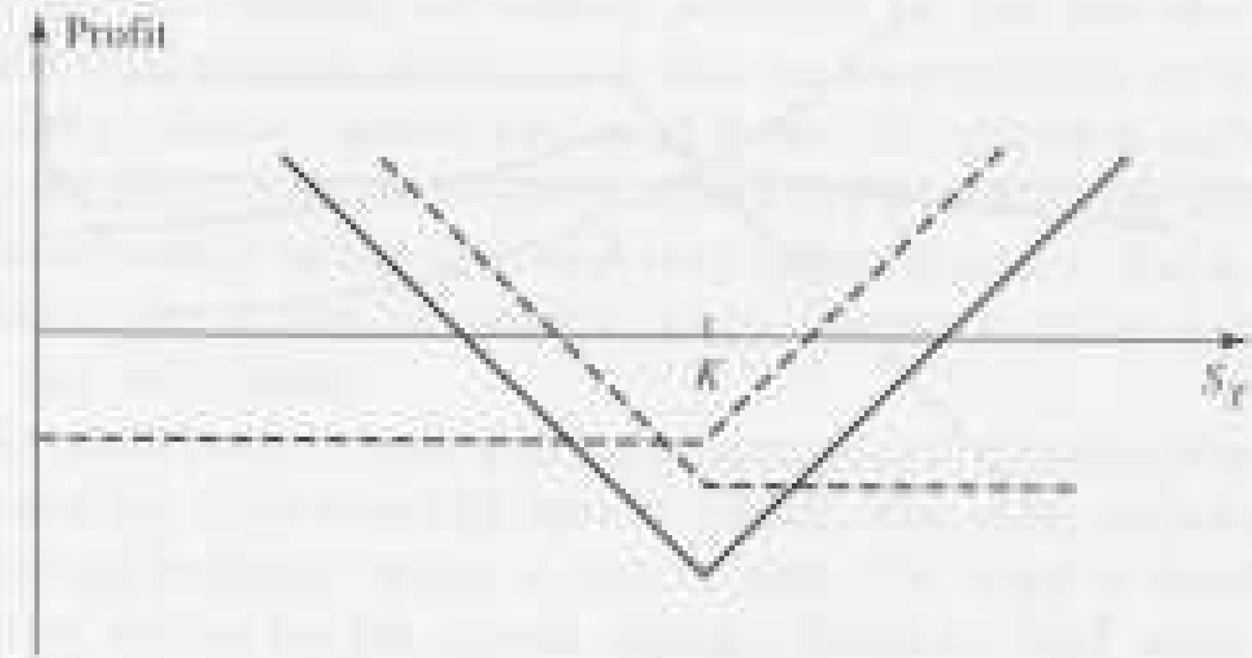
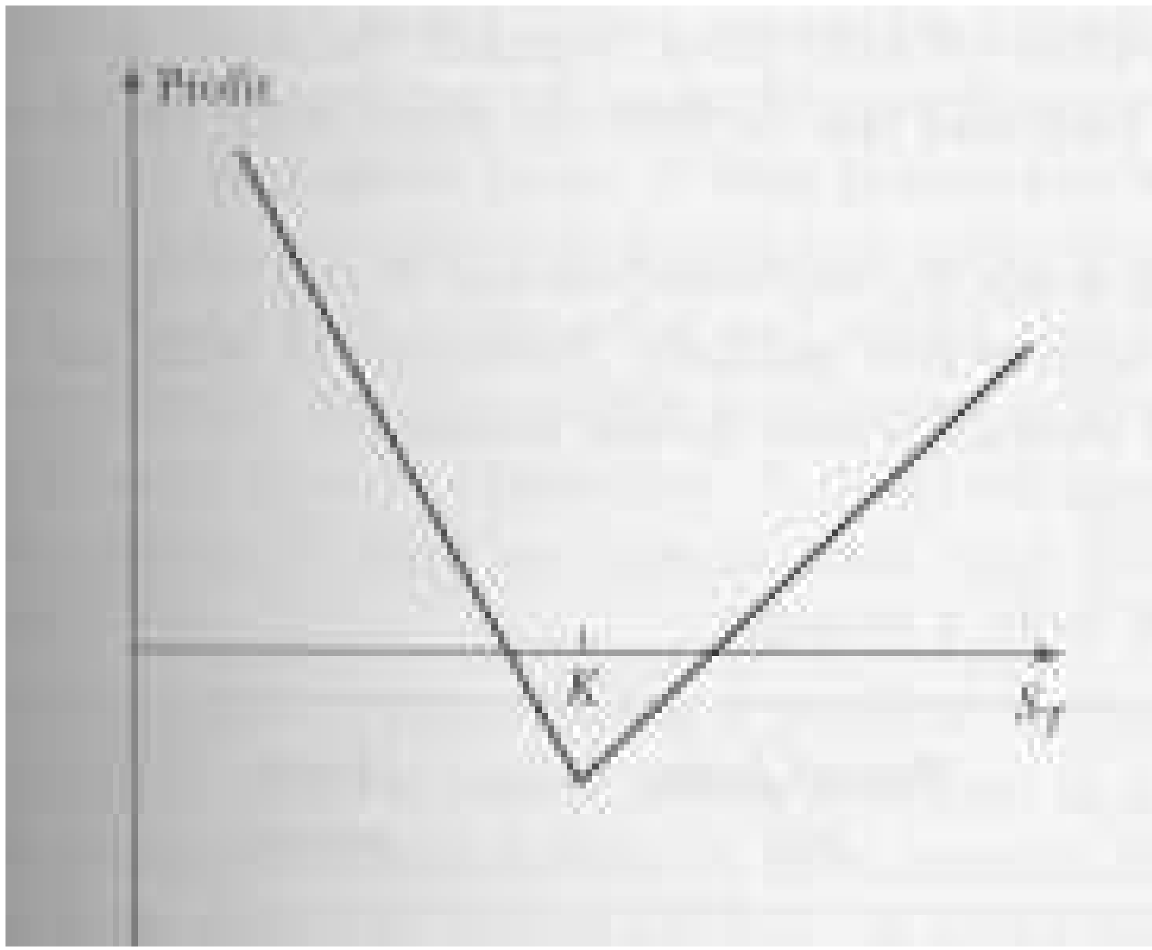


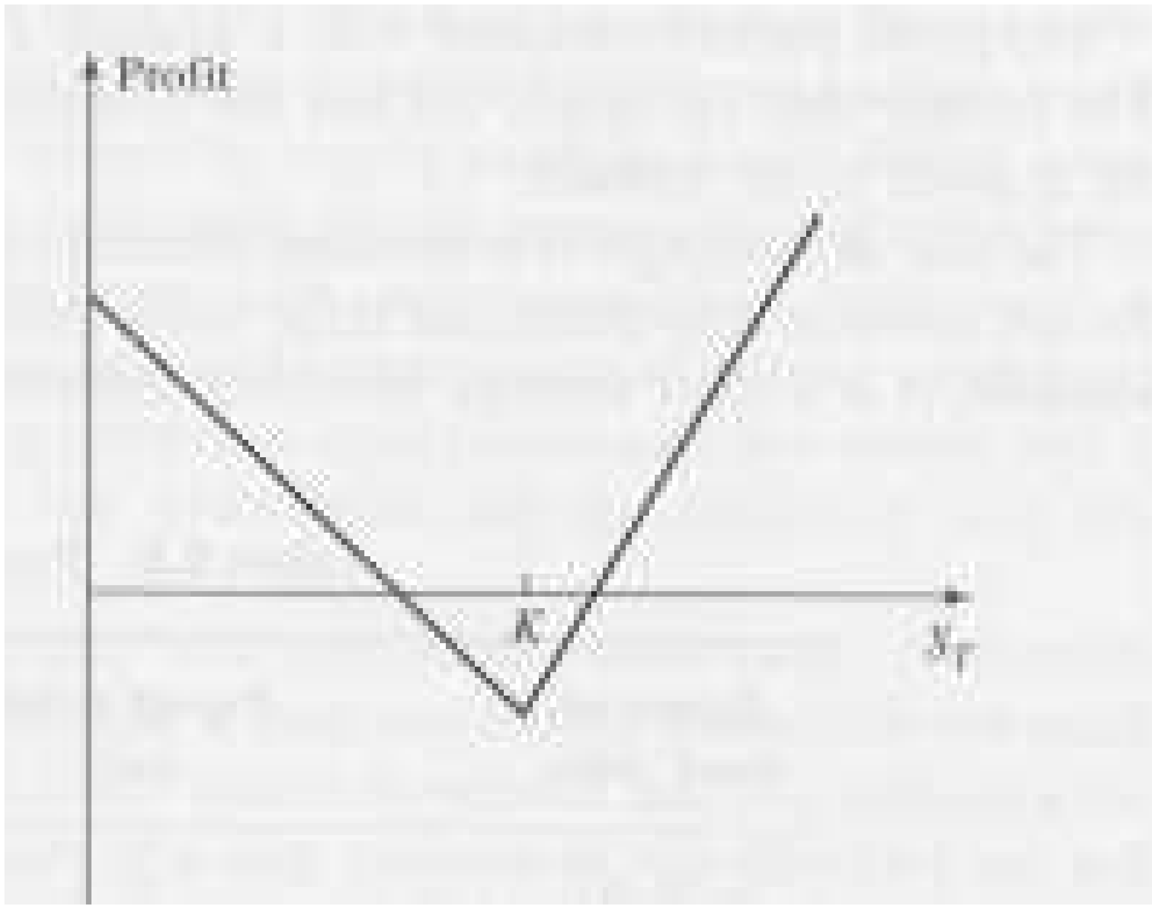
Figure 9.10 A straddle

(b) **Strips and straps**

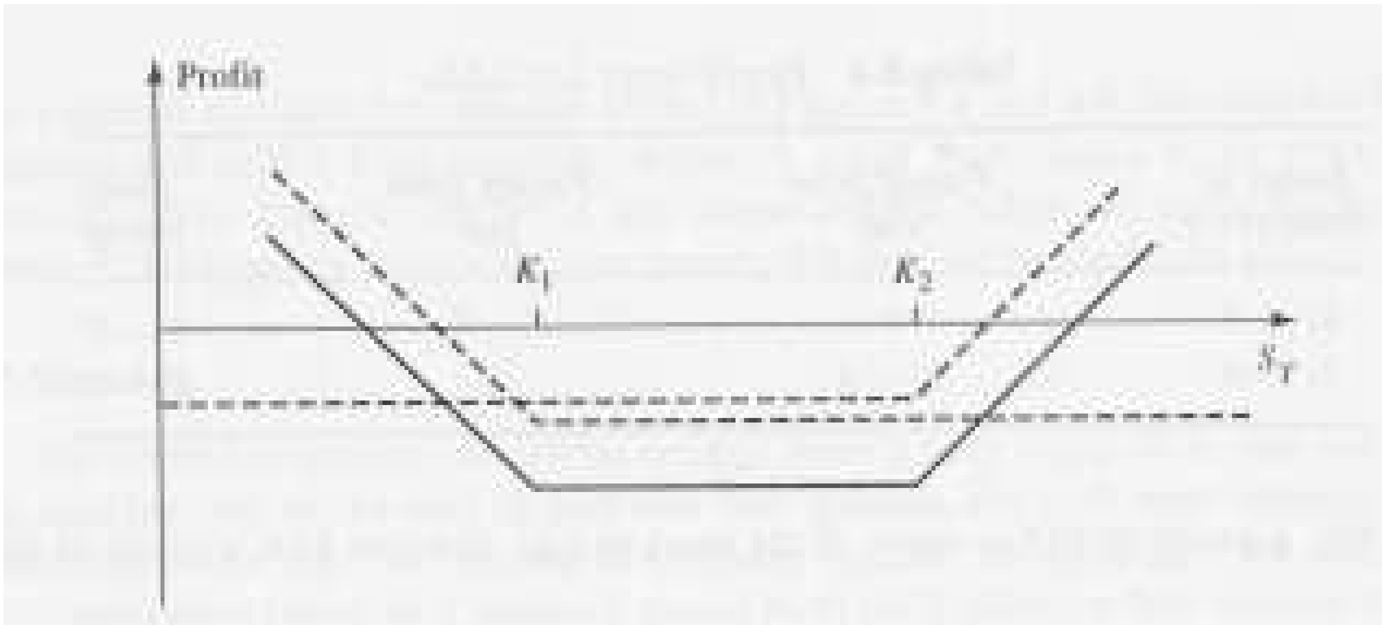
- Strip: one long call, two long puts, all with same K and T. Similar to straddle, but used when belief is that a rise in S_T is less likely than a fall.



- Strap: two long calls and one one long put, with the same K and T.



(c) **Strangle** (also called a bottom vertical combination). It's a long put and a long call with the same T but different values of K.



(d) Nearly infinite possible patterns of payoff can be obtained by suitable combinations.