

1 Introduction to various derivatives

What is a stock?

Ownership of share of the company.

Why is it worth something? You have a claim on:

- the assets
- current and future profits

What is a derivative?

A derivative is a financial instrument whose value depends on (or derives from) the values of other, more basic underlying variables.

1.1 Two types of markets

- Exchange traded markets
 - Individuals trade standardized contracts, defined by the exchange.
 - Chicago Board Of Trade (CBOT), Chicago Mercantile Exchange (CME), Chicago Board Options Exchange (CBOE).
- Over-the-counter markets (OTC)
 - Financial institutions, or a financial institution and a corporate client, trade non-standardized contracts.
 - Typical trade is much larger than in an exchange
 - Total volume is much larger than in exchanges.

1.2 Forward Contracts

- **Definition:** An agreement to buy or sell an asset at a specific future time at a specified price. Traded in the over-the-counter market.
- In contrast with a *spot contract*, which is an agreement to buy or sell an asset today.
- **Terminology:**
 - long position – agrees to **buy** the asset
 - short position – agrees to **sell** the asset
 - spot price – denoted S_t
 - delivery price – denoted K
 - forward price – denoted $S_{t,T}$

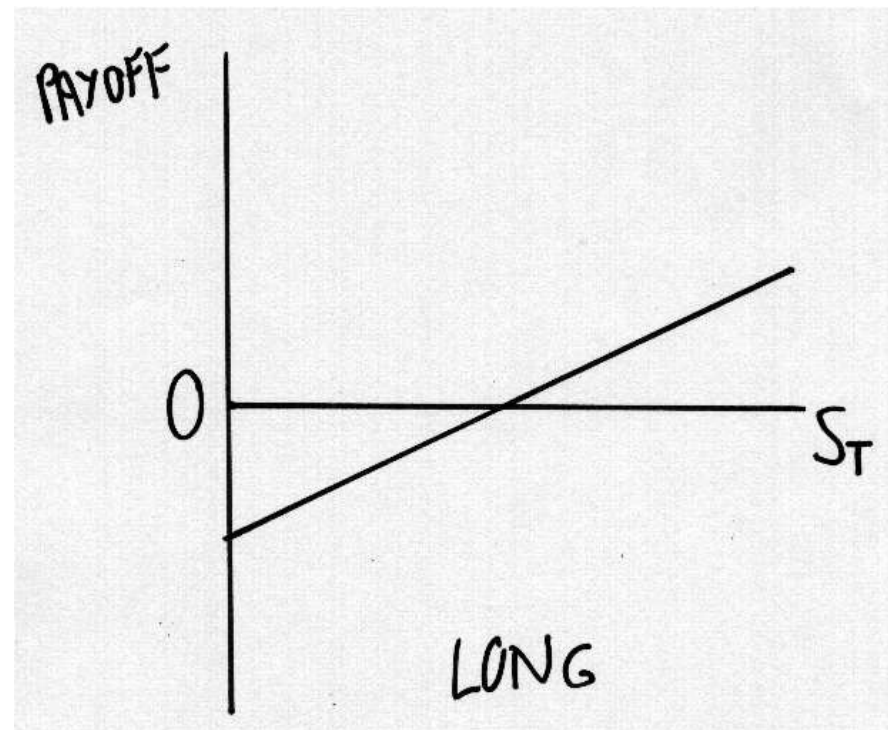
- **Payoffs – long position**

Payoff is equal to $S_T - K$ where:

T = delivery date

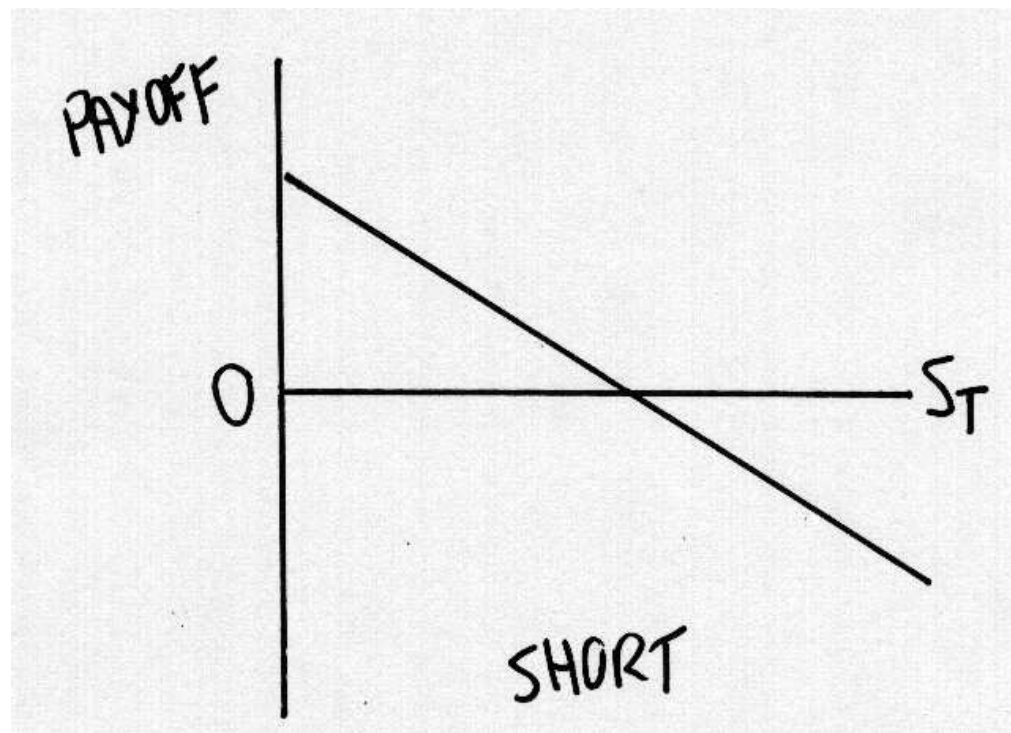
S_t = spot price at time t

K = delivery price (varies over time)



- **Payoffs – short position**

Payoff is equal to $K - S_T$



Notice that these are linear functions (and the slope is 1) of S_T .

- **Forward prices, spot prices and interest rates**

Suppose in the gold market

$$S_t = \$300 \text{ /oz}$$

$$K_{t+1} = \$340 \text{ /oz}$$

$$R = 0.05$$

Then a trader can do the following to earn a sure profit:

1. Borrow \$300 for 1 year at 5% interest
2. Buy one ounce of gold
3. Enter into a short forward contract to sell 1 oz. of gold in one year at \$340.
4. Repay the loan plus \$15 interest and earn \$25 profit.

Everybody would try to do this, which would raise demand for gold today and drive up today's price to the point where there was no profit from this scheme:

$$\begin{aligned}S_t \times (1.05) &= \$340 \\S_t &= \$340/1.05 \\ &= \$323.81\end{aligned}$$

In the converse case, suppose

$$S_t = \$300 / \text{oz}$$

$$K_{t+1} = \$300 / \text{oz}$$

$$R = 0.05$$

Then traders can make a profit by:

1. Selling gold now for \$300
2. Lend the proceeds at 5%
3. Enter into a long forward contract to buy 1 oz. of gold in one year at \$300.
4. Get repaid \$315 in one year
5. Use \$300 to buy back the gold leaving \$15 payoff.

1.3 Futures Contracts

Much like a forward contract but standardized and subject to more rules, such as:

- Traded on exchanges
- Fixed size of contracts
- Exact delivery date usually not specified (delivery month usually is fixed and the exchange specifies the period during the month when delivery must be made)
- Margin accounts with daily settlement required to mark to market.

1.4 Options

Options give the holder the right to buy or sell (depending on the type of option) at a specified price on (or perhaps before) a specified date.

- **Types**

1. Buy or sell

- Buy – call option
- Sell – put option

2. Timing

- European (have to wait until the expiration)
- American (don't have to wait until the expiration)

- **Terminology**

- expiration date or maturity
- strike price or exercise price

- **Call options**

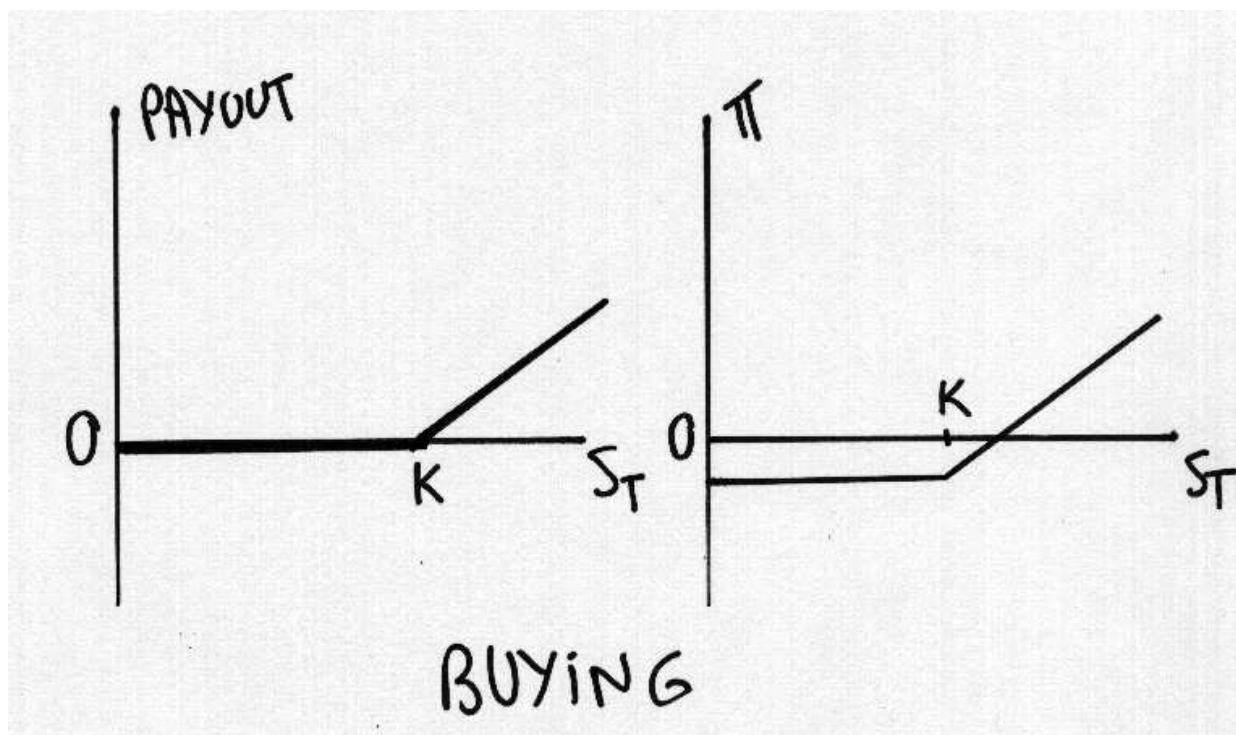
- Payout at exercise: $\max(S_T - K, 0)$

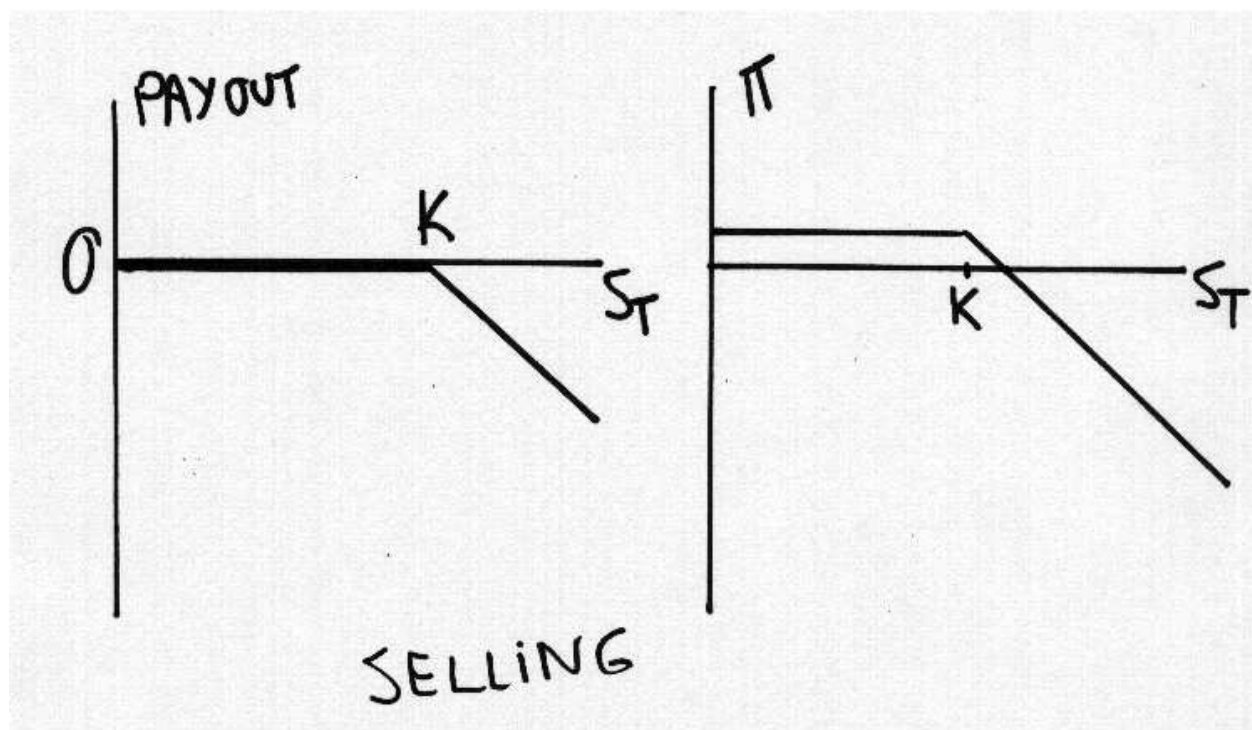
- Profit from buying a call (long position):

$$\begin{aligned}\Pi &= \text{payout} - (\text{option price} + \text{foregone interest}) \\ &= \max(S_T - K, 0) - P_t(1 + R)^{T-t}\end{aligned}$$

- Profit from selling/writing a call (short position)

$$\begin{aligned}\Pi &= P_t(1 + R)^{T-t} - \max(S_T - K, 0) \\ &= P_t(1 + R)^{T-t} + \min(K - S_T, 0)\end{aligned}$$





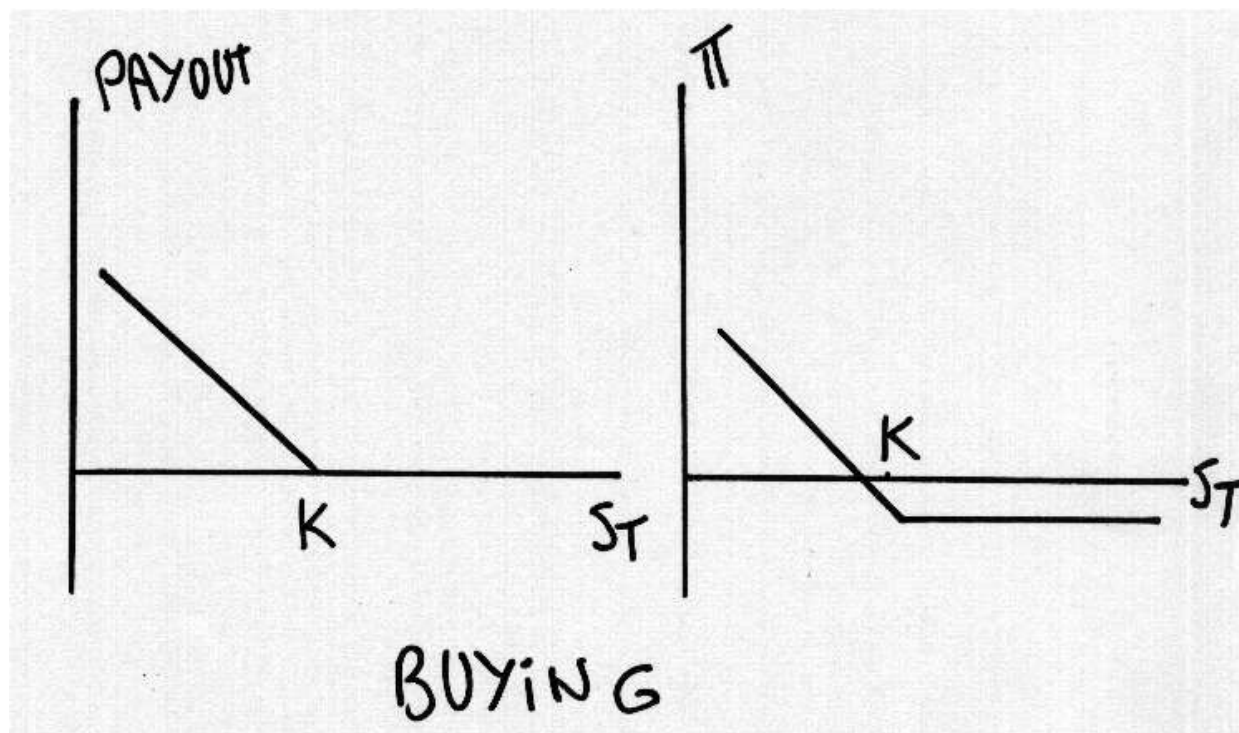
- **Put options**

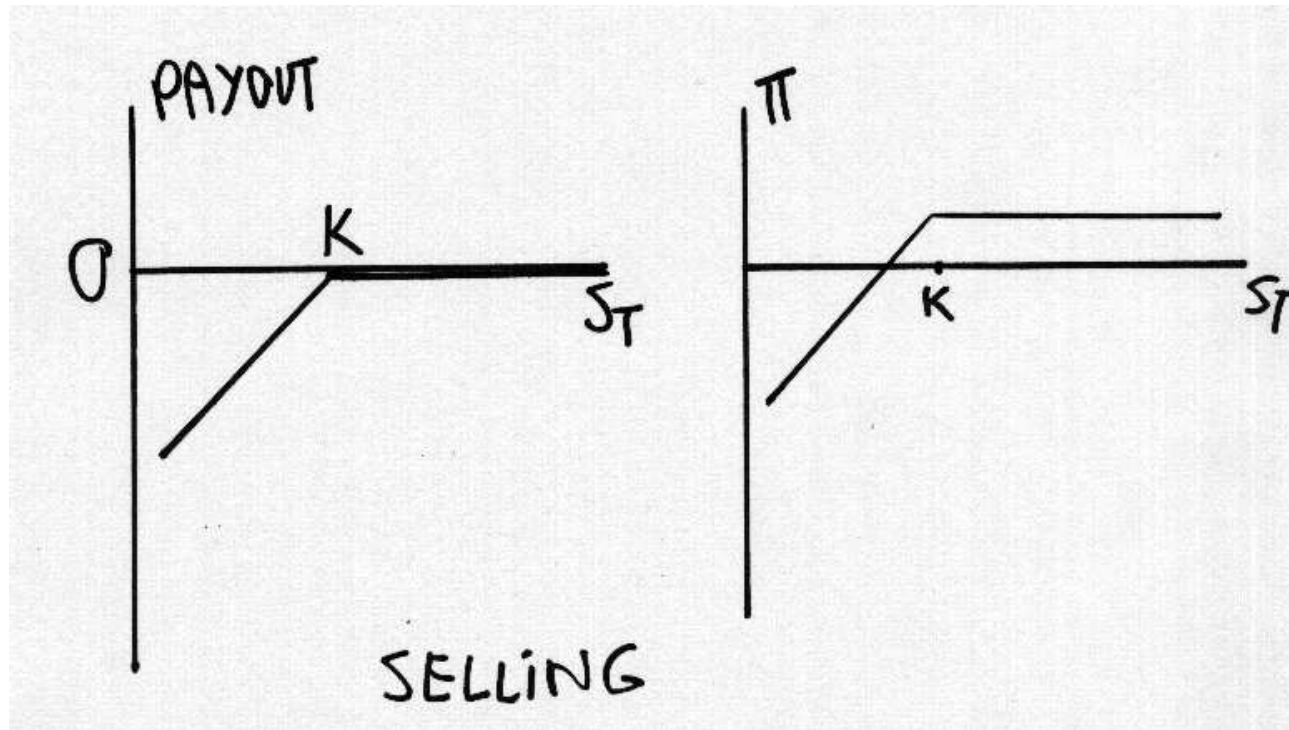
- Payout at exercise: $\max(K - S_T, 0)$
- Profit from buying a put (long position)

$$\Pi = \max(K - S_T, 0) - P_t(1 + R)^{T-t}$$

- Profit from selling/writing a put (short position)

$$\Pi = P_t(1 + R)^{T-t} + \min(S_T - K, 0)$$





Notice that all these functions are non-linear in S_T .

1.5 Hedging

To hedge a risk is to hold assets with opposite payout characteristics in response to change in the terminal spot price S_T .

- Purpose: reducing our exposure to this risk
- Mechanics – an example
 1. Suppose a US company knows it will pay £1 million 6 months from now (perhaps to buy a machine made in Britain).
 2. Suppose the forward price now for pounds in term of dollars (in six months from now) is \$1.4456 / £1.
 3. However, over next 6 months the exchange rate could change, thus changing the dollar price of the machine.
 4. To avoid this exchange rate risk, the American company can buy £1 million forward at today forward price of 1.4456, thus **guaranteeing** a dollar price of \$1.4456 million.

1.6 Types of traders

- **Hedgers** – previous example.
- **Arbitrageurs** – taking advantage of arbitrage opportunities
 - Consider a stock traded on two exchanges (NYSE and LSE)
 - Suppose the stock price is \$152 in New York and £100 in London, and the exchange rate is \$1.5500 per pound.
 - We could buy 100 shares in NY and sell them in London:

$$\Pi = 100 \times [(\$1.55 \times 100) - \$152] = \$300$$

assuming no transaction cost.

- **Speculators** – built-in leverage
 - Suppose the stock price of Red Hat is currently \$20, a two-month call option with a \$25 strike price is selling for \$1. I have \$4,000 to invest
 - Strategy 1: buying 200 share of Red Hat
 - Strategy 2: buying 4000 call options
 - Outcome 1: in two months the stock price is \$35 dollars:

$$\Pi_1 = 200 \times (\$35 - \$20) = \$3,000$$

$$\Pi_2 = 4000 \times \$10 - \$4,000 = \$36,000$$

- Outcome 2: in two months the stock price is \$15:

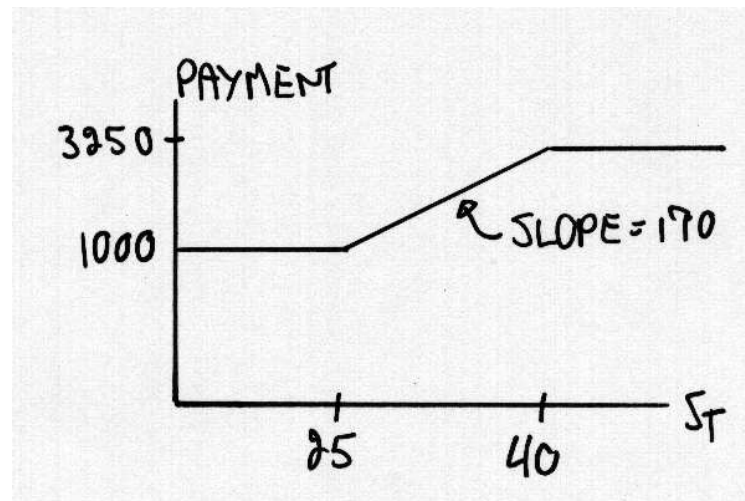
$$\Pi_1 = 200 \times (\$15 - \$20) = -\$1,000$$

$$\Pi_2 = 4000 \times \$0 - \$4,000 = -\$4,000$$

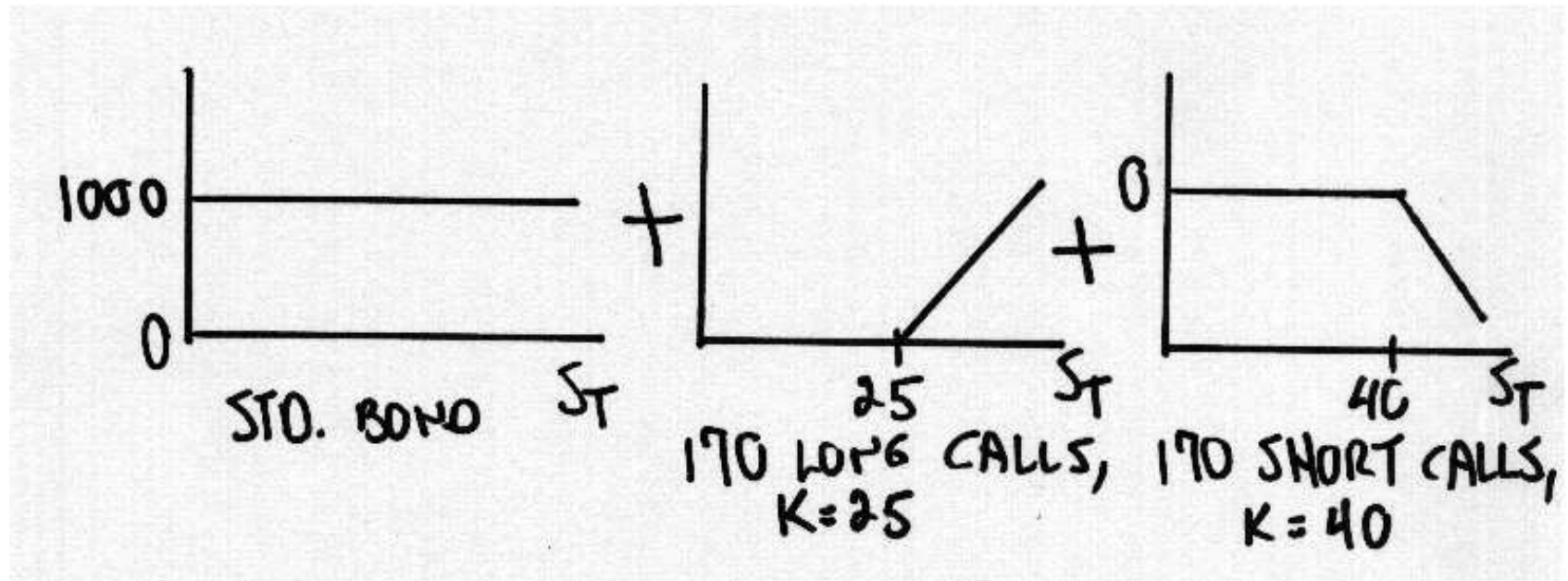
1.7 Nonstandard options (“exotics”)

1.7.1 Standard Oil’s bond issue

- At maturity, pays \$1,000 plus $170(S_T - 25)$ up to \$2,250 [= $170(40 - 25)$] where S_T is the price of a barrel of oil.
- Idea was that if price of commodity rose, company was in good position to pay extra return on bond.
- Payment scheme:

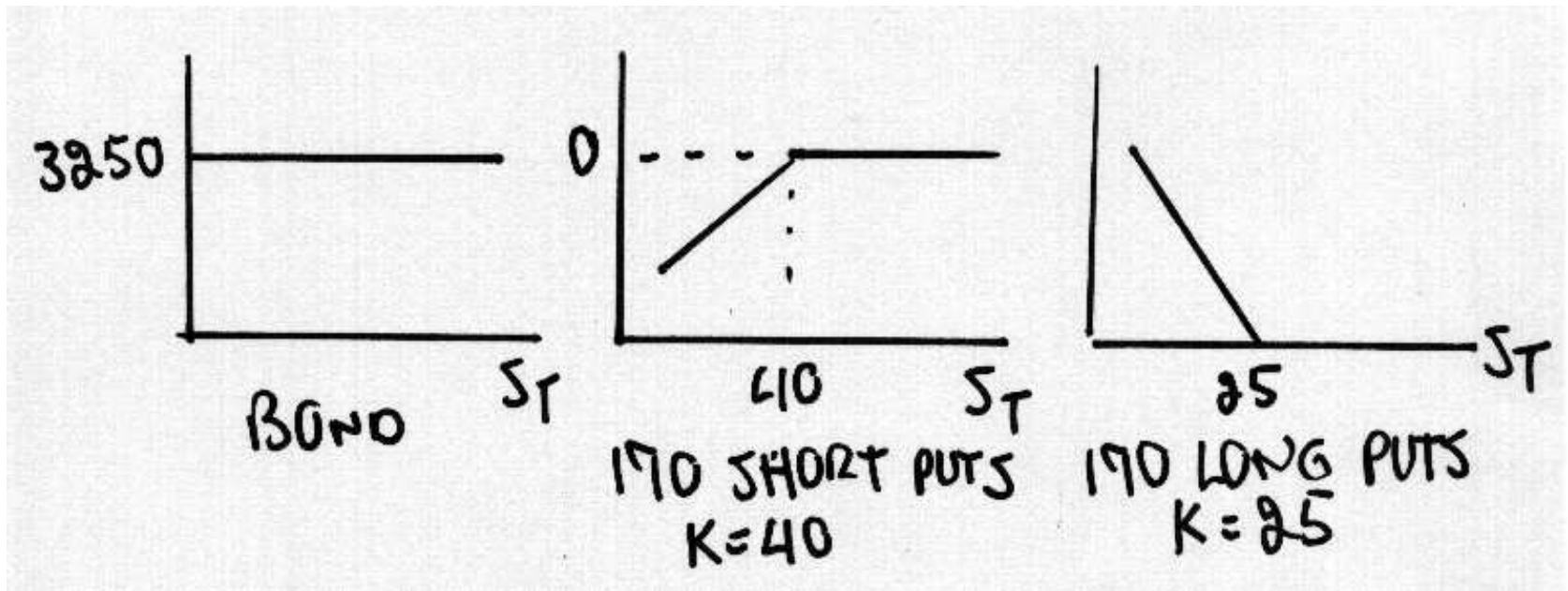


- Bond contract is equivalent to a nonstandard option



$$\text{Payment} = \$1,000 + \max[170(S_T - 25), 0] - \max[170(S_T - 40), 0]$$

- An equivalent option using puts:



$$\text{Payment} = \$3,250 - \max[170(40 - S_T), 0] + \max[170(25 - S_T), 0]$$

1.7.2 Index Currency Option Notes (ICON)

- Bonds whose payment varies with a foreign exchange rate.
 - Specify two exchange rates, $K_1 > K_2$.
 - If $S_T > K_1$, bond holder receive full face value.
 - If $S_T < K_2$, bond holder receive nothing.
 - If $K_1 \geq S_T \geq K_2$, bond holder receives part of the face value according to a predetermined formula.

- For example:

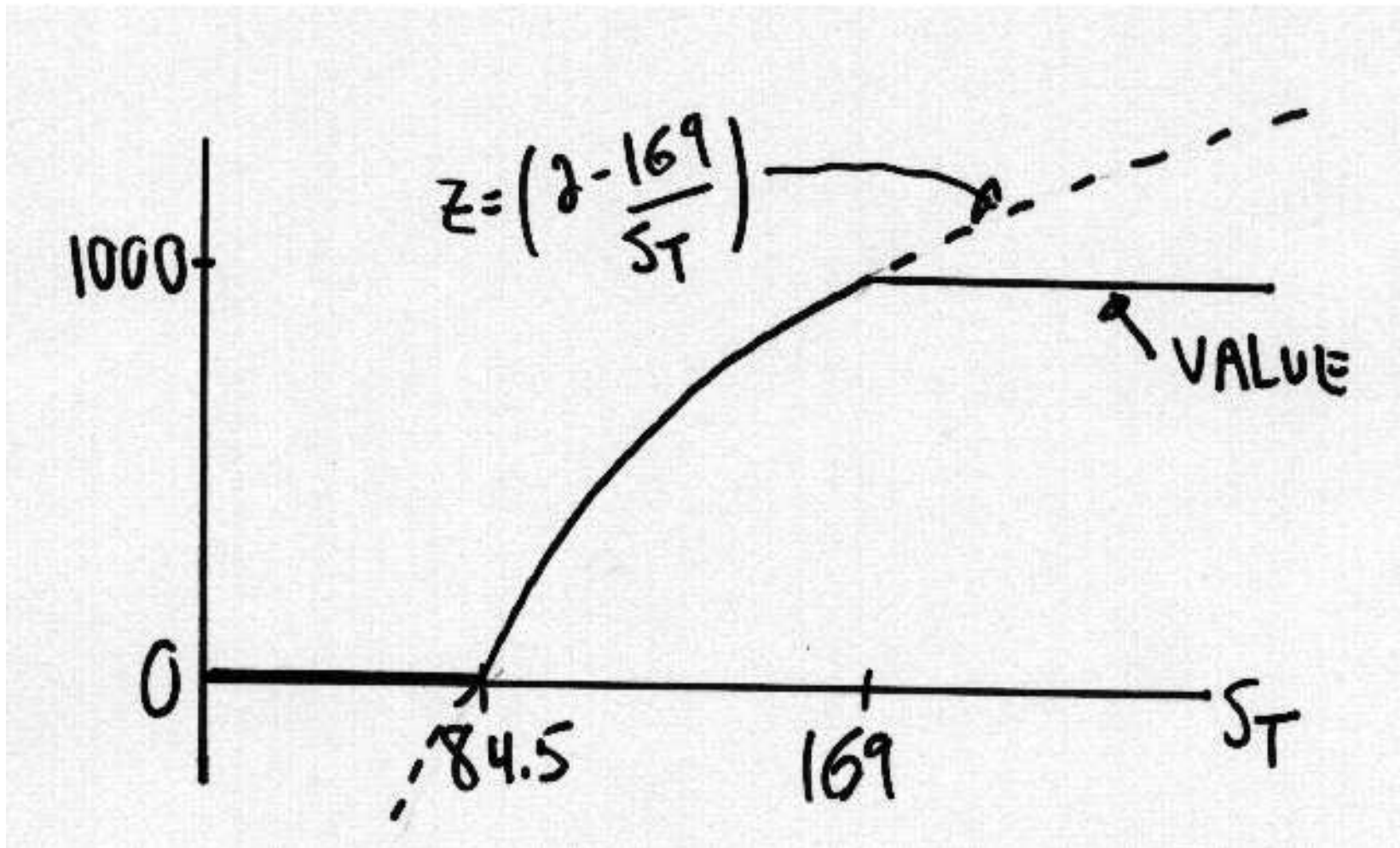
$$F \equiv \text{face value} = \$1,000$$

$$K_1 = 169 \text{ ¥}$$

$$K_2 = 84.5 \text{ ¥}$$

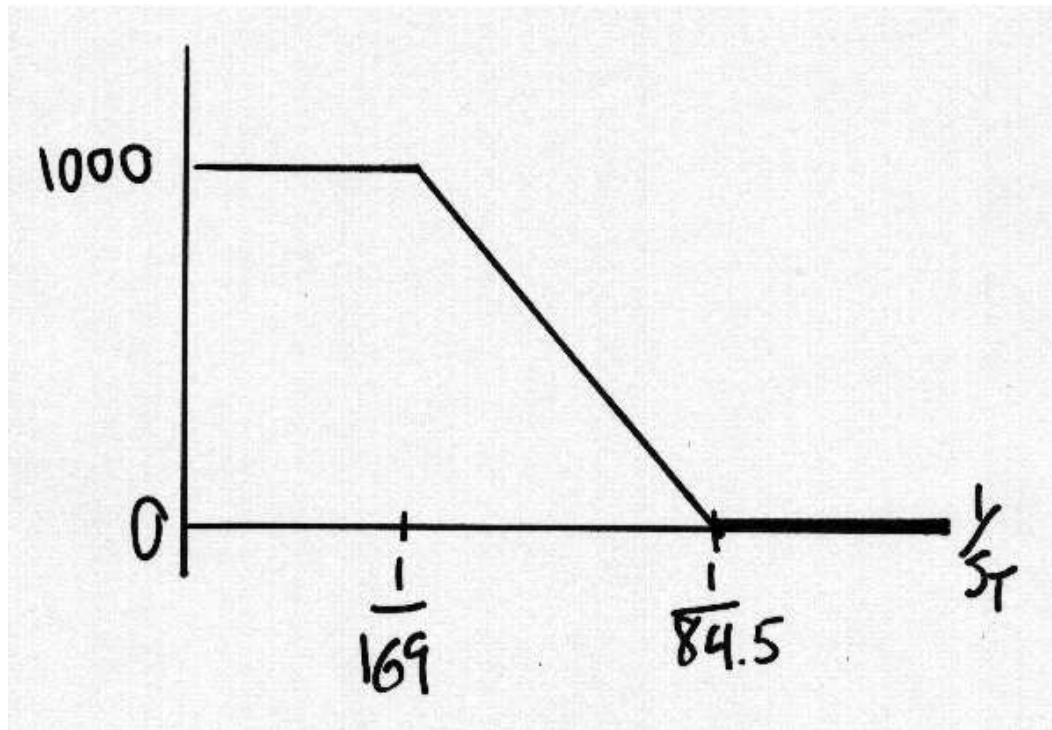
$$\text{Partial value} = \$1,000 - \max \left[0, 1000 \left(\frac{169}{S_T} - 1 \right) \right]$$

$$V = \begin{cases} 0 & \text{for } S_T < 84.5 \\ 2000 - 1000 \left(\frac{169}{S_T} \right) & \text{for } 84.5 \leq S_T \leq 169 \\ 1000 & \text{for } S_T > 169 \end{cases}$$



V is nonlinear in S_T , which makes it difficult to write a simple option formula in term of S_T

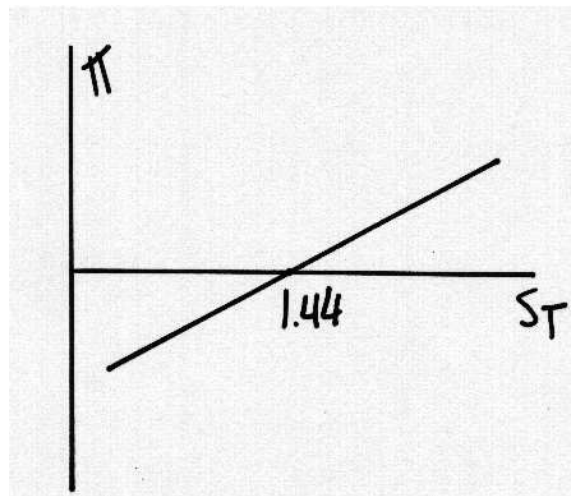
- Overcome this difficulty by treating $1/S_T$ as the price. Then V is linear in S_T



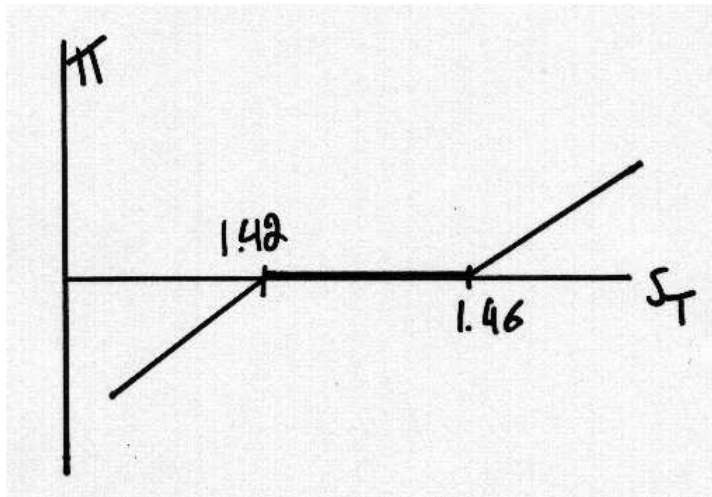
- ICON is equivalent to
 - Regular bond worth \$1,000
 - 169,000 short calls worth $\min\left[\frac{1}{169} - \frac{1}{S_T}, 0\right]$
 - 169,000 long calls worth $\max\left[\frac{1}{S_T} - \frac{1}{84.5}, 0\right]$
 - $V = 1000 + 169,000 \min\left[\frac{1}{169} - \frac{1}{S_T}, 0\right] + 169,000 \max\left[\frac{1}{S_T} - \frac{1}{84.5}, 0\right]$

1.7.3 Range forward contract

- Popular in foreign exchange markets
- Fix the upper and lower bounds on the exchange rate that the holder of the contract will pay.
- The bounds typically straddle today's forward rate
- For example, suppose the forward rate for 3 months is \$1.44 per pound. Firm needs pounds in 3 months
 - Firm could buy a future contract



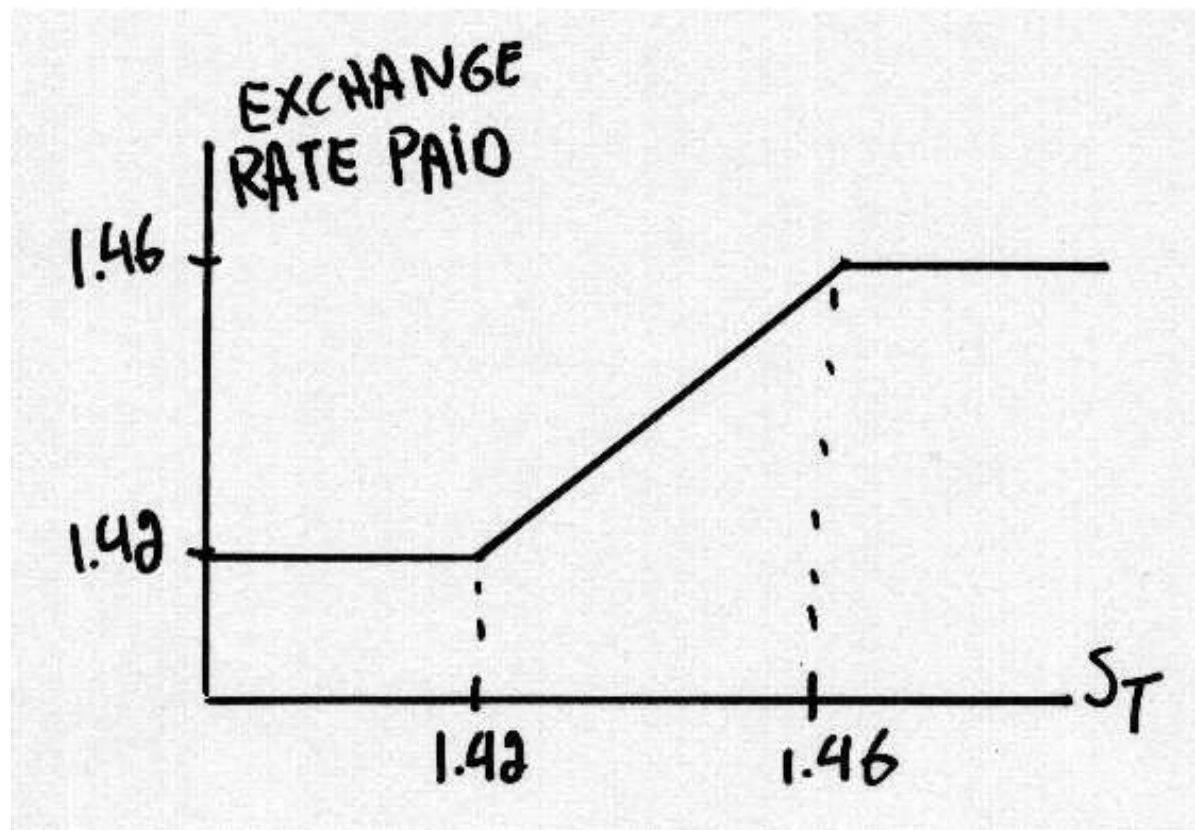
- Firm could buy a RFC. Suppose the contract has upper and lower bounds of \$1.46 and \$1.42. Then the payoff from the RFC is



which is a combination of a short put with $K = 1.42$ and a long call with $K = 1.46$.

Dollar cost D of obtaining L pounds is

$$D = L \times 1.42 + \max[L(S_T - 1.42), 0] - \max[L(S_T - 1.46), 0]$$



1.8 Short Sales

- **Definition:** selling an asset that is not owned
- **Mechanics**
 1. Borrow the asset
 2. Sell it now
 3. Repay it later
 4. Must pay any income on the asset, such as a dividend, to the lender
 5. Profit?
 - Positive if the price falls
 - Negative if price rises