

Optimization

- Optimization is derived from the Latin word "optimus", **the best**.
- Optimization characterizes the activities involved to find "the best".
- People have been "optimizing" forever, but the roots for modern day optimization can be traced to the Second World War (the born of Operations Research).

Optimization is a process of achieving the *best* outcome of given objectives while *satisfying certain restrictions*.

Optimization Models for Design

- The selection of a set of (*design*) *variables* to describe the design alternatives
- The selection of an *objective* expressed in terms of the design variables, which we seek to minimize or maximize
- The determination of the a set of *constraints* expressed in terms of design variables which we seek to satisfy

Formulation of an optimum design problem involves transcribing a verbal description into a well-defined mathematical statement.

Elements of Optimization Models

- **Design Space** - the set of considered possible design options, described using *design variables*, over which we have a direct choice.
 - Design variables must be controllable by the designer.
 - Design variables are desired to be independent.
- **Design parameters** - quantities that are given specific value in any particular model statement.
- **Constant parameters** - quantities fixed by underlying phenomenon.
- **Performance variables** - measurable quantities to express constraints and objectives
- **Mathematical relations** - equalities and inequalities that relate the above.

Basic Formulation of Optimization

Given: Design parameters p

Find: The optimal solution x

Subject to: Constraints $g(x,p) \leq 0$

or $g(x,p) (\leq, =, \text{ or } \geq) b$

Bounds $x \in R^n$ or $x^l \leq x \leq x^u$

Min or Max: objective function $z = f(x,p)$

Example Formulations

1. Design a beer can that can hold at least 400 cm^3 volume of beer. The objective is to reduce the cost of manufacturing the can.
2. Design a minimum weight tubular column of length L supporting a load P . The column is fixed at the base and free at the top.

Classifications of Optimization Problems

- **By model:** single variable or multiple variable, constrained or unconstrained, linear or nonlinear problem
- **By solution method:** Linear programming, Sequential quadratic programming, integer programming, golden section, steepest descent, geometry programming, dynamic programming, etc.
- **By derivatives:** gradient-based method, nongradient based (random, simplex method, Genetic Algorithms) etc.
- **By nature of information:** deterministic vs. probabilistic models