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MODEL VALIDATION VIA UNCERTAINTY PROPAGATION USING RESPONSE SURFACE MODELS

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ABSTRACT

Model validation has become a primary means to evaluate accuracy and reliability of computational simulations in engineering design. Mathematical models enable engineers to establish what the most likely response of a system is. However, despite the enormous power of computational models, uncertainty is inevitable in all model-based engineering design problems, due to the variation in the physical system itself, or lack of knowledge, and the use of assumptions by model builders. Therefore, realistic mathematical models should contemplate uncertainties. Due to the uncertainties, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with the confidence of using a model. In this paper, a generic model validation methodology via uncertainty propagation is presented. The approach reduces the number of physical testing at each design setting to one by shifting the evaluation effort to uncertainty propagation of the computational model. Response surface methodology is used to create metamodels as less costly approximations of simulation models for uncertainty propagation. The methodology is illustrated with the examination of the validity of a finite-element analysis model for predicting springback angles in a sample flanging process.

Key words: Model validation, Uncertainty propagation, Response surface models, Sheet metal flanging.

1. INTRODUCTION

During the last several decades, computer simulation models have been used in analysis and design of engineering systems. These models provide designers with a flexible and cheaper means (in terms of money, time and computational complexities) to explore design alternatives before physical part deployment. The increased dependence on computer models arises a critical issue of confidence in modeling and simulation accuracy. Model verification and validation are the primary methods for building and quantifying confidence, as well as for the demonstration of correctness of a model (Hill and Trucano, 1999; Oberkampf and Trucano, 2000). Briefly, *model verification* is the assessment of the solution accuracy of a mathematical model (AIAA guide, 1998). Model validation, on the other hand, is the assessment of how accurately the mathematical model represents the real world application (DoD, 1994; DoD, 1996; AIAA, 1998). Thus, in verification, the relationship of the simulation to the real world is not an issue, while in validation, the relationship between the computation and the real world, i.e., experimental data, is the issue. The fundamental strategy of model validation is to estimate uncertainties in the computational model, to quantify the numerical error in the computational solution, to estimate the experimental uncertainties, and to compare the computational results with the physical experiments. The utmost interest of this paper is model validation under uncertainty.

One limitation of the existing model validation approaches is that they are restricted to the validation at a particular design setting. There is no guarantee that the conclusion can be extended over the whole design space. In addition, model validations are frequently based on comparisons between the output from *deterministic simulations* and that from single or repeated experiments. The existing statistical approaches, for which the physical experiment has to be repeated a sufficient number of independent times, is not practical for many applications, simply due to the cost and time commitment associated with experiments. Furthermore, deterministic

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simulations for model validation do not consider uncertainty at all. Although recent model validation approaches propose to shift the effort to propagating the uncertainty in model predictions, which implies that a model validation should include all relevant sources of uncertainty and variation, little work has been accomplished in this area (Hill and Trucano, 1999; Sargent, 1999; Oberkampf and Trucano, 2000). Since realistic mathematical models should contemplate uncertainty, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with the confidence of using a model.

Traditionally, a model has been considered valid if it reproduces the results with adequate accuracy. The two traditional model validation approaches are: 1) subjective and 2) quantitative comparisons of model predictions and experimental observations. Subjective comparisons are through visual inspection of x-y plots, scatter plots and contour plots. Though they show the trend in data over time and space, subjective comparisons depend on graphical details. Quantitative comparisons use mathematical measures of correlation coefficients, which are dependent on the magnitude of confidence level. To quantify model validity from a stochastic perspective, researchers have proposed various statistical inference techniques, such as χ^2 test on residuals between model and experimental results (Freese, 1960; Reynolds, 1984; Gregoire and Reynolds, 1988). These statistical inferences require multiple evaluations of the model and experiments, and many assumptions that are difficult to satisfy. Therefore, *there is a need for a model validation approach that takes the least amount of statistical assumptions and requires the minimum number of physical experiments.*

In this paper, we present a rigorous and practical approach for model validation (*Model Validation via Uncertainty Propagation*) that utilizes the knowledge of system variations along with computationally efficient uncertainty propagation techniques to provide stochastic assessment of the validity of a modeling approach for a specified design space. The approach evaluates various sources of uncertainties in modeling and in physical tests and reduces the number of physical testing at each design setting to ONE. The approach also uses response surface methodology to create metamodels of a true model, therefore reducing the computational effort for uncertainty propagation. Even though the proposed methodology is demonstrated for validating a finite-element model for simulating sheet metal forming, namely a flanging process, it can be generalized to other engineering problems.

This paper is organized as follows. In Section 2, the technical background of this research is provided. The major types of uncertainties in modeling are first introduced and classified into three categories. Existing techniques on uncertainty propagation are then reviewed and the background of the response surface methodology is provided. Our proposed model validation approach is described in Section 3. In Section 4, our proposed approach is demonstrated by a case study in sheet metal forming. Finally, conclusions are provided in Section 5.

2. TECHNICAL BACKGROUND

2.1 Classification of Uncertainties

Various types of uncertainties exist in any physical system and in its modeling process and can affect the final experimental or predicted system response. Different ways of classifying uncertainties have been seen in the literature (Apostolakis, 1994; Trucano, 1998; Hazelrigg, 1999). In this work, we classify uncertainties into three major categories:

- Type I: Uncertainty associated with the inherent *variation* in the physical system or environment that is under consideration. For example, uncertainty associated with incoming material, initial part geometry, tooling setup, process setup, and operating environment.
- Type II: Uncertainty associated with *deficiency* in any phase or activity of the simulation process that originates in lack of system knowledge. For example, uncertainty associated with the lack of knowledge in the laws describing the behavior of the system under various conditions, etc.
- Type III: Uncertainty associated with *error* that belongs to recognizable deficiency but is not due to lack of knowledge. For example, uncertainty associated with the limitations of numerical methods used to construct simulation models.

When providing the stochastic assessment of model validity, all these three types of uncertainties should be taken into account.

2.2 TECHNIQUES FOR UNCERTAINTY PROPAGATION

The use of an analysis approach to estimate the effect of uncertainties on model prediction is referred to as uncertainty propagation. Several categories of methods exist in the literature. The first category is the conventional sample-based approach such as Monte Carlo Simulations (MCS). Although alternative sampling techniques such as Quasi Monte Carlo Simulations including Halton sequence (Halton, 1986), Hammersley sequence (Hammersley, 1960), and Latin Supercube Sampling (Owen, 1997; 1998) have been proposed, none of these techniques are computationally feasible for problems that require complex computer simulations, each taking at least a few minutes or even hours. Validating a modeling approach at multiple design settings becomes computationally infeasible. The second category of uncertainty propagation approach is based on sensitivity analysis. Most of these methods only provide the information of mean and variance based on approximations. The level of accuracy is not sufficient for applications in model validation. We propose to use a response surface model (or metamodel) to replace the numerical model for uncertainty propagation. The response surface model is generated as a function of both the deterministic variables and parameters subject to variations. Monte Carlo Simulations are later performed using the response surface model as a surrogate of the original numerical program.

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Details of response surface methodologies are provided in the next section.

2.3 RESPONSE SURFACE METHODOLOGIES

Response surface methodologies are well known approaches for constructing approximation models based on either physical experiments or computer experiments (simulations) (Box et al., 1978; Montgomery, 2000). Our interest in this work is the latter where computer experiments are conducted by simulating the to-be-validated model to build response surface models. They are often referred to as metamodels as they provide a "model of the model" (Kleijnen, 1986), replacing the expensive simulation models during the design and optimization process. In this paper, response surface models based on simulation results from finite element analysis (FEA) models are constructed and tested for model validation by using two response surface modeling methods: Polynomial Regression (PR) and Kriging Methods (KG).

PR models have been applied by a number of researchers (Engelund et al., 1993; Chen et al., 1996; Simpson et al., 1997) in designing complex engineering systems. In optimization, the smoothing capability of polynomial regression allows quick convergence of noisy functions (see, e.g., Giunta et al., 1994). In spite of the advantages, there is always a drawback when applying PR to model highly nonlinear behaviors. Higher-order polynomials can be used; however, instabilities may arise (Barton, 1992), or it may be too difficult to take sufficient sample data to estimate all coefficients in the polynomial equation, particularly in large dimensions.

A Kriging model (Sacks et al., 1989; Booker et al., 1999) postulates a combination of a polynomial model and departures from it, where the latter is assumed to be a realization of a stochastic process with mean zero and spatial correlation function. A variety of correlation functions can be chosen (Simpson et al., 1998); the Gaussian correlation function proposed in (Sacks et al., 1989) is the most frequently used. In addition to being extremely flexible due to the wide range of the correlation functions, the Kriging method has advantages in that it provides a basis for a stepwise algorithm to determine the important factors, and the same data can be used for screening and building the predictor model (Welch et al., 1992). The major disadvantage of the Kriging process is that model construction can be very time-consuming. Fitting problems have been observed with some full factorial designs and central composite designs when using Kriging models (Meckesheimer et al., 2000).

In earlier work, the advantages and limitations of various metamodeling techniques have been examined using multiple modeling criteria and multiple test problems (Jin et al., 2001a; Jin et al. 2001b).

3. OUR PROPOSED MODEL VALIDATION APPROACH

Our proposed model validation approach is illustrated in Figure 1. The whole process includes four major phases, in which Phase II and Phase III can be implemented in parallel. Phase I is the *Problem Setup* stage. Here, uncertainties of all types described in Section 2.1 are investigated; probabilistic descriptions of model inputs are established. With the aim of model validation over a design space rather than at single design point, sample design settings, represented by x^i, $i = 1 \ldots n$, are formed using different combinations of values of design variables. The sampling can be based on the knowledge of critical combinations of design variables at different levels, the standard statistical techniques such as Design of Experiments (DOE) (Box et al., 1978), or other methods for efficient data sampling (e.g., optimal Latin Hyper Cube (McKay et al., 1979)). These techniques will be useful in reducing the size of samples when the number of design variables considered is large.

Phase II is the (Physical) *Experiments* stage. *One of the cornerstones of this proposed approach is the minimum number of physical tests required.* Physical experiments will be performed only **once** at each design setting identified in Phase I. Measurements are taken for the model responses that are of interest. The results of experiments are denoted as Y^i, $i = 1, \ldots, n$. Errors of measurements are predicted.

Phase III (*Model Uncertainty Propagation*) is the stage for uncertainty propagation based on the to-be-tested (computational) model. For computationally expensive models, we propose to first construct the response surface models based on samples of numerical simulation results. Next, the total uncertainty of the response prediction is analyzed using the response surface model for each of the sample design settings identified in Phase I.

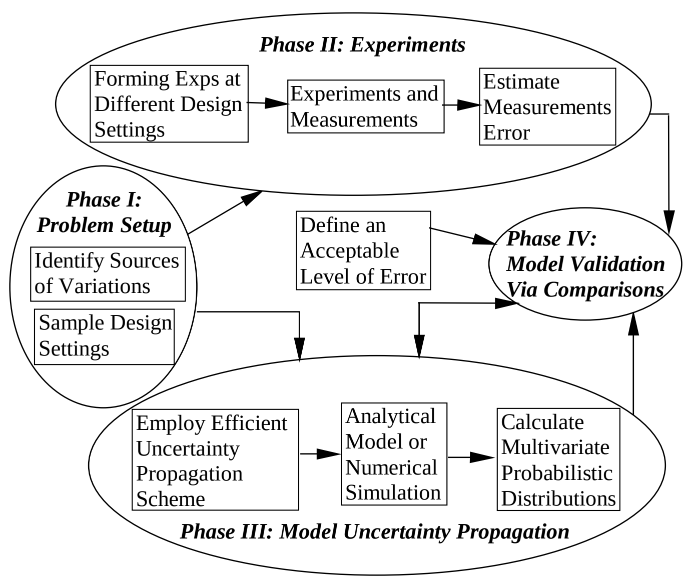

Figure 1. Proposed Procedure for Model Validation

The uncertainty of the (computational) model prediction can be evaluated by uncertainty propagation using Monte Carlo Simulation (MCS) applied to the metamodel (in this case, response surface model), following the uncertainty descriptions identified in Phase I. When a sufficient number of simulations

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are performed, the MCS is robust in a sense that it provides good estimates of uncertainty in the predicted parameters, no matter whether the model is highly nonlinear or not. The MCS also provides estimates of the shape of the probability density functions (pdf), which are used further in Phase IV for model validation.

Phase IV is the *Model Validation* phase, when the stochastic assessment of model validity is provided based on the comparisons of the physical experimental results from Phase II and the computational results from Phase III. Details of model validation strategies for single and multiple design points are discussed next. The strategies introduced by Hill et al. (1999) are followed.

Model validation at a single design point

Hill's method states that for a given confidence limit (say 100*(1-α)%), if the physical experiment falls within the performance range obtained from the computer model (here, the probability density function (pdf)) obtained from the MCS in Phase III), it indicates that the model is consistent with the experimental result (however, we can not say the model is valid for the confidence limit). On the other hand, if the physical experiment is outside of the performance range, then we would reject the model for that specified confidence level (100*(1-α)%). *Our strategy of model validation is to identify at which critical limit of confidence level the physical experiment falls exactly at the boundary of the performance range obtained from the computer model* (see Figure 2). Therefore if the given confidence level is lower than the critical limit of confidence level, the model will be rejected, and vice versa.

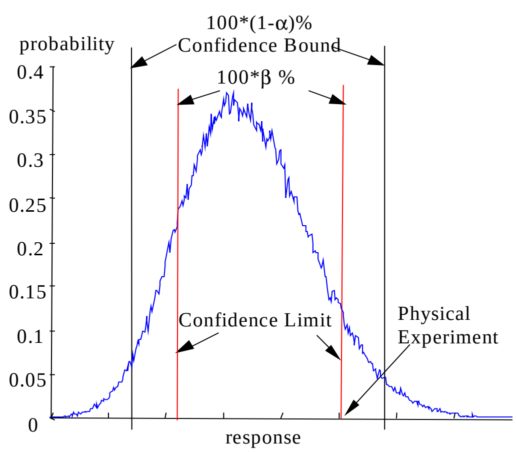

Figure 2. Model Validation for a Single Design Point.

As shown illustratively in Figure 2, the dark probability density function describes the distribution of a response based on the (computational) model for the given uncertainty description at a single design point. The confidence limit with which one cannot reject the simulation model is the area under the pdf curve that bounds exactly on the physical experiment, includes the mean of the pdf, and excludes the two equally sized tails that depend on the location of the physical experiment. If the confidence limit is identified as β% which is smaller than the given confidence bound (e.g., 100*(1-α)% in Figure 2), we cannot reject the model for an experiment that falls on the boundary of β%. If the confidence bound (100*(1-α)%) is given at a value smaller than β%, we can reject the model since the physical experiment falls outside the distribution range. When a model is rejected, it indicates that a new model needs to be constructed and the whole procedure of model validation should be carried out again. It should be noted that since stochastic assessments are provided for model validity, there are certain risks associated with the error of hypothesis testing (Kleinbaum. et al., 1998). In our case, the false positive error (commonly referred to as a Type I error) is the error of rejecting a model while the true state is that the model is indeed valid. The probability of leading to this outcome is α%. We note that providing a higher confidence bound (lower α%) would widen our acceptance region, while it will reduce our chances of rejecting a valid model, it would also increase our chance of accepting an invalid model, i.e., increasing the probability of making the false negative error (referred to as a Type II error). Indications of Type I and Type II errors were discussed by Oberkampf and Trucano (2000), where they related the Type I error to a model builder's risk and Type II error to model users' risk.

Model validation at multiple design settings

When a model needs to be validated at multiple design settings, the physical experiment results need to be compared against the joint probability distributions of a response at multiple design settings. The probability distributions of y^i at multiple design settings (n) are used to generate the joint probability distributions (multidimensional histogram). The contours of the joint probability distributions are used to define the boundary of a given confidence level for model validation and compared with the results from physical tests. Provided in Figure 3 is an illustrative example of model validation for a problem with two physical tests (corresponding to two design settings). The joint *pdf* of y^1 and y^2 is first obtained for the same response, and then the boundary with 1-α confidence level is determined by the iso-count contour that contain 100(1-α)% samples of Monte Carlo Simulations conducted over the RSM. Theoretically, if the experiment result Y in an n-dimensional space (n = 2 in this example) falls within the boundary, it indicates that we cannot reject the model with a confidence level of (1-α). If the point falls outside of the boundary, then we can reject the model with confidence level (1-α). Note the results of single experiments at multiple design settings now become a single point in the multivariate histogram space.

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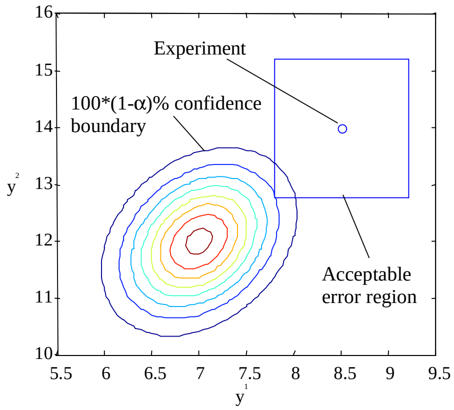

Figure 3. Model validation at two design settings

For multivariate distributions symmetric about their means, contours of constant probability are given by ellipses determined with r^2. r^2 can be related to normal probability through the chi-square distribution for 100*(1-α)% confidence with *n* degrees of freedom (*n* design settings). The prediction model can be rejected at 100*(1-α)% if the combination of multiple design points measured from physical experiments is outside of 100*(1-α)% confidence region.

According to Hills, a constant probability is given by the following ellipses where r is constant for iso-probability curves.

$$r^2 = [p_1 - p_{mean1} \quad p_2 - p_{mean2} \quad \cdots\cdots \quad p_n - p_{meann}] V^{-1} \begin{bmatrix} p_1 - p_{mean1} \\ p_2 - p_{mean2} \\ \cdots\cdots \\ p_n - p_{meann} \end{bmatrix} \tag{1}$$

In the above equation, p_i, i =, 1..n, stands for the single physical experiment result for each design setting i. p_{meani} is the mean of the random samples obtained from the computer model at each testing point i. The *V* matrix is the *n* by *n* co-variance matrix based on the random samples.

For model validation, the critical value of r^2 is obtained as:

$$r_{critical}{}^2 = l^2_{1-\alpha}(n), \tag{2}$$

where *l* is the value associated with the 100*(1-α)% confidence for n testing points through the chi-square distribution. If the value of r^2 from Eqn. (1) is less than the critical value of r^2 from Eqn. (2), then we do not possess statistically significant evidence to declare our model invalid, and vice versa.

Measurement error, response surface model error, and acceptable level of error

In the proposed model validation procedure, it is also important to consider various uncertainties (errors) that cannot be predicted by the uncertainty propagation based on the computational model. These errors include the measurement errors, the response surface model error, and the acceptable level of error. To simplify the process, we count the measurement errors and response surface model error by adding them directly to the prediction uncertainty obtained through uncertainty propagation. Specifying an acceptable level of error is practically significant because the discrepancy between the simulation and experiment results indicates the errors associated with the model structure and numerical procedures (Types II and III uncertainties discussed in section 2). Approximated models should not be declared invalid if they provide predictions within an error that the user finds acceptable for a particular application. The acceptable error is modeled as a box around the physical test point in Figure 3. Figure 3 shows a situation in which the confidence region of the model prediction and the acceptable error region overlap. This indicates that we cannot declare that the model is invalid for the given confidence level considering the acceptable level of error.

4. VALIDATING A FINITE-ELEMENT MODEL OF SHEET METAL FLANGING PROCESS

4.1 Sheet Metal Flanging Process and its Modeling

Sheet metal forming is one of the dominant processes in the manufacture of automobiles, aircraft, appliances, and many other products. As one of the most common processes for deforming sheet metals, flanging is used to bend an edge of a part to increase the stiffness of a sheet panel and (or) to create a mating surface for subsequent assemblies. The popularity of flanging is mainly due to the high degree of design flexibility it offers and the high strength, low-weight and typically low cost components it yields. As the tooling is retracted, the elastic strain energy stored in the material recovers to reach a new equilibrium and causes a geometry distortion due to elastic recovery (see Figure 4), the so-called 'springback' (Taylor et al., 1995; Song et al., 2000; Li et al., 2002). Springback refers to the shape discrepancy between the fully loaded and unloaded configurations as shown in Figure 4.

Springback depends on a complex interaction between material properties, part geometry, die design, and processing parameters. The capability to model and simulate the springback phenomenon early in the new product design process can significantly reduce the product development cycle and costs. However, many factors influence the amount of springback in a physical test. Prediction and experimental testing of springback is particularly sensitive to the three types of uncertainties (see Figure 5) as discussed in Song et al. (2000), Hu and Walters (1999), Shi and Zhang (1999), Zhang et al. (1999), Esche and Kinzel (1998), Li and Wagoner (1998), Saran and Demeri (1998), and He and Wagoner (1996). The quantifications of these uncertainties are detailed in the next section.

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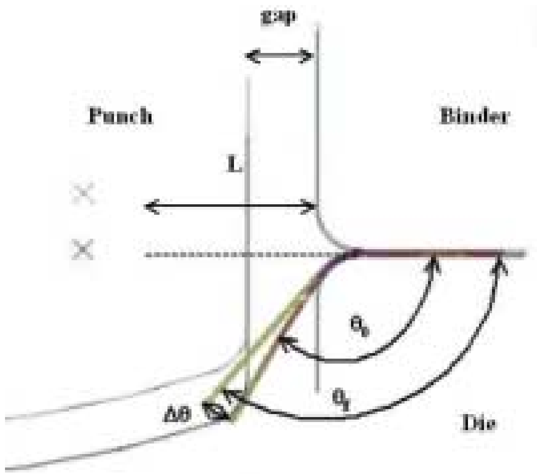

Figure 4. Schematic of the springback in flanging; *g* is the gap between the die and the punch, θ_0 is the flange angle at the fully loaded configuration, θ_f is that of the unloaded configuration, and $\Delta\theta$ is the springback.

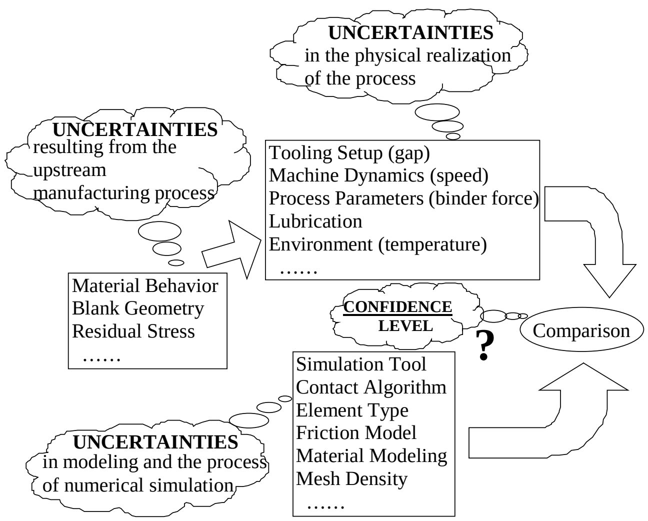

Figure 5. Types of Uncertainties in a Sheet Metal Forming Process

Various modeling approaches have been used to model the flanging process. These models include both analytical models and finite element analysis-based models. In this study, we illustrate how the proposed model validation approach can be applied to validate a finite element analysis model that models the blank plasticity with the combined hardening law (ABAQUS^{TM} manual). The modeling approach uses an implicit integration method in which the sheet metal blank is modeled by 1440 2D continuum elements with reduced integration. The angle at fully unloaded configuration (see Figure 4) has been considered as a process output.

4.2 Problem Setup, Experiments, and Uncertainty Propagation in Validating a Sheet Metal Forming Process Model

We illustrate in this section how the major phases in the proposed model validation approach are followed for our case study.

Phase I – Problem Setup

To accomplish Phase I, design variables and design parameters that affect the process output (springback angle) are determined. Primarily, two design variables that are related to the process setup are considered, i.e. flange length, L; and gap space, g; and design parameters that are related to the material are selected, i.e. sheet thickness, t; and material properties (namely, Young Modulus, *E*; Strain Hardening Coefficient, *n*; Material Strength Coefficient, *K*; and Yield Stress, *Y*) (see Figure 6). Design parameters are uncontrollable (given) while design variables can be controlled over the design space to achieve the desired process output.

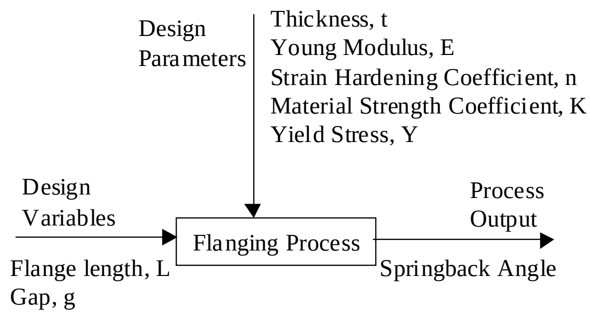

Figure 6. System Diagram for Flanging Process

To form sample design settings, different combinations of values of design variables, i.e., L and g, are used. Five sample design settings are formed with all possible combinations of low and high levels of flange length (3 and 5 inches) and gap (5 and 30 mm) plus a design point close to the middle (4, 10) (see Figure 7).

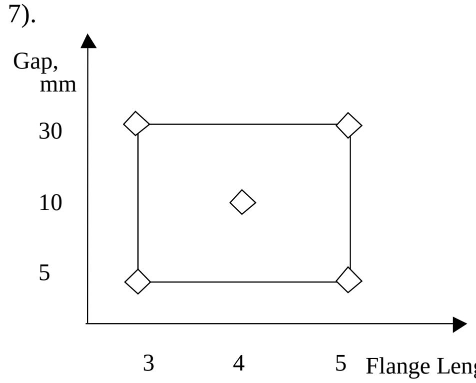

Figure 7. Sample Design Settings of Flanging Process for Model Validation

The variations of design variables and design parameters are identified in this phase. Based on statistical experimental data, obtained by tensile tests, the relationships among *K*, *n*, and

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Y for carbon steel sheet metals used in the tests are approximated as

$$K(n) = 1128.5n + 499.97 \text{ , and} \quad (3)$$
$$Y(n) = -779.8n + 484.85 \text{ .} \quad (4)$$

Therefore among the four parameters describing the material property, two are independent parameters (*n* and *E*), and the other two (*K* and *Y*) are dependent. Also, the statistical descriptions for these material parameters are obtained. The distribution of Young Modulus (*E*) is assumed to be a normal distribution with 197949.7 MPa and 12914.7 MPa as the mean and standard deviation, respectively. The distribution of strain-hardening exponent (*n*) is assumed to be a uniform distribution from 0.10 to 0.18. The distributions of the strength coefficient (*K*) and yield stress (*Y*) depend on *n* as shown in equations 3 and 4. The distribution of sheet thickness (*t*) is assumed to be a normal distribution with 1.5529 mm and 0.0190 mm as the mean and standard deviation, respectively. Similarly, the variation of the design variable gap space (*g*) is assumed to be normally distributed with a standard deviation of 0.6 mm; note that the mean of the gap will change based on the location of the design point. The variation of flange length (L) is ignored.

Phase II – Experiments and Measurements

In this phase, physical experiments are conducted and measurement errors are estimated. The dimensions of the sheet blank used in the physical experiments are 203.2 mm. x 203.2 mm. (or 8 inches x 8 inches). The flanging process uses a punch, a binder, a draw die, and a blank. The experiments have been performed by the 150-ton computer controlled HPM hydraulic press in the Advanced Materials Processing Laboratory at Northwestern University. The unloaded configurations, i.e., the angles between two planes in degrees (see Figure 4), have been measured by a coordinate measuring machine (Brown & Sharpe MicroVal Series Coordinate Measuring Machine B89) in the metrology laboratory at Northwestern University.

Phase III – Model Simulation and Uncertainty Propagation

The flanging process has been numerically simulated based on the finite element code that employs the combined hardening law. The blank has been modeled by an implicit and static nonlinear finite element code, ABAQUS/Standard. 1440 of eight-node, two-dimensional (plane strain) continuum elements with reduced integration have been used in this problem to model the sheet blank (ABAQUS element type CPE8R). The sheet thickness has been modeled with six layers. Tools have been modeled as rigid surfaces. The coefficient of friction is set to 0.125. The interface between the tooling and the sheet has been modeled by interface elements (IRS22) while the penalty-based contact algorithm has been used. To have a better convergence rate, the surface interaction is modeled by a soft contact. The analysis is performed in six steps: moving the binder toward the blank; developing the binder force; moving the punch down to flange the blank; retracting the punch up; releasing the binder force, and finally, moving the binder up. The following two cases are considered in simulation experiments for creating the response surface models for model validation.

Case 1: Validation at a single design point. 81 simulation experiments have been conducted to create a response surface model for model validation at a single design point (3, 30), i.e., flange length at 3 inches and gap at 30 mm. Since there is randomness associated with gap, we consider [25, 35] inches for gap when building the model. The response surface model represents the springback angle as a function of (*g*, *t*, *E*, *n*, *K*, *Y*) at L =3 inches. The 81 simulation experiments are designed based on various combinations of (*g*, *t*, *E*, *n*, *K*, *Y*), where three levels are considered for both gap (*g*) and thickness (*t*) and a full factorial design of these two factors are combined with nine settings of (*E*, *n*, *K*, *Y)* that capture a wide range of the material property.

Case 2: Validation for multiple design settings. 243 simulation experiments have been conducted to create the response surface model for model validation at five design points, i.e., the following combinations of design variables (flange length in inches, and gap in millimeters): (3, 5), (3, 30), (4, 10), (5, 30) and (5, 5). The response surface model represents the springback angle as a function of (*L*, *g*, *t*, *E*, *n*, *K*, *Y*). The 243 simulation experiments are designed based on various combinations of (L, *g*, *t*, *E*, *n*, *K*, *Y*). Similar to the strategy used for designing the experiments in Case 2, three levels are considered for flange length (*L*), gap (*g*), and thickness (*t*) and a full factorial design of these three factors are combined with nine settings of (*E*, *n*, *K*, *Y)* that capture a wide range of the material property.

Both second order Polynomial Regression (PR) and Kriging (KG) approximation models (Jin et al. 2001) are used to create response surface models for Cases 1 and 2. The accuracy of each model for Cases 1 and 2 is assessed by examining the sum of squares of error (SSE) based on a set of confirmation tests. The results are obtained as:

Case 1: SSE for PR is 0.0212, while SSE for KG is 2.8596.
Case 2: SSE for PR is 0.9862, while SSE for KG is 375.0888.

The confirmation tests show that the polynomial regression can better represent the actual simulation model since it has lower values of the SSE for both Cases 1 and 2. Considering that the magnitude of the springback angle is in the range of 100 to 150 degrees, the achieved SSE from PR is quite satisfactory. Therefore, for uncertainty propagation and model validation, only the results from the polynomial models will be used in both Cases 1 and 2.

Once the response surface models are created, the MCS have been used to efficiently predict the distributions of the springback angle under uncertainty using 200,000 random sample points based on the polynomial models for both Cases 1 and 2. The uncertainty descriptions identified in Phase I are

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followed for random sampling. The predicted distributions of the springback angle will be presented together with the validity results next.

Phase IV – Model Validation via Comparisons

Normality Check

To simplify the model validation process, the predicted distributions of springback angle (for single design point and each individual design point in multiple design settings) have been checked for normality. The following method for checking normality has been used (Montgomery, 2000): For a normal distribution, 68.26% of the points should fall within [mean-1 sigma, mean+ 1 sigma], 95.44% of the points should fall within [mean-2 sigma, mean+ 2 sigma], and 99.72% of the points should fall within [mean-3 sigma, mean+ 3 sigma] limits.

The resulting probability distributions are plotted in Figure 8 (case 1) and Figure 9 (case 2).

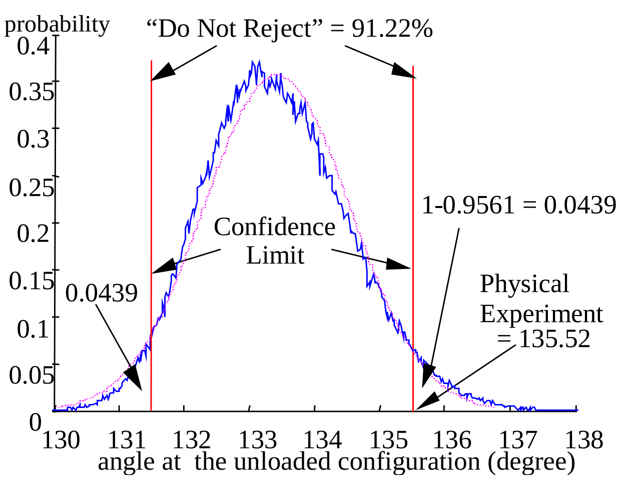

Figure 8. Confidence Limits based on Polynomial Model at Single Design Point (3, 30).

In Figures 8 and 9, the light pdf curve is the fitted normal distribution. It is noted that in general, the predictions (considered separately for each design point) based on polynomial models are all very close to normal. The normality assumption can greatly simplify the validation process, which is introduced next.

Model Validation

Case 1: For the single design point (3, 30), the results of the predicted springback angles based on MCS using the response surface model are compared with the result from a single physical experiment. As shown in Figure 8, the angle obtained from the experiment is 135.52, and 95.61% of the angles predicted with simulation based on polynomial model, are smaller than the value of 135.52 (the left tail with the middle "Do not reject" area in Figure 8, together equal to 0.0439+0.9122=0.9561). Thus, [0.9561-(1-0.9561)]=0.9122 (the "Do not reject" area in Figure 8) is the confidence level with which one cannot reject the simulation model. The two tails (each equal to 1-0.9561=0.0439) are the "Reject the model" area.

Based on the identified critical confidence limit, we can say that if the confidence level is given at 90% (<91.22%), we can reject the model. If the confidence level is given at 95%, we cannot reject the model. We note that providing a higher confidence level, say 99%, would widen our acceptance region, while it will reduce our chances of rejecting a valid model, it would also increase our chance of accepting an invalid model. Note, in general, the result of lower confidence limit is preferred, which indicates that there is a lower confidence to reject the model.

Case 2: From the results of normality check conducted earlier, it is assumed that the total model uncertainty for five design points could be modeled by jointly distributed normal probability density functions. One physical experiment at each design point has been considered (see Figure 9), and the angles obtained from the experiments at each design point are provided in Table 1. Note that the physical experiments fall within the 95% confidence level at each design point. This means that the polynomial models considered separately at each individual design point cannot be rejected at 95% confidence level.

Table 1. Angles (Degree) from Physical Experiments at Five Design Settings

Design Points	Angles from Experiments
(3, 30)	134.9287
(3, 5)	106.5019
(4, 10)	111.6919
(5, 30)	135.2204
(5, 5)	106.7697

Equations 1 and 2 have been used to calculate r^2 for the polynomial model. r^2 for the polynomial model is 7.4462. For 95% confidence level, the critical value of r^2 is obtained as

$$r_{critical}{}^2 = l_{95\%}^2(5) = 11.07 \qquad (5)$$

Since the r^2 from polynomial model is smaller than the critical r^2, there is not enough statistical evidence to conclude that polynomial is not valid. We find that for the polynomial model, the critical confidence limit lies on about 80% contour because r^2 for 80%, for 5 dof = 7.289.

We should note that we have not yet considered the errors of the response surface model, the experimental error, and the inaccuracy tolerance in the model validation process introduced so far. If considered, the modified confidence limit is expected to be lower.

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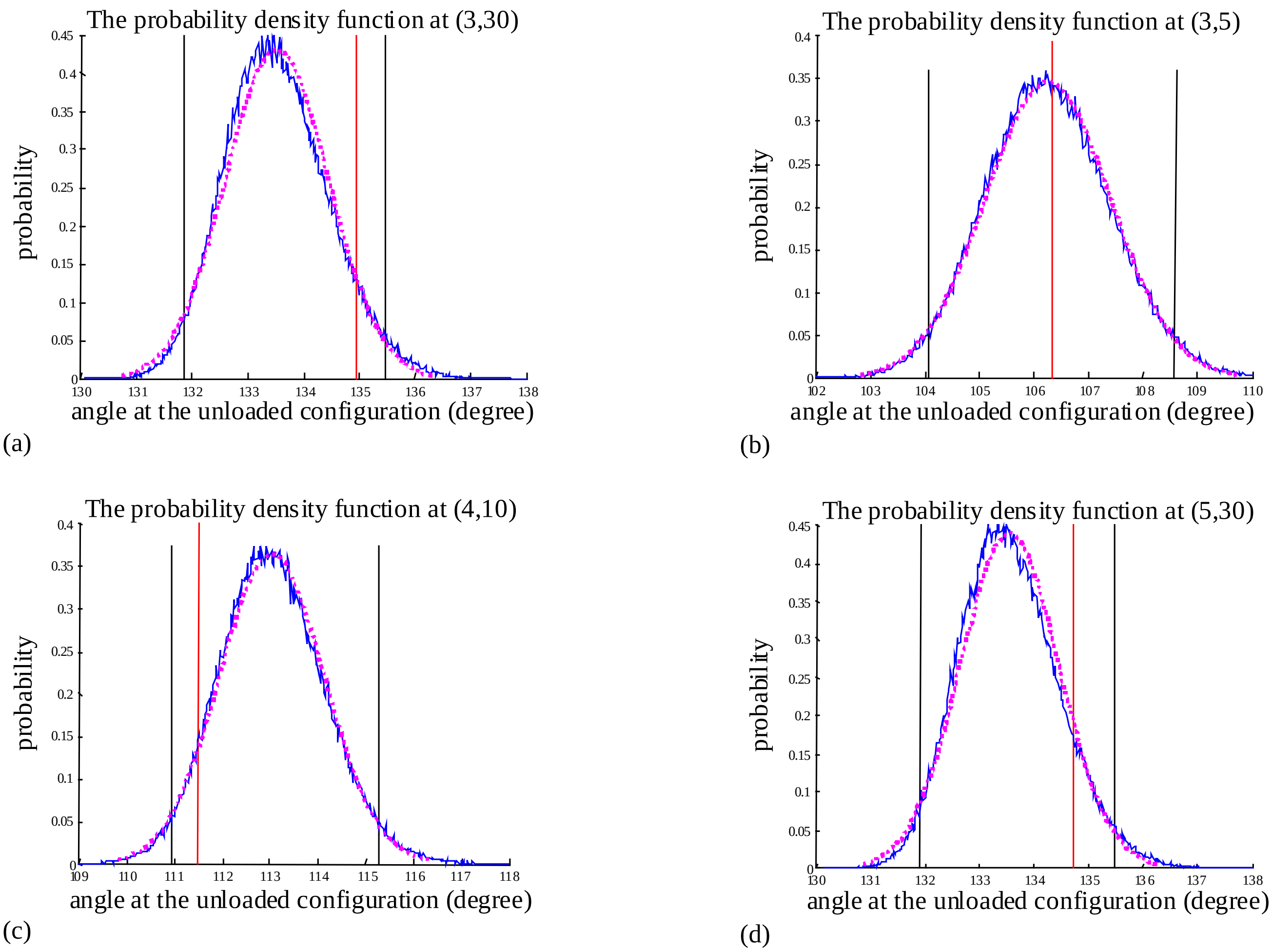

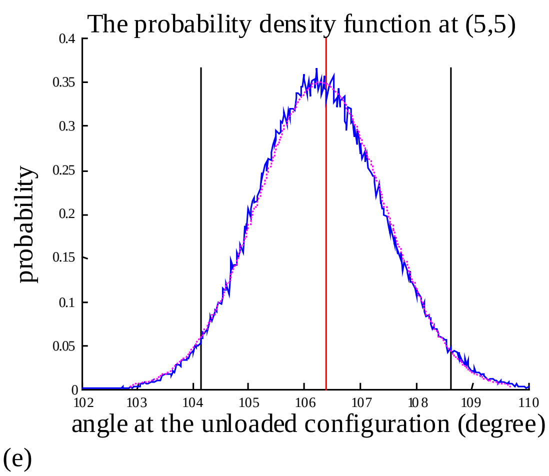

Figure 9. Pdf Plots for Multiple Design Points; polynomial model at design points: (a) (3,30), (b) (3,5), (c) (4, 10), (d) (5,30), and (e) (5, 5). The light pdf curve is the fitted normal distribution at each design point. The two vertical lines are 95% confidence level and the line between them is the angle obtained from physical experiment at each design point.

4.3 Considering Measurement Error, Response Surface Model Error, and Acceptable Level of Error

Following the description in Section 3.1, the measurement error and the response surface model error are added directly to the predicted values of springback angle using MCS samples. The mean and the variance of the response surface model error are estimated as the mean and the variance of the differences between the angles obtained from the simulation and those predicted with the response surface model for 200 samples. The samples are obtained with Optimum Latin Hypercube Sampling (OLHS) (Sudjianto et al., 1998). As the result, normal distribution N (-0.7120776, 0.605479) is used to describe the error of the polynomial model in Case 1, and N (-0.6311915, 1.0657299) is used to describe the error of the polynomial model in Case 2. The mean and the variance of the measurement error are obtained based on the specification of the CMM machine, represented as N (0, 0.04193576). The

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acceptable level of error is set to ±0.5 degrees and incorporated by adjusting the result from the physical experiment. Thus, for the value of 135.52 (degrees) obtained from the physical experiment, the model cannot be rejected if the response distribution falls to the left of the minimum acceptable value of the experiment, i.e. 135.52 - 0.5 = 135.02 (degrees) (see Figure 10). In Figure 10, the curve with upper tails and lower pick reflects the modified pdf, which is checked against the lower limit of acceptable error range. Note that considering different errors reduces the possibility of rejecting a model.

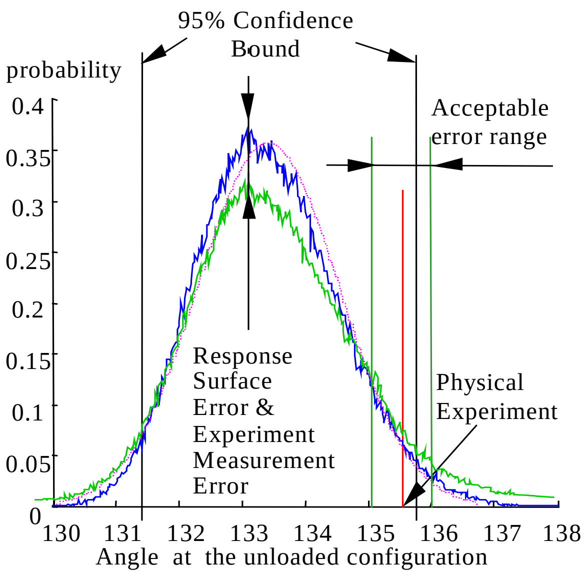

Figure 10. Considering Various Types of Errors.

Case 1: Single design point: the confidence limit for 135.02 (degrees) is 87.95% for the polynomial model after the modification.

Case 2: Multiple design settings: r^2 for the polynomial model is 3.67 for the minimum acceptable values of the experiments for all design points, i.e., 134.4287, 106.0019, 111.1919, 134.7204, and 106.2697. As mentioned earlier, for 95% confidence level, the critical value of r^2 is 11.07. Thus, the polynomial model for multiple design points cannot be rejected at 95% confidence level. The critical limit for the polynomial model lies on about 40% since r^2 for 40%, for 5 dof = 3.6555.

5. CONCLUSIONS

In this paper an approach for model validation via uncertainty propagation using the response surface methodology is presented. The approach uses response surface methodology to create metamodels as less costly approximations of simulation models for uncertainty propagation. Our proposed model validation procedure incorporates various types of uncertainties involved in a model validation process and significantly reduces the amount of physical experiments. The proposed approach can be used to provide stochastic assessment of model validity across a design space instead of a single point. The approach has been illustrated with an example of sheet metal flanging process, for two cases: for a single design point and multiple design settings. Polynomial and Kriging response surface models are created for both cases. Since the polynomial models are confirmed to be more accurate than the Kringing models, they are used for uncertainty propagation in both cases. The critical confidence levels are identified by comparing the performance distribution obtained from uncertainty propagation with the results from the single experiments. The polynomial models have not been statistically declared as invalid if the given significant level is set at 95% for both single and multiple design points. The results are adjusted after considering the response surface model error, the measurement error, and the acceptable level of error. For the tested finite element model based on the combined hardening law, the model cannot be statistically declared as invalid if the given significant level is set at 95% for both single and multiple design points.

Future work will be directed toward validating other alternative models in simulating the sheet metal flanging process. Efficient methods for identifying the critical limit of confidence level in model validation are to be developed for situations under which the response distributions do not necessarily follow the normal distribution.

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