

A Revision of the Distillation Theory (Part I)

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ABSTRACT

The McCabe-Thiele graphical method is considered as a basic and simple technique to determine the number of theoretical stages in a distillation column. However, it is demonstrated that there are a number of weaknesses in this method when it is used to the distillation of subcooled liquid (or superheated vapor) feed. There is also the problem of determining the optimum location of the feed stage in this graphical method.

A revision of the distillation theory is presented in this paper which is free from such weaknesses. It is demonstrated that this revised distillation theory is applicable to graphical binary distillation as well as to the analytical stage-to-stage calculation method, the Ponchon-Savarit method and the short-cut approximate method of binary and multi-component distillations.

Considering the fact that the theory of distillation can be also used in other separation processes such as absorption, stripping, leaching and extraction the present revised theory could have widespread applicability in such other separation processes.

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INTRODUCTION

The classical McCabe-Thiele graphical method^[1-3] for analysis of binary distillation was developed in 1925. From then on, this method has been used as a fundamental and simple technique to calculate the number of theoretical stages in a distillation column. In this paper we point out the weaknesses associated with this graphical method and we propose a revised graphical distillation calculation method.

THE McCABE-THIELE GRAPHICAL METHOD

A continuous fractionating distillation column is shown in Figure 1. According to constant-molal-overflow (CMO) assumption ($V_1 = \dots = V_{n-1} = V_n = V$, $\bar{V}_1 = \dots = \bar{V}_{m-1} = \bar{V}_m = \bar{V}$), Where V and \bar{V} are the flow rates of vapor in the enriching and stripping sections of the column, respectively, V_n is a general stage in the enriching section, and V_m is a general stage in the stripping section. Similarly for liquid flow rates ($L_1 = \dots = L_{n-1} = L_n = L$, $\bar{L}_1 = \dots = \bar{L}_{m-1} = \bar{L}_m = \bar{L}$) where L and \bar{L} are the flow rates of liquid in the enriching and stripping sections, respectively). It is also assumed that the column is adiabatic and steady state and the pressure is constant throughout the column.

One then can write the following overall total and component material balance equations for a binary mixture distillation.

$$F = D + W \tag{1}$$

$$F x_F = D x_D + W x_W \tag{2}$$

where F , D , W are feed, overhead, and bottom flow rates and x_F , x_D , x_W are the related compositions of the more volatile component of the binary mixture, respectively.

The material balances over the enriching section of the column are:

$$V = L + D \tag{3}$$

$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D \tag{4}$$

By introducing the reflux ratio $R = \frac{L}{D}$, and knowing that $\frac{L}{V} = \frac{L}{L+D} = \frac{R}{R+1}$ and $\frac{D}{V} = \frac{1}{R+1}$, eq. (4) becomes:

$$y_{n+1} = \frac{R}{R+1}x_n + \frac{1}{R+1}x_D \quad (5)$$

This is known as the equation of the operating line for the enriching section. Similarly, the material balances for the stripping section for the column are:

$$\bar{V} = \bar{L} - W \quad (6)$$

$$y_{m+1} = \frac{\bar{L}}{\bar{V}}x_m - \frac{W}{\bar{V}}x_W \quad (7)$$

Equation (7) is the equation of the operating line for the stripping section.

The feed types is characterized by using a factor q , defined as the number of moles of liquid flow in the stripping section that results from the introduction of each mole of feed, i.e.,

$$q = (\bar{L} - L)/F \quad \text{or} \quad \bar{L} = L + qF \quad (8)$$

Similarly it can be shown that,

$$1-q = (V - \bar{V})/F \quad \text{or} \quad V = \bar{V} + (1-q)F \quad (9)$$

In order to locate the intersection of the operating lines of the enriching and stripping sections, let $y_{n+1} = y_{m+1} = y$, $x_n = x_m = x$. Then from eq's (5) and (7) and the definition of q we get:

$$y = \frac{q}{q-1}x - \frac{1}{q-1}x_F \quad (10)$$

This is known as the equation of the feed line.

The two operating lines and the feed line are plotted on the x-y diagram as shown by Figure 2. The number of theoretical stages can be determined by using these lines and the equilibrium line. The step by step construction can start at either the bottom of the stripping section operating line or the top of the enriching section operating line. Assuming that we start at the top of the enriching section operating line, we draw triangular cascades between the operating line and the equilibrium curve. Each of these triangles represents a theoretical stage. In order to get a minimum number of theoretical stages, we switch from enriching operating line to stripping operating line when the triangle straddles the intersection of the operating lines. By using this method, the number of theoretical stages is defined.

Minimum reflux ratio: The minimum reflux is obtained when the feed line, equilibrium

curve and the operating line of the enriching section meet at one point.

The minimum reflux is the minimum allowable reflux for the separation. For saturated feed, the operating lines for different values of reflux is shown in Figure 2. The slope of the enriching-section operating line is $R/(R+1)$. For the four different cases of operating lines demonstrated on Figure 2 the slope, $R/(R+1)$, decreases as we move from 1 to 4 ($R_1 > R_2 > R_3 > R_4$). For a case like number 4, where the two operating lines cross each other on the left side of the equilibrium curve, distillation separation can not be achieved. So the lowest value of R (minimum reflux ratio= R_3) is where the two operating lines meet with the equilibrium curve. As a result the minimum reflux ratio can be calculated from the following equation

$$\frac{R_{\min}}{R_{\min} + 1} = \frac{x_D - y_f}{x_D - x_f} \quad \text{or} \quad R_{\min} = \frac{x_D - y_f}{y_f - x_f} \quad (11)$$

where x_f and y_f are concentrations at the feed stage. The above review is needed in order to discuss the shortcomings of the McCabe-Thiele method and present a revised graphical method.

SHORTCOMINGS OF THE McCABE-THIELE METHOD

A number of shortcomings exist in the McCabe-Thiele method when it is applied to the cases of subcooled (or superheated) feed:

1. When the feed is a subcooled liquid (or a superheated vapor), on the assumption that x_F , x_B , x_D , L and D are all constants, the number of theoretical stages tends to a minimum according to the McCabe-Thiele method, which is not correct. As an example in Figure 3a the McCabe-Thiele graphical method is demonstrated when the feed is a subcooled liquid (when the temperature of the feed drops, the slope of the operating line will be close to the diagonal line of the x - y diagram). According to this figure, the number of theoretical stages (N_{\min}) tends to a minimum. Meanwhile, the number of theoretical stages (N_n) for the enriching section tends to zero. Similarly, when the feed is superheated, the slope of the operating line will be very close to the diagonal line, the number of theoretical stages again tends to a minimum, and the number of theoretical stages (N_m) for stripping section tends to zero, as shown in Figure 3b. Since the separation process starts only when the feed is saturated, the more the thermal state of feed deviates from the saturated state, a larger number of theoretical stages will be needed. The McCabe-Thiele graphical method does not indicate this fact.

2. The McCabe-Thiele method sometimes predicts negative minimum reflux ratio which is physically meaningless. For example, according to Figure 3c, for the more volatile component in a binary mixture, the concentrations of feed, bottom product and overhead product are, respectively, $x_F=0.50$, $x_B=0.10$, $x_D=0.90$. When feed is a subcooled liquid, the coordinates of the intersection of feed line and the equilibrium

curve are $x_q = 0.83$, $y_q = 0.93$. The minimum reflux ratio can be calculated from this data (Figure 3c)

$$R_{\min} = \frac{x_D - y_q}{y_q - x_q} = \frac{0.90 - 0.93}{0.93 - 0.83} = -0.3$$

which is negative and meaningless.

According to the McCabe-Thiele method the limits of the minimum reflux ratio are

$$-\infty \leq R_{\min} \leq \infty$$

which is also physically meaningless. These limits are derived when the feed is either a superheated vapor or a subcooled liquid (when the enriching section operating line coincides with the diagonal line on the x-y diagram).

3. Some conceptual errors will appear in the derivation of q-line, equation (10), when the feed is a subcooled liquid (or a superheated vapor): The equation of the feed line is derived by combining the equations of the two operating lines. The McCabe-Thiele method uses the hypothesis that the intersection of the two operating lines always exists. This is not the case for a subcooled liquid (or a superheated vapor) feed because the feed stage is not a theoretical stage in these cases. There must be a heating (or cooling) section to bring the feed to its saturated state in order to start the separation process. The stages of heating (or cooling) section are only heat-exchange stages, so neither of these two operating lines can control this section. Apparently the operating lines of the enriching and stripping sections will not meet together when the feed is superheated (or subcooled).

A NEW GRAPHICAL METHOD

In this section we propose a new graphical method which is free from the shortcomings discussed above. To do this we first present the relationship between T-x,y and x-y diagrams needed in this new technique. Then we will define a new factor (K) which replaces the q factor used in the McCabe-Thiele method. Also, the operating lines and feed line equations will be revised by using the K factor. This new technique will be applicable for all cases of feed conditions whether saturated, superheated, or subcooled.

Relationship between T-x,y and x-y diagrams: In the new graphical method we define a graphical relationship between T-x,y and x-y diagrams. Figure 4a demonstrates how we relate x-y and T-x,y diagrams. For a given point on one diagram, we can find its

corresponding point on the other diagram. For example we define points M', N' corresponding to points M, N for the cases of subcooled and superheated feed, respectively. Figures 4b and 4c show how to draw points M' and N' on the x-y diagram corresponding to points M and N that represents the same state on the T-x,y diagram. Point M represents a subcooled liquid state and N represents a superheated vapor state. Provided points M' and N' are the feed conditions then lines (eM' for Figure 4b and eN' for Figure 4c) will represent the feed lines in the two cases of subcooled and superheated feed.

Definition of the Phase change line : In the T-x,y diagram (at a given pressure) the two saturated lines divide the diagram into three parts (subcooled, saturated, and superheated sections). For a given feed composition, x_F , the feed temperature could be at T_c (subcooled feed), T_b (feed at bubble point), T_d (feed at dew point), or T_h (superheated feed). For points M, J, H and N on the T-x,y diagram we locate the corresponding points M', J', H' and N' on the x-y diagram. The straight line \overline{AMJHN} and the path $\overline{A'M'J'H'N'B'}$ are called *phase change line* and *phase change path*, respectively.

Material balance and operating lines for two component systems: For a continuous fractionating distillation column, Figure 1, the same material balance equations described in the McCabe-Thiele method are still valid. However, in order to characterize the feed types, we define a new factor K as:

$$K = \frac{\text{number of moles of liquid in the feed}}{\text{number of moles of the feed}}$$

K will have different numerical values with respect to the thermal conditions of the feed as it is shown in Table 1

Table 1. The values of K with different feed types.

feed condition	phase	K
Subcooled liquid	Liquid	=1
Liquid at its bubble point	Vapor and liquid	≈1
Vapor-liquid mixture	Vapor and liquid	= 0~1
Vapor at its dew point	Vapor and liquid	≈0
Superheated vapor	Vapor	=0

The material balance over the enriching section of the column, Figure 1, will give the operating line equation for the enriching section:

$$x_n = \frac{R+1}{R}y_{n-1} - \frac{1}{R}x_D \quad (12)$$

Considering that the contribution of feed to the internal flow of liquid is Kx_F , the following mass balance equation (instead of eq. 8) will hold in the stripping section:

$$\bar{L}_m = L_n + KF \quad (13)$$

By substituting equation (13) and (6) into (7) and with the assumption of constant molar overflow, we derive a new operating line equation in the stripping section as the following

$$y_m = \frac{L + KF}{L + KF - W}x_{m-1} - \frac{W}{L + KF - W}x_W \quad (14)$$

Similarly considering that the contribution of feed to the internal flow of vapor is $(1-K)F$, the following mass balance equation (instead of eq. 9) holds in the enriching section:

$$V = \bar{V} + (1 - K)F \quad (15)$$

In order to find the intersection of the two operating lines we set $y_{n+1}=y_{m+1}=y$ and $x_n=x_m=x$. Then from eq's (5) and (15) and the definition of K we get:

$$y_K = \frac{K}{K-1}x_K - \frac{1}{K-1}x_F \quad (16)$$

This is the new feed line equation. We then plot the feed-line on the x - y phase diagram for various feed conditions as shown in Figure 5:

1. For a subcooled liquid feed, $K=1$, the feed-line (or K -line) is line eM' .
2. For feed at its bubble point, $K \approx 1$, the feed-line (or K -line) is line eJ' .
3. For feed consisting of a mixture of liquid and vapor, $0 < K < 1$, the
4. For feed at its dew point, $K \approx 0$, the feed-line (or K -line) is line eH' .
5. For a superheated vapor feed, $K=0$, the feed-line (or K -line) is line eN' .

Minimum reflux ratio: The minimum reflux is again obtained when the feed line, equilibrium curve and the operating line of the enriching section meet at one point. Line SI' is the operating line of the enriching section. (point S is the intersection of the vertical line at x_D and the diagonal line). According to Eq. (11) the slope of line SI' is $R/(R + 1)$. For all the possible feed conditions, the slope of the feed-line would vary between zero and infinity. Thus the range in which line SI' can vary is in between lines SH' and SJ' .

Slope of line SJ' gives us $R_{\min,b}$ the minimum reflux ratio at the bubble point and slope of line SH' gives us $R_{\min,d}$ the minimum reflux ratio at the dew point. Since lines SJ' and SH' are the extremes of the enriching section operating line then the minimum reflux ratio will have finite bounds (shown on Figure 5),

$$R_{\min,b} \leq R_{\min} \leq R_{\min,d}$$

which is physically meaningful. This is not the case for the McCabe-Thiele method,

Concentrations at the feed stage: In order to calculate the minimum reflux ratio from equation $R_{\min} = \frac{x_D - y_f}{y_f - x_f}$, we need to find the concentrations of vapor and liquid phases at the feed stage.

(i) In the case when the feed is at its bubble point the feed stage compositions are:

$$x_f = x_F \quad \text{and} \quad y_f = \frac{\alpha x_F}{1 + (\alpha - 1)x_F}$$

where the equation for y_f is derived from the definition of the relative volatility,

$$\alpha = [y(1-x)]/[x(1-y)].$$

(ii) In the case when the feed is a liquid - vapor mixture we can calculate concentration at the feed stage by using the relative volatility definition and the K-line equation, Eq. (16) to get

$$x_f = \frac{(\alpha - 1)x_F + \alpha(K - 1) - K + \sqrt{[K - (\alpha - 1)x_F - \alpha(K - 1)]^2 + 4K x_F(\alpha - 1)}}{2K(\alpha - 1)} \quad \text{and}$$

$$y_f = \frac{\alpha x_f}{1 + (\alpha - 1)x_f}$$

(iii) In the case when the feed is at its dew point the feed stage compositions are:

$$x_f = \frac{y_F}{a - (a - 1)y_F} \quad \text{and} \quad y_f = y_F$$

At the minimum reflux ratio, when the two operating lines and the phase equilibrium curve meet at one point the number of theoretical stages becomes infinity. There will be an invariant zone at the bottom of the enriching section and the top of the stripping section.

(iv) In the case when the feed is a subcooled liquid the minimum reflux ratio of

distillation column will be controlled by the enriching section operating line. At the minimum reflux, there will be an invariant zone at the bottom of the enriching section, so the feed line and enriching section operating line will meet the phase equilibrium curve at one point. Therefore, the calculation of minimum reflux ratio will be the same as with saturated liquid feed. The concentrations of the vapor entering the enriching section and the liquid entering the stripping section are the same as when the feed is at its bubble point.

(v) In the case when the feed is a superheated vapor the reflux ratio will be controlled by the operating line of the stripping section. At the minimum reflux, there will be an invariant zone at the top of the stripping section. So the feed line and the stripping section operating line will meet the phase equilibrium curve at one point. Therefore, the calculation of minimum reflux ratio will be the same as that with saturated vapor feed. The concentrations of the vapor entering the enriching section and the liquid entering the stripping section are the same as when the feed is at its dew point.

PROPOSED REVISED GRAPHICAL METHOD

In this section, a graphical method for the revised McCabe-Thiele method is presented where we also define the concentrations of liquid and vapor, and the temperatures at each theoretical stage by using the x-y diagram along with the T-x,y diagram. Here again we assume constant-molal-overflow (CMO) for a steady state column operating adiabatically and at constant pressure.

Feed at its Saturated State:

On the x-y diagram, Figure 6b, we plot the feed-line using Eq. (16) with $0 < K < 1$, the operating line of the enriching section, Eq. (7), and the operating line of the stripping sections, Eq. (14). Then, we draw cascades starting from the intersection of the feed line and the equilibrium curve towards, both, the enriching and stripping sections. The stage which covers the intersection of the feed-line and phase equilibrium curve corresponds to the feed stage and also the first stage of the stripping section. The stages above the feed-line are for the enriching section, and the ones below the feed-line are for the stripping section.

On Figures 6a and 6c the revised cascade method is presented when the feed is at its bubble point ($K \approx 1$) and its dew point ($K \approx 0$), respectively.

Feed at its Subcooled Liquid State:

In this case the feed temperature is below its bubble point ($T_F < T_b$). Distillation process will begin only when the feed is heated up to its bubble point temperature. For this a heating section is needed in the column located below the feed entrance and above the stripping section. As a result the enriching and stripping sections will be separated. Then the two operating lines will no longer meet together. In what follows

we will first present the calculation procedure for the number of heating stages and then we discuss the graphical method for calculation of the number of theoretical stages in the enriching and stripping sections.

Calculation of the heating requirements: This is performed by using the following equation:

$$\dot{Q}_h = FC_{PL}(T_b - T_F)$$

where \dot{Q}_h = rate of heat transfer at heat exchange stages, F = molar flow rate of feed, C_{PL} = heat capacity of liquid at constant pressure, T_b = bubble point temperature, and T_F = feed temperature. No matter what kind of heat transfer is involved in the heating section, we can always expect the heat flux to be proportional to a driving force. In this case, the driving force is the the temperature difference of a cold liquid and a hot vapor. For a hypothetical total heat exchange area, A_c , we can write

$$A_c = \frac{\dot{Q}_c}{K_c \Delta T_m}$$

where K_c = local overall heat-transfer coefficient (determined empirically) and

$$\Delta T_m = \frac{T_2 - T_1}{\ln(T_2/T_1)}$$

is a logarithmic mean temperature difference. One may define $\Delta T_1 = T_{ho} - T_{ci}$ and $\Delta T_2 = T_{hi} - T_{co}$, where T_{hi} = vapor temperature entering heating section from stripping section, T_{ho} = temperature of the liquid which is condensed from vapor temperature entering heating section from stripping section, T_{co} = temperature of a mixture made up of feed and liquid entering heating section from enriching section

which is approximated as $T_{co} = \frac{FT_F + L_n T_{ci}}{F + L_n}$, T_{ci} = temperature of liquid leaving the enriching section (see Fig. 7b). From this one can calculate the number of heating stages using the following equation

$$N_c = \frac{A_c}{A_a}$$

where N_c = number of heating stages and A_a = effective heat transfer area for each heat transfer stage determined empirically. The more the feed condition deviates from the saturated state, a larger number of heating stages will be needed represented by a longer length of line J'M' on Figure 7a.

Enriching and stripping sections: On the x-y diagram, Fig. 7a, we plot a vertical feed-line ($K=1$) and the operating lines for the enriching, Eq. (12), and stripping, Eq. (7),

sections. In Eq. (7) we replace \bar{L} and \bar{V} with the following expressions

$$\bar{L} = L + F + L_m' \quad (17)$$

$$\bar{V} = \bar{L} - W = L + F + L_m' - W \quad (18)$$

In the above expressions L_m' is due to condensation of vapor in the heating section due to subcooled feed. Eq. (17) is obtained by the material balance over the heating section shown in Figure 7b. L_m' is calculated by the following equation:

$$L_m' = \frac{\dot{Q}_c}{\lambda} = \frac{FC_L(T_b - T_F)}{\lambda}$$

where λ is the effective latent heat of evaporation for the vapor condensing in the heating section. Substituting Eq.s (17) and (18) in equation (7), we get the operating line equation in the stripping section:

$$y_{m+1} = \frac{L + F + L_m'}{L + F + V_m' - W} x_m - \frac{W}{L + F + V_m' - W} x_W$$

A representative figure corresponding to this case is shown by Figure 7a. The number of theoretical stages in the enriching section are determined by starting at the intersection point of the feed line and the operating line of the enriching section and stepping upwards in the enriching section. The number of theoretical stages in the stripping section are determined by starting at the intersection point of the feed line and the operating line of the stripping section and stepping downwards in the stripping section. For the case of Figure 7 the number of stages in the enriching section are 3 and in the stripping section are 5. The heating section is represented as the subcooled line AB on the T-(x,y) diagram and the subcooled line A'B' on the x-y diagram.

Feed at its Superheated Vapor State

In this case the feed temperature is higher than its dew point ($T_F > T_d$), distillation process will begin only when the feed is cooled down to its dew point temperature. For this a cooling section is needed in the column located above the feed entrance and below the enriching section. As a result the enriching section and the stripping section will be separated. Then the two operating lines will no longer meet together. In what follows we will first present the calculation procedure for the number of cooling stages and then we discuss the graphical method for calculation of the number of theoretical stages in the enriching and stripping sections.

Calculation of the cooling requirement: This is performed by using the following equation:

$$\dot{Q}_h = FCPV(T_F - T_d)$$

where \dot{Q}_h = rate of heat transferred at heat exchange stages, F = molar flow rate of feed, CPV = heat capacity of vapor at constant pressure, T_d = dew point temperature, T_F = feed temperature. Similar to the subcooled liquid feed, for a hypothetical total heat exchange area, A_h , we can write

$$A_h = \frac{\dot{Q}_h}{K_h \Delta T_m}$$

where K_h = overall heat-transfer coefficient, (which should be determined empirically)

and $\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$ is a logarithmic mean temperature difference. One may define $\Delta T_1 = T_{ho} - T_{ci}$ and $\Delta T_2 = T_{hi} - T_{co}$, T_{hi} = temperature of a mixture made up of feed and the vapor entering the cool section from the stripping section which is approximated as $T_{hi} = \frac{FT_F + V_m T_{ci}}{F + V_m}$, T_{ho} = temperature of the vapor leaving the stripping section, T_{ci} = temperature of the liquid entering the cooling section from the enriching section, T_{co} = temperature of vapor which is vaporized from the liquid entering the cooling section from the enriching section.

From this information one can calculate the number of cooling stages in the cooling section by using the following equation

$$N_h = \frac{A_h}{A_a}$$

where N_h = number of cooling stages and A_a = effective heat transfer area for each heat transfer stage which can be determined empirically. The more the feed condition deviates from the saturated state, the larger number of the cooling stages will be which is represented by a longer length of line C'D' in Figure 8a.

Enriching and stripping sections: On the x-y diagram, Fig. 8a, we plot a horizontal feed-line ($K=0$) and the operating line for the enriching, Eq. (12), and stripping, Eq.(7), sections. In Eq. (7) we replace \bar{L} and \bar{V} with the following expressions

$$\bar{L} = L + L'_n \tag{19}$$

$$\bar{V} = \bar{L} - W = L + L_n' - W \quad (20)$$

In the above expressions L_n' is due to evaporation of liquid in the cooling section due to superheated vapor feed. it can be calculated by the following equation:

$$L_n' = \lambda \frac{\dot{Q}_h = FC_v(T_F - T_d)}{\lambda}$$

where λ is an effective latent heat of evaporation for the liquid evaporating in the cooling section. In Eq. (12) we replace R with $(R_d + R_h)$, where $R_h = L_n'/D$. Eq. (19) is obtained by the material balance over the cooling section shown in Figure 8b. Substituting Eq.s (19) and (20) in eq. (7), we get the operating line equation in the stripping section:

$$y_m = \frac{L - L_n'}{L - L_n' - W} x_{m-1} - \frac{W}{L - L_n' - W} x_w$$

A representative figure corresponding to this case is shown by Figure 8a. For the case of Figure 8 the number of stages in the enriching section are 4 and in the stripping section are 4.

The heating section is represented as the subcooled line CD on the T-(x,y) diagram and the subcooled line C'D' on the x-y diagram.

REFERENCES

- [1]. McCabe, W.L. and Thiele, E. W., "Graphical Design of Fractionating Columns," *Ind. Eng. Chem.*, Vol. 17, pp605-612, 1925.
- [2]. Thiele, E. W., "Computation of Distillation," *ind. Eng. Chem.*, Vol. 25, PP289-95, 1932.
- [3]. McCabe, W. L., Smith, J. C. and Harriott, P., "Unit Operations of Chemical Engineering," New York, 4th Edition, McGraw-Hill Book Company, 1980.
- [4]. Ponchon, M., "Application of Graphs to the Study of Commercial Fractional Distillation," *Tech. moderne*, Vol. 13, p20, p55, 1921.
- [5]. Savarit, R., "Arts et Metiers", pp.65, 142, 178, 241, 266, 307, 1922.
- [6]. Fenske, M. R., "Fractionation of Straight-Run Pennsylvania Gasoline," *Ind. Eng. Chem.*, Vol. 24, pp482-485, 1932.
- [7]. Gilliland, E. R., "New Design Calculation for Multicomponent Rectification," *Ind. Eng. Chem.*, Vol. 27, pp260-265, 1935.
- [8]. Gilliland, E. R., "Multicomponent Rectification," *Ind. Eng. Chem.*, Vol. 32, p1220, 1940.
- [9]. Liddle, C.J., "Improved Shortcut Method for Distillation Calculations," *Chem. Eng.*, Vol. 75, P137, 1968.
- [10]. Underwood, A. J. V., "Design of Fractionating Columns for Complex Mixtures," *J. Soc. Chem. Ind.*, Vol. 52, p223-7T, 1933.
- [11]. Underwood, A. J. V., "Fractional Distillation of Ternary Mixtures," *J Inst. Petrol.*, Vol. 32, p598, 614, 1946.
- [12]. Underwood, A. J. V., "Fractional Distillation of Multicomponent Mixtures," *Chem. Eng. Prog.*, Vol. 44: p603, 1948.

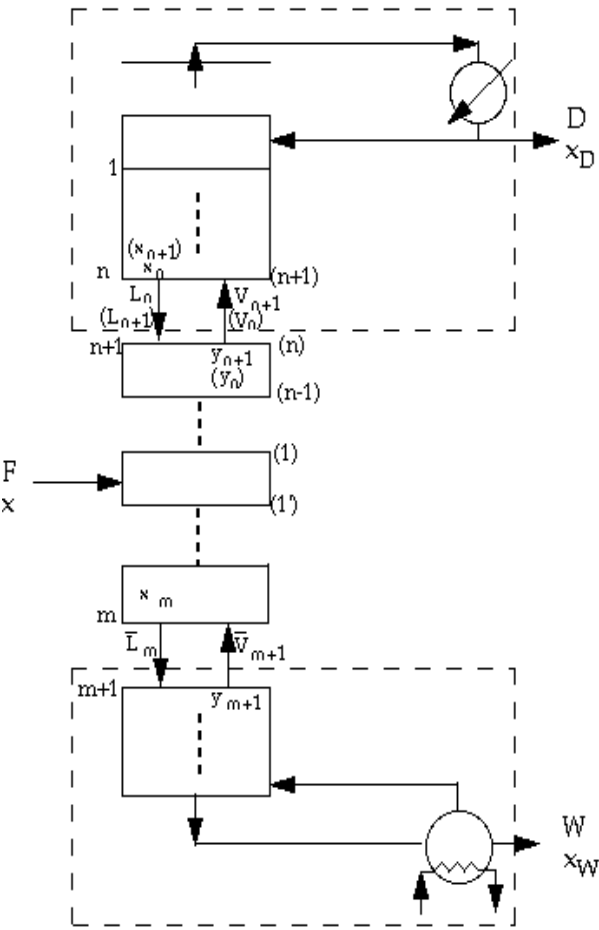


Figure 1. Material-balance diagram of continuous fractionating column for McCabe-Thiele method (the flow rates, liquid and vapor concentrations for the revised method are shown in the bracket).

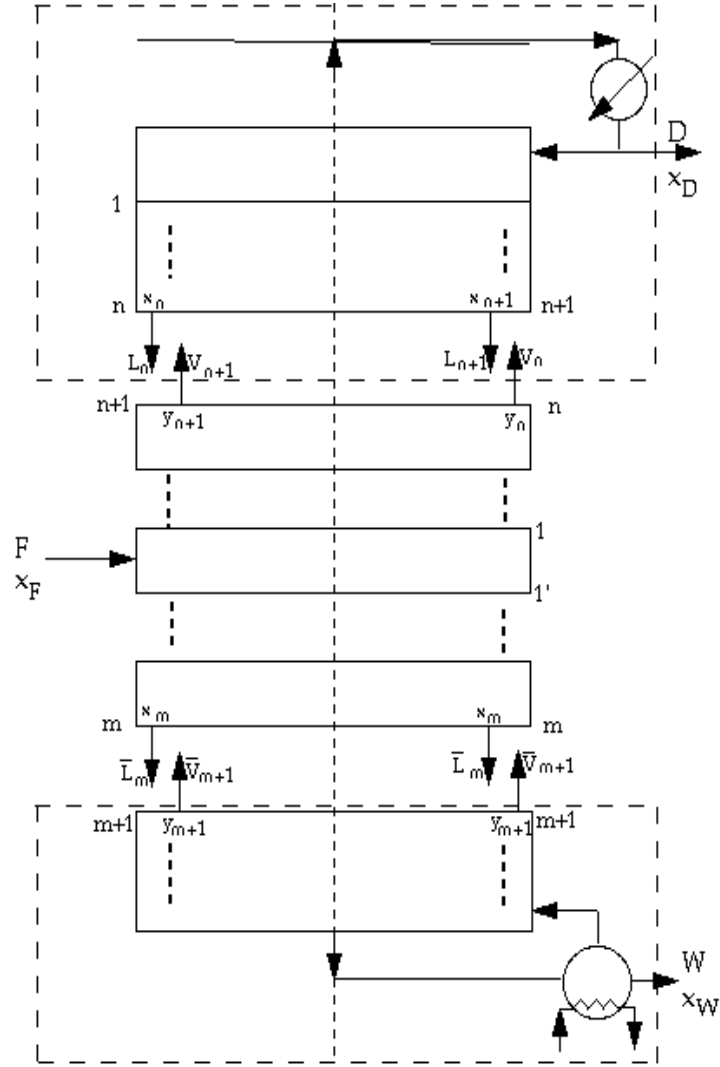


Figure 1. Material-balance diagrams of continuous fractionating column for McCabe-Thiele method (the left-side notations) and the revised method (the right-side notations).

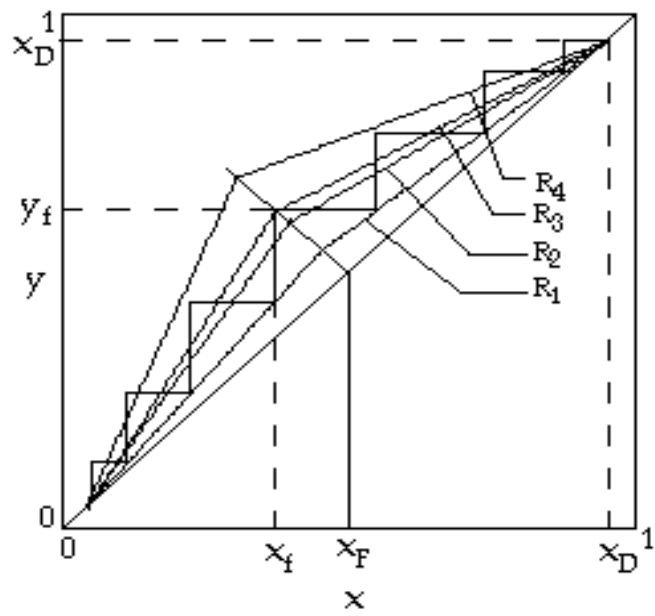
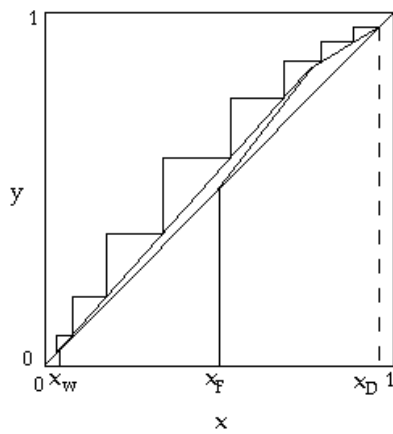
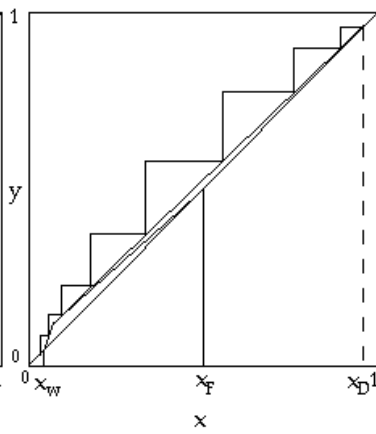


Figure 2. Reflux and McCabe-Thiele diagram.



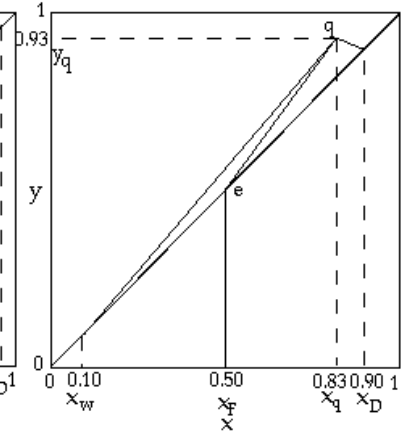
(a)

Figure 3a. McCabe-Thiele graphical calculation for a subcooled liquid feed.



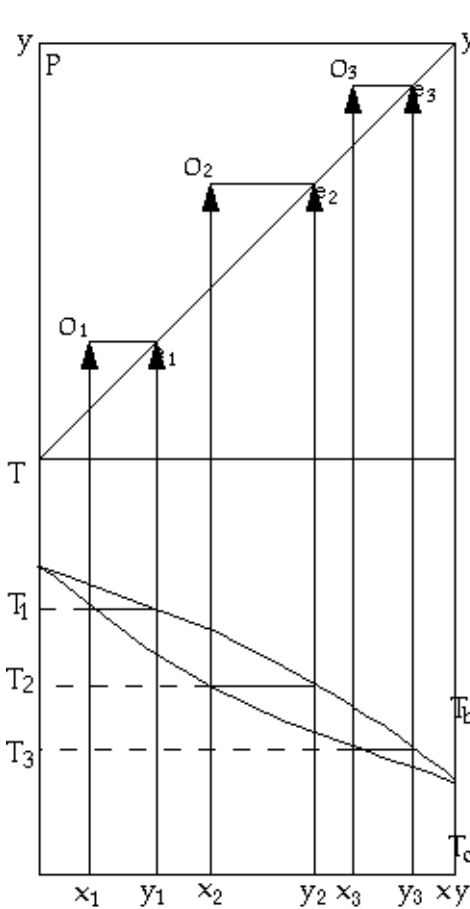
(b)

Figure 3b. McCabe-Thiele graphical calculation for a superheated vapor feed.



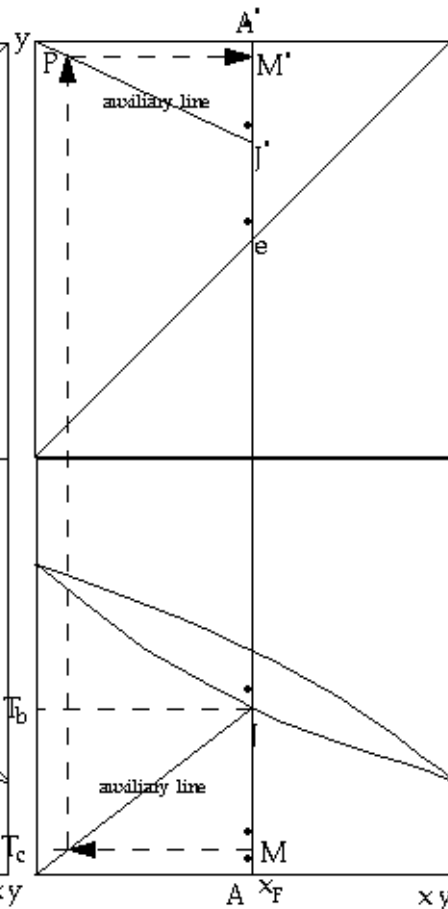
(c)

Figure 3c. Minimum reflux



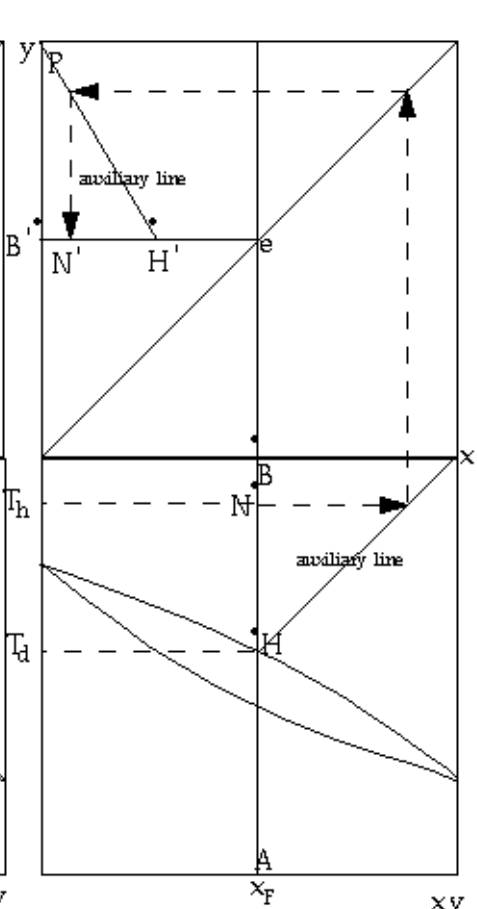
(a)

Figure 4a. T-x,y and x-y diagrams.



(b)

Figure 4b. T-x,y and x-y diagrams for a subcooled liquid feed.



(c)

Figure 4c. T-x,y and x-y diagrams for a superheated vapor feed.

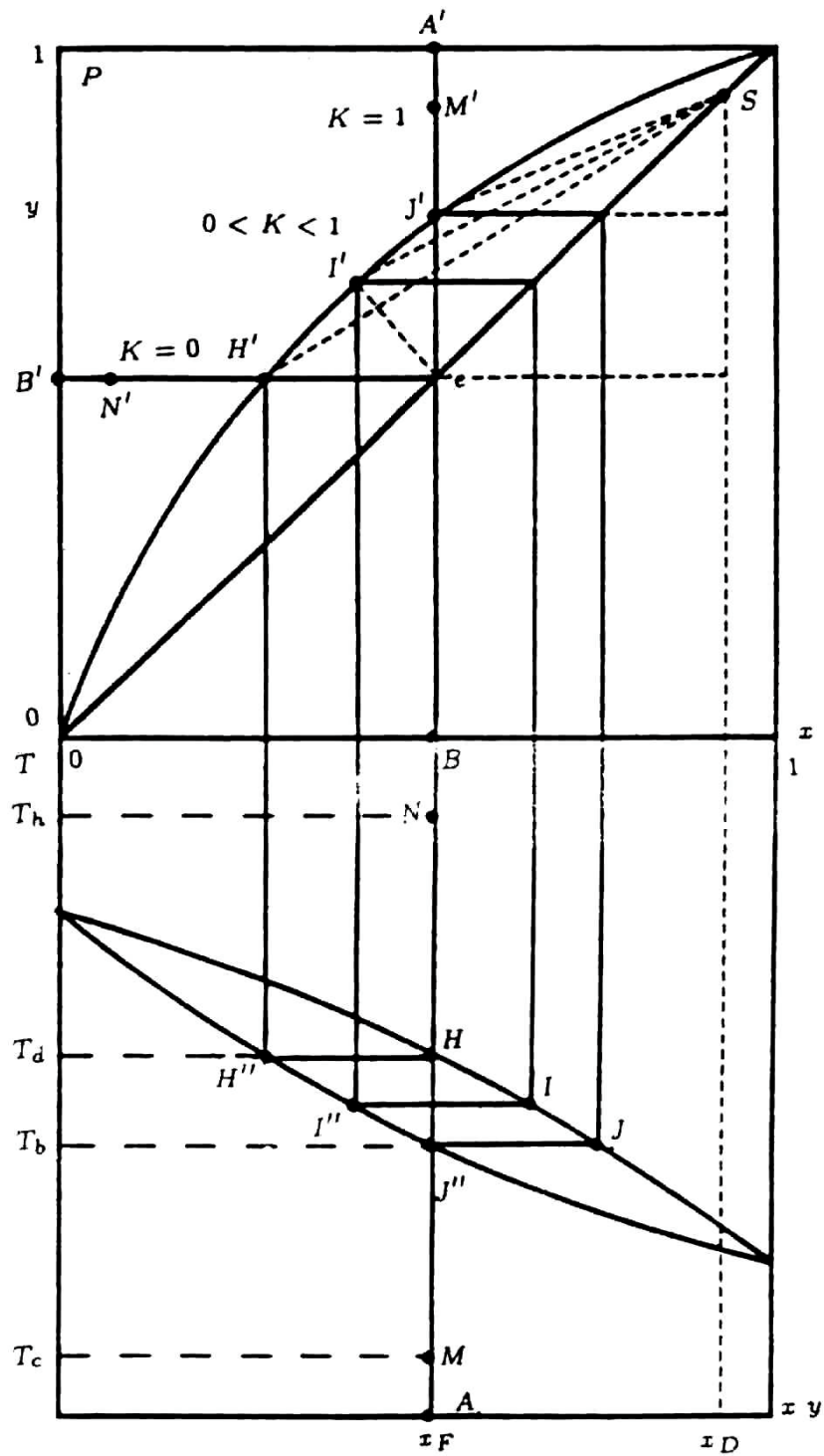


Figure 5. Range of minimum reflux ratio and phase change line of the feed on $T - x, y$ and $x - y$ diagrams.

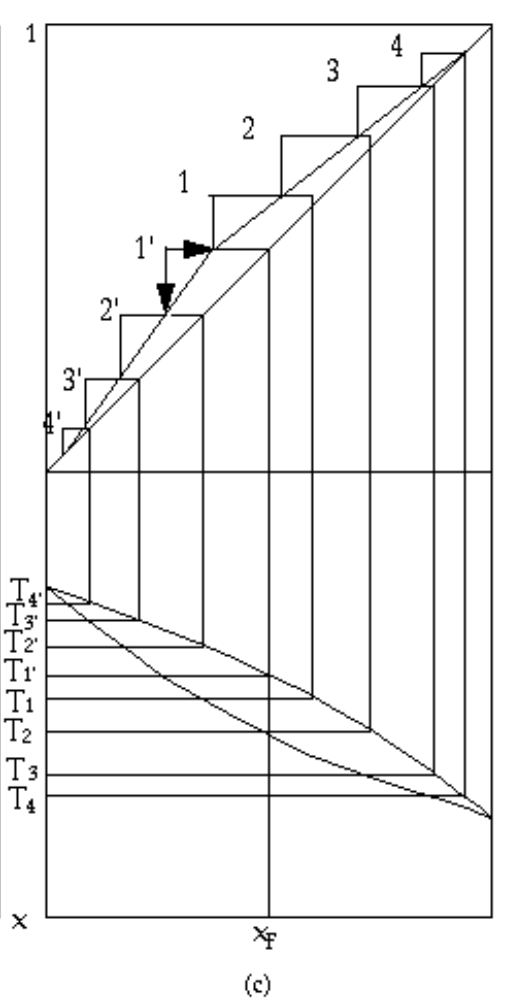
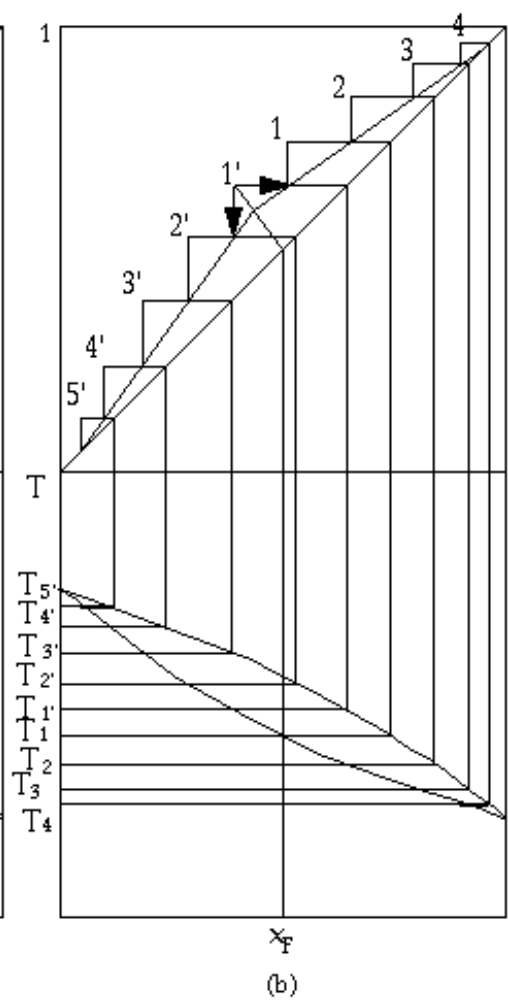
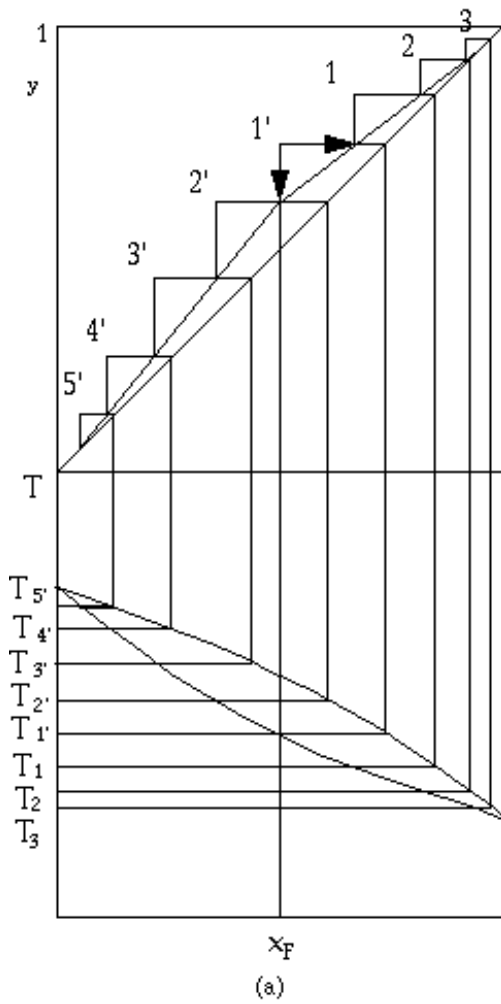


Figure 6a. T-x, y diagram and the revised graphical construction on the x-y diagram for the saturated liquid feed.

Figure 6b. T-x, y diagram and the revised graphical construction on the x-y diagram for the liquid-vapor mixture feed.

Figure 6c. T-x, y diagram and the revised graphical construction on the x-y diagram for the saturated vapor feed.

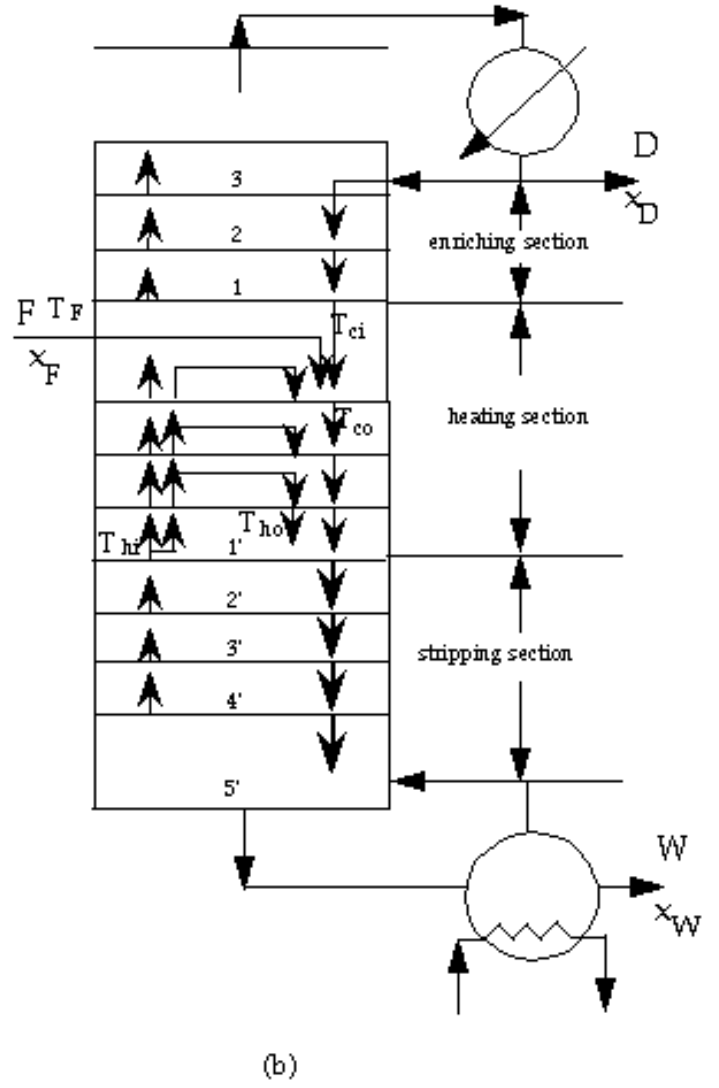
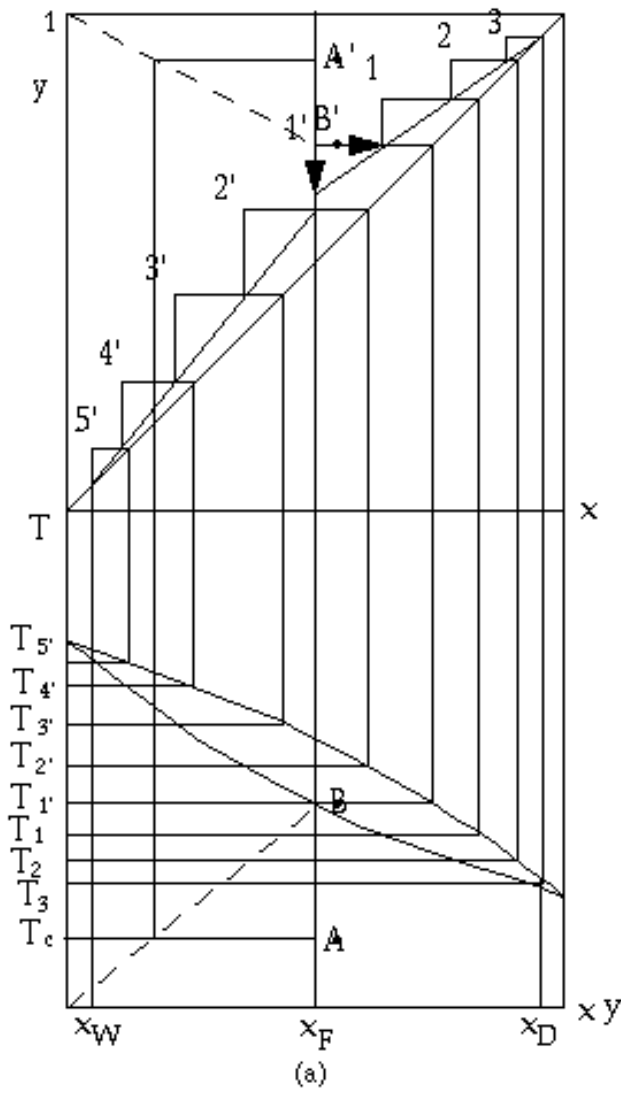


Figure 7. Proposed graphical method in the case of a subcooled liquid feed.

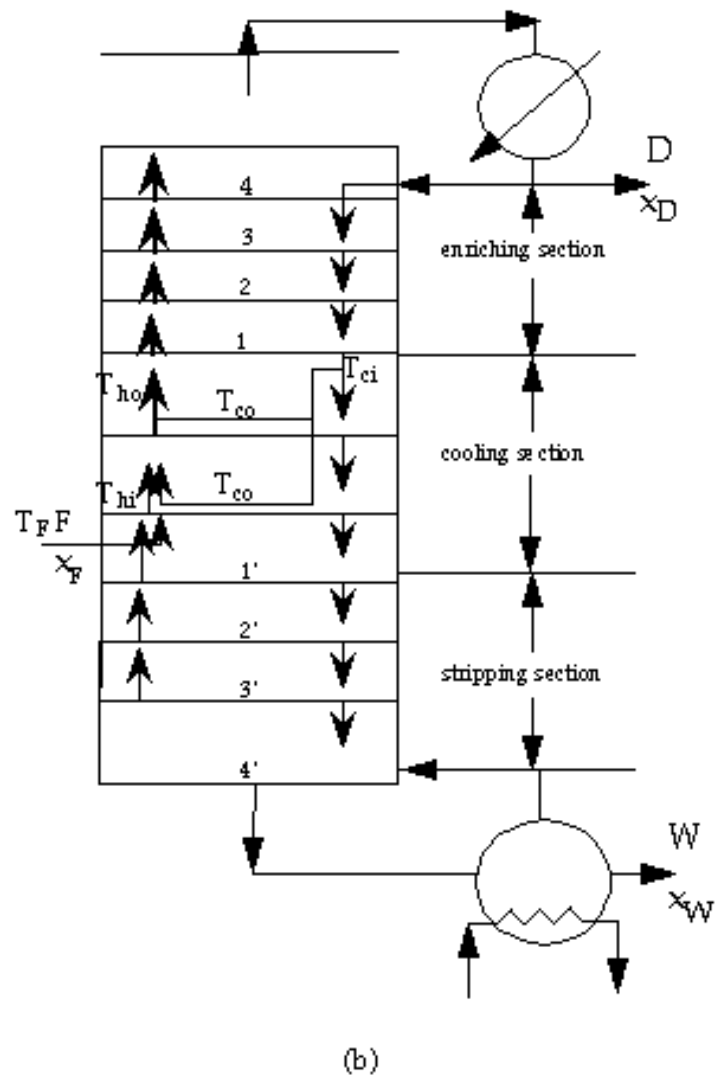
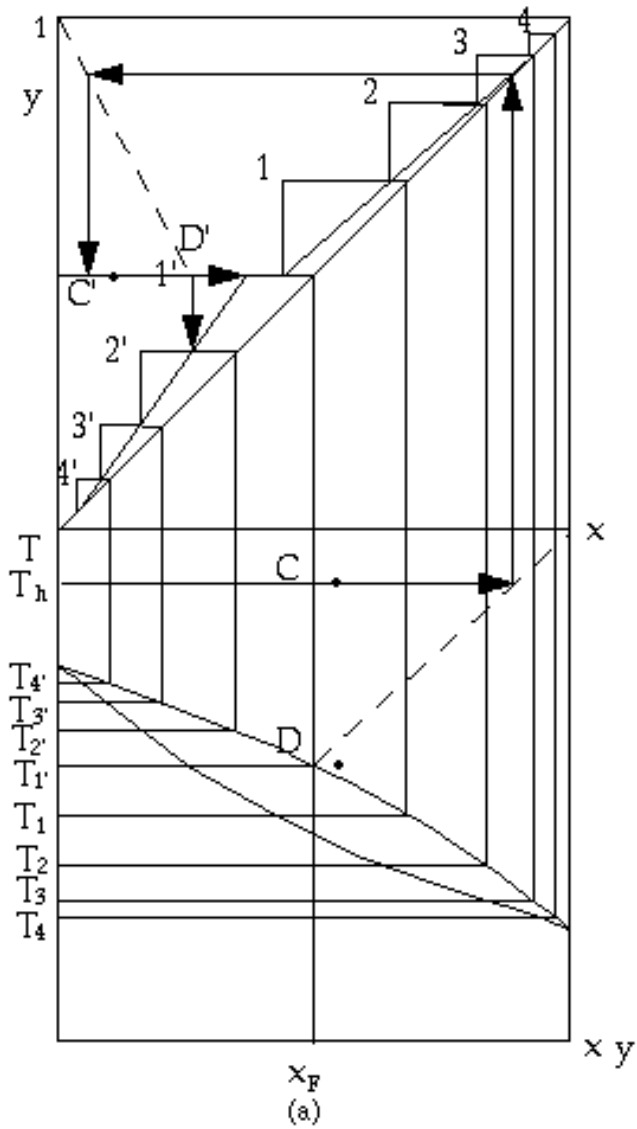


Figure 8. Proposed graphical method in the case of a superheated vapor feed.