

## Buried Gas Transmission Pipelines: Temperature Profile Prediction through the Corresponding States Principle

MOHSEN EDALAT  
G. ALI MANSOORI

Chemical Engineering Department  
University of Illinois  
Chicago, IL 60680

**Abstract** *A new analytic technique for the prediction of temperature profile of buried gas pipelines is developed based on the corresponding states principle. The resulting equations are tested through prediction of the actual experimental data. It is demonstrated that the new technique can predict temperature profile quite accurately without using any additional chart or table. This technique can also be used for prediction of gas mixture temperature profile flowing in a buried pipeline.*

Knowledge about the variation of temperature along a gas transmission pipeline and the capability of its prediction is of major importance in the design and operation of such pipelines. There exist a number of analytic equations that are used in gas industry to predict temperature profile in buried gas pipelines. The Schorre equation (Schorre 1954) is one that has been used widely. Forrest (1978) interpreted and modified the Schorre equation to show that the Joule-Thomson coefficient  $\mu$  varies with respect to distance. Coulter (1979) solved the energy equation by using the compressibility factor  $z$  and assuming that heat capacity at constant pressure  $C_p$  remains constant along the pipeline, in order to derive the temperature profile along gas pipelines.

While there has been extensive activity in the development of such equations, little attention has been paid to the fact that the Joule-Thomson coefficient and the heat capacity at constant pressure of a gas are not constants: They are functions of temperature as well as pressure. In the present report we introduce a new concept for the development of an analytical technique through the corresponding states principle, while considering the fact that  $\mu$  and  $C_p$  are functions of both temperature and pressure.

### Derivation of the New Analytic Technique

Differential formulation of the first law of thermodynamics has been considered for a control volume segment  $dx$  of a pipeline which is

$$A(dq_x/dx) + \rho v A(dh/dx) + q_r \mathcal{P} = 0 \quad (1)$$

where  $q_x$  is the axial heat conduction flux,  $h$  is the axial enthalpy flow rate, and  $q_r$  is the radial heat conduction flux. In derivation of Eq. 1, changes in kinetic and potential energies across the control volume are neglected. The first term in the above equation

represents the axial heat conduction, the second term represents the axial enthalpy flow, and the third term represents the radial heat conduction across the control volume.

The Peclet number is defined as the ratio of the axial enthalpy flow and the axial heat conduction. One can neglect the axial heat conduction when the Peclet number is very large, greater than 100 (Arpaci 1966). Generally, in natural gas transmission pipelines, as can be shown by any case study, the Peclet number is larger than 100. As a result, the axial conduction term may be neglected in such calculations.

The Biot number  $Bi$  is defined as the ratio of the internal and external resistance to heat transfer in a system. The Biot number for a gas transmission pipeline can be defined as

$$Bi = h_{\text{gas}} R_{\text{pipeline}} / K_{\text{solid}}$$

where  $h_{\text{gas}}$  is the heat transfer coefficient of the gas inside the pipeline,  $R_{\text{pipeline}}$  is the radius of the gas pipeline, and  $K_{\text{solid}}$  is the thermal conductivity of the solid surrounding the gas. Heat conduction of the natural gas can be considered radially lumped (Arpaci 1966) when the Biot number is less than 0.1. Study of heat transfer in buried natural gas transmission pipelines shows that the Biot number is always less than 0.1 and therefore the following relation is used for  $q_r$ :

$$q_r = U(T - T_g) \quad (2)$$

where the overall heat transfer coefficient  $U$  is chosen to be (Eckert and Drake 1972)

$$U = 2k/D[1/\ln(4z/D)] \quad (3)$$

The enthalpy of gas is generally a function of temperature and pressure and it is defined as

$$dh = C_p dT - \mu C_p dP \quad (4)$$

By substituting Eqs. 3 and 4 into Eq. 1 we can obtain

$$dT/dx + \xi(T - T_g) = \mu dp/dx \quad (5)$$

where  $\xi = \pi DU / (\rho v A C_p)$ . Furthermore, Eq. 5 can be reduced to a dimensionless form:

$$dq_r/dx^* + \xi^* q_r = \mu (P_c/T_c) (dP_c/dx^*) \quad (6)$$

where  $\xi^* = \xi D$ ,  $q_r = (T - T_g)/T_c$ , and  $x^* = x/D$ . The Joule-Thomson coefficient  $\mu$  is obtained from the following relation (Reid et al. 1977):

$$\mu C_p P_c / T_c = T_r d[B_m(T) P_c / RT_c] / dT_r - [B_m(T) P_c / RT_c] \quad (7)$$

where

$$B_m P_c / RT_c = (0.1445 + 0.073 w_m) - (0.33 - 0.46 w_m) T_r^{-1} - (0.1385 + 0.5 w_m) T_r^{-2} - (0.012 + 0.097 w_m) T_r^{-3} - 0.0073 w_m T_r^{-8} \quad (8)$$

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In the above expressions  $B_m$  is the gas mixture second virial coefficient,  $P_c$  and  $T_c$  are critical pressure and temperature, respectively, and  $w_m$  is the gas mixture acentric factor. By substituting Eq. 8 into Eq. 7 and upon linearization of the resulting expression one obtains

$$\mu C_p P_c / RT_c = -(0.338 + 1.302w_m) + (1.278 + 2.01w_m)T_r^{-1} + O(T_r^{-2}) \quad (9)$$

Since the second- and higher-order terms of the above expression have little or no effect on the calculation of the temperature profile they can be neglected for all practical purposes.

Heat capacity is generally a function of temperature and pressure (Reid et al. 1977)

$$C_p = C_p^0(T) + \Delta C_p(P, T) \quad (10)$$

where

$$C_p^0(T) = A_1 + A_2T + A_3T^2 + A_4T^3$$

and

$$\Delta C_p(P, T) = P_r R [(0.66 - 0.92w_m)T_r^{-2} + (0.83 + 3w_m)T_r^{-3} + (0.145 + 1.1w_m)T_r^{-4} + 0.526w_m T_r^{-9}]$$

where  $C_p^0(T)$  is the ideal-gas heat capacity and  $\Delta C_p(P, T)$  is not ideality deviation from ideal gas heat capacity. Equations 9 and 10 have been substituted into Eq. 6 and upon integration of the resulting equation, the following expression has been derived for the temperature profile:

$$\begin{aligned} T_2/(T_2 - T_1) \ln[(T - T_2)/(T_0 - T_2)] - T_1/(T_2 - T_1) \ln[(T - T_1)/(T_0 - T_1)] \\ = \pi D a / (m^0 b) \ln[a/(a + bx)] \end{aligned} \quad (11)$$

where

$$\begin{aligned} T_1 &= -c_1/2 + T_g/2 - [(T_g - c_1)^2/4 + c_2]^{0.5} \\ T_2 &= -c_1/2 + T_g/2 + [(T_g - c_1)^2/4 + c_2]^{0.5} \\ c_1 &= (0.34 + 1.30w_m)RT_c P_c (dP/dx) m^0 / (\pi D U) \\ c_2 &= (1.28 + 2.01w_m)RT_c^2 / P_c (dP/dx) m^0 / (\pi D U) \\ a &= a_1 + a_2 P \\ b &= a_2 (dP/dx) \\ a_1 &= C_p^0(\bar{T}) \\ a_2 &= DC_p(P, \bar{T})/p \end{aligned}$$

Equation 11 can be used to obtain the temperature in a pipeline at a specific distance. In case the temperature is known, Eq. 11 can be written explicitly with respect to distance in the following form:

$$\begin{aligned} x &= a/b \{ [\exp(m^0 b / \pi D a)] T_2 / (T_2 - T_1) \ln[(T - T_2)/(T_0 - T_2)] - T_1 / (T_2 - T_1) \\ &\quad \times \ln[(T - T_1)/(T_0 - T_1)]^{-1} - 1 \} \end{aligned} \quad (12)$$

This equation can then be used to locate a point in the pipeline that would possess a specified temperature.

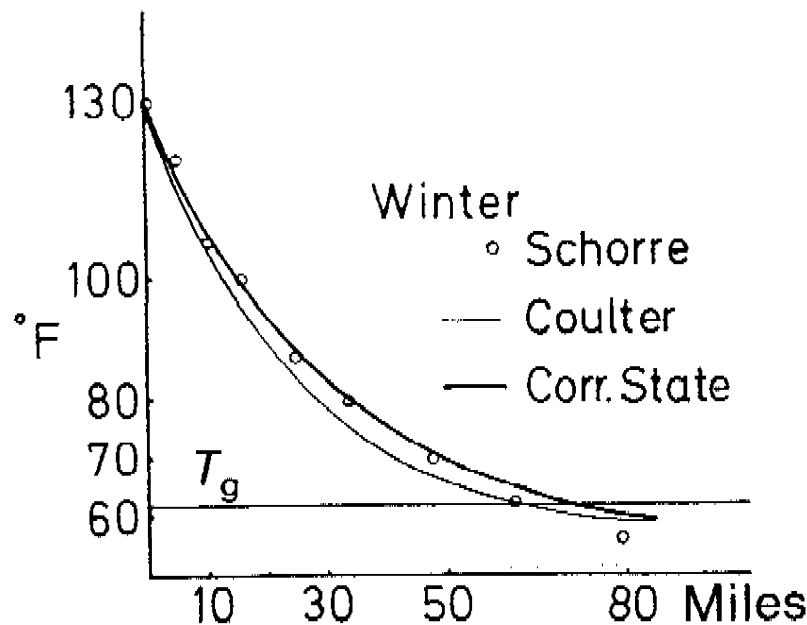
### Calculations and Results

Equation 11 is nonlinear with respect to temperature. To obtain temperature at a specific distance of a gas pipeline the Newton-Raphson (or any other) regression method can be used to solve this equation numerically. Equation 12 is used to calculate the distance for a specific temperature.

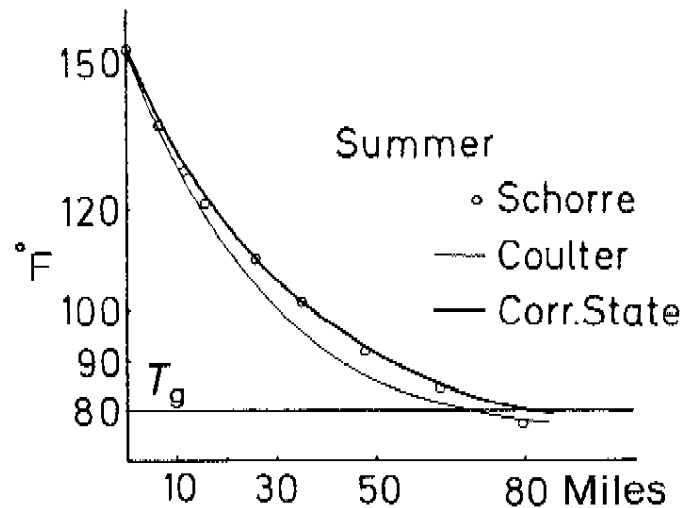
The proposed analytical technique can predict temperature profiles more precisely than the other existing techniques. Figures 1 and 2 show that for the same environmental conditions, the analytical technique presented here gives more accurate results than other existing techniques. In addition, since the present technique is based on the theorem of corresponding states, it is readily extended to the mixtures. Since the temperature profile prediction technique developed in this work is quite accurate it can be expected that one would be also able to predict the pressure drop in a gas pipeline accurately. For this purpose the present technique must be joined with an accurate gas-flow-rate equation.

### Nomenclature

$A$	Pipe cross section ( $m^2$ )
$A_1, A_2, A_3, A_4$	Ideal gas heat capacity coefficient
$B_m$	The gas mixture second virial coefficient
$C_p$	Heat capacity (cal/mol K)
$D$	Pipe diameter (m)
$h$	Enthalpy of fluid
$k$	Soil thermal conductivity (cal/s $m^2$ K)
$m^0$	Molar flow rate (mol/s)



**Figure 1.** Temperature profile in a buried gas pipeline as a function of distance considering a winter ground temperature  $T_g$  of 60 °F.



**Figure 2.** Temperature profile in a buried gas pipeline as a function of distance considering a summer ground temperature  $T_g$  of 80 °F.

$\mathcal{P}$	Pipe perimeter
$P_c$	Reduced pressure
$q_x$	Axial heat conduction flux
$q_r$	Radial heat conduction flux
$R$	Universal gas constant (cal/mol K)
$T$	Gas temperature (K)
$T_0$	Initial gas temperature (K)
$T_c$	Critical temperature (K)
$T_r$	Reduced temperature (K)
$T_g$	Ground temperature (K)
$\bar{T}$	Average temperature (K)
$U$	Overall heat transfer coefficient (cal/s m <sup>2</sup> K)
$v$	Velocity of fluid (m/s)
$x$	Pipe length (m)
$dp/dx$	Average pressure drop (atm/m)
$\rho$	Density of fluid (mol/m <sup>3</sup> )
$\mu$	Joule-Thomson coefficient (K/atm)
$w_m$	Acentric factor

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