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On Some Aspects of Diallel Cross Designs with Correlated Observations

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Abstract: The purpose of this article is to present some theoretical results for analysis of diallel cross designs in blocked situations with a possible correlation structure within each block.

Key words and phrases: Combining Ability Parameters, Complete Symmetry, Correlated Data, Diallel Crosses, Information Matrix, Linear Models, Parental Lines.

1. Introduction: Diallel Cross Designs (DCDs)

Diallel Cross Designs [DCDs] have been extensively studied in the literature during the last few decades. Complete and Partial Diallel Crosses have been studied from sampling and genetical experiment perspectives. Generally experiments are planned to accommodate all crosses in randomized blocks which make the block sizes too prohibitive. Therefore incomplete block designs as well as partial diallel crosses have been recommended. We point to Curnow (1963), Hinkelman (1975), Singh and Hinkelman (1990), Gupta and Kageyama (1994), Dey and Midha (1996) and Mukerjee (1997) for some references.

We assume that there are V parental lines $1, 2, \ldots, V$ and that a typical cross is denoted by $(i, j); 1 \leq i < j \leq V$. We do not differentiate between (i, j) and (j, i). As is well-known, the purpose of such experiments is to compare the parental lines with respect to their general combining abilities. A diallel cross experiment is said to be complete if it accommodates all $V(V-1)/2$ crosses; otherwise, it is partial. There has been a great deal of emphasis and study on optimal partial diallel crosses, in both unblocked and blocked situations.

The objective of this paper is to study diallel cross designs in blocked experiments, after introducing a natural correlation structure among the observations within each block and the implications on data analysis thereof.

In the next section [Section 2], we discuss an approach for generating diallel cross data within blocks under a linear fixed effects additive model without interactions and without any correlation structure. We also provide a general representation of the joint information matrix for the combining ability parameters of the parental lines. Next, in Section 3, we carry out the representation of the joint information matrix for blocks of sizes 3, 6 and 10, involving a correlated structure. We believe these results on representation are important for a study of optimality in the choice of designs, as well as for data analysis. In section 4, we compare two competing diallel cross designs using the same correlation structure as in the previous section for blocks of size 6. In section 5, we have some concluding remarks. Finally, all technical details of results are reserved for the Appendix in Section 6.

2. Information Matrix for DCDs in an uncorrelated model

In this section, we quickly review a well-known result for DCDs in an uncorrelated model. We only discuss about the nature of the joint information matrix for the general ability parameters of the parental lines. In a very general setting, suppose there are b blocks, with block sizes $k_1^*, k_2^*, .., k_b^*$. There are V parental lines to be compared in respect of their combining abilities, by forming diallel crosses involving them. We assume that each k_i^* has a representation of the form $k_i^* = k_i(k_i - 1)/2 < V(V - 1)/2$ where each k_i is an integer, $k_i < V$. For the i^{th} block, we select k_i distinct parents, $i_1, i_2, ..., i_{k_i}$ and assume that these parents form a *line*. Next, we develop this into diallel crosses of the type $(i_u, i_v), 1 \leq u < v \leq k_i$ and thereby generate k_i^* observations within the i^{th} block. Next we assume a linear model for the resulting data which we denote by $Y(i; i_u, i_v); 1 \leq u < v \leq k_i$:

$$Y(i; i_u, i_v) = \mu + \beta_i + \tau(i_u) + \tau(i_v) + e_{i_u, i_v}; 1 \leq u < v \leq k_i. \tag{2.1}$$

Taken over all the blocks, this is a linear model, additive in the combining ability parameters of the parental lines, denoted by $\tau_1, \tau_2, ..., \tau_V$. In block i, only k_i of these parameters are involved. Moreover, μ represents the “general mean” and β_i is the effect of the block i. Usually, it is assumed that the errors are uncorrelated and have the same variance σ^2. Throughout the paper, we will take $\sigma^2 = 1$. In such a case, the joint information matrix of the parental combining ability parameters for the design with b blocks is given by $I(\tau) = ((I_{(u,v)}; 1 \leq u \leq v \leq V))$ where

$$I_{(uu)} = \sum_i I(u,i)(k_i-1)(k_i-2)/k_i; I_{(uv)} = \sum_i I(u,i)I(v,i)(-1)(k_i-2)/k_i$$

where $I(u,i)$ is the usual indicator function relating to parental line u and block i. Specializing to proper block designs for which $k_i = k$, say we would end up with

$$I_{(uu)} = r_u(k-1)(k-2)/k; I_{(uv)} = (-1)\lambda_{u,v}(k-2)/k$$

where r_u is the over-all replication of parental line u and $\lambda_{u,v}$ is the joint incidence of the pair of parental lines (u,v) among the blocks of the design.

Remark 1. It also follows from the above representation that for proper block designs, the study of optimal cross-over designs may borrow its tools and techniques from such studies of optimal designs as are readily available in standard block design set-up; see Shah and Sinha (1989).

3. Information Matrix for DCDs in a correlated model

In this section, we propose to study the nature of the information matrix of the treatment effects when the errors within each block are assumed to be correlated. Consider, as before, diallel crosses formed of pairs like (i_u, i_v), thereby resulting in data of the type $Y(i_u, i_v)$.

3.1. Information Matrix for block size 3

Consider the case of block size 3 and take the parental lines to be 1, 2, 3. Then under diallel crosses, we have three observations $Y(1,2)$, $Y(1,3)$, $Y(2,3)$. Since between any pair of observations, one parental line is common, we may expect certain dependence in the responses. Thus we consider the same model as above but this time we assume a *common* correlation ρ between any two observations. [The situation will be different for block sizes more than 3]. Thus, under this setup, we have the following result.

Result 1. For DCDs with block size 3, the information matrix for 3 treatments, arising out of 3 correlated observations in one block, is given by

$$\mathrm{I}(\boldsymbol{\tau}) = \frac{1}{1-\rho}[I - J/3]. \tag{3.1}$$

Further to this, considering each of the b available blocks to be of size 3, it turns out that the joint information matrix for all V parental lines is given by

I(τ) with elements

$$\begin{aligned} I_{(uu)} &= 2(1-\rho)^{-1} r_u/3; \\ I_{(uv)} &= (-1)(1-\rho)^{-1}\lambda_{u,v}/3 \end{aligned} \tag{3.2}$$

where, as before, r_u is the over-all replication of parental line u and $\lambda_{u,v}$ is the joint incidence of the pair of parental lines (u,v) among the blocks of the design.

3.2. Information Matrix for block size 6

Now let us study the case of $k^* = 6$, i.e., $k = 4$. We consider the first block and take 1, 2, 3, 4 to be the parental lines, to start with. This time we have to deal with 6 observations arising out of 6 pairs of parental lines viz., Y(1,2), Y(1,3), Y(1,4), Y(2,3), Y(2,4), Y(3,4). However, we note that although $Y(1,2)$ is correlated with each of $Y(1,3)$, $Y(1,4)$, $Y(2,3)$, $Y(2,4)$, it is *not* correlated with $Y(3,4)$. Similarly, $Y(1,3)$ is not correlated with $Y(2,4)$ and the like. Hence in the postulated linear model $[Y, X\theta, \sigma^2 I]$, we need to replace I by the matrix $\Sigma(\rho)$ of order 6 defined by

$$\Sigma(\rho) = \begin{pmatrix} 1 & \rho & \rho & \rho & \rho & 0 \\ \rho & 1 & \rho & \rho & 0 & \rho \\ \rho & \rho & 1 & 0 & \rho & \rho \\ \rho & \rho & 0 & 1 & \rho & \rho \\ \rho & 0 & \rho & \rho & 1 & \rho \\ 0 & \rho & \rho & \rho & \rho & 1 \end{pmatrix}. \tag{3.3}$$

Under this setup, we have the following result:

Result 2. For DCDs with block size 6, the information matrix for 4 treatments, arising out of 6 correlated observations in one block, is given by

$$\mathrm{I}(\boldsymbol{\tau}) = 2[I - J/4]. \tag{3.4}$$

Details of the proof of this result are given in the appendix.

Remark 2. We find that for the case of block size 4 and diallel crosses with correlated errors, the information matrix for the treatment effects remains the same as the one without any correlation structure! It is now a routine task to form the over-all information matrix for all the V parental lines from b blocks, each of size 4, in terms of the design parameters r_u's and $\lambda_{u,v}$'s.

3.3. Information Matrix for block size 10

Next, we proceed to establish an analogous result for block size $k^* = 10$, i.e., $k = 5$. In continuation to the study of 4 parental lines, when an additional parental line is brought in, we will start with the observational sequence Y(1,2), Y(1,3), Y(1,4), Y(2,3), Y(2,4), Y(3,4) and then append Y(1,5), Y(2,5), Y(3,5), Y(4,5) to it. That way, the new $\Sigma(\rho)$ matrix of order 10 may be written in the form of a partitioned matrix as $\Sigma_{1,2}(\rho) = \begin{pmatrix} \Sigma_{1,1}(\rho) & \Sigma_{1,2}(\rho) \\ \Sigma_{2,1}(\rho) & \Sigma_{2,2}(\rho) \end{pmatrix}$ where $\Sigma_{1,1}(\rho)$ is the matrix of order 6 considered earlier. It follows from our assumptions of the correlation structure that

$$\Sigma_{1,2}(\rho) = \begin{pmatrix} \rho & \rho & 0 & 0 \\ \rho & 0 & \rho & 0 \\ \rho & 0 & 0 & \rho \\ 0 & \rho & \rho & 0 \\ 0 & \rho & 0 & \rho \\ 0 & 0 & \rho & \rho \end{pmatrix} \text{ and further that } \Sigma_{2,2}(\rho) = \begin{pmatrix} \rho & \rho & 0 & 0 \\ \rho & 0 & \rho & 0 \\ \rho & 0 & 0 & \rho \\ 0 & \rho & \rho & 0 \\ 0 & \rho & 0 & \rho \\ 0 & 0 & \rho & \rho \end{pmatrix}.$$

After tedious calculations (details are in the appendix), we have the following result:

Result 3. For DCDs with block size 10, the information matrix for 5 treatments, arising out of 10 correlated observations in one block, is given by

$$\mathrm{I}(\boldsymbol{\tau}) = \frac{3}{1+\rho}[I - J/5]. \tag{3.5}$$

Remark 3. It turns out that in each of the cases studied above,the information matrix under a correlated model is a multiple of that under uncorrelated model. We have also derived simple forms of the multipliers in each case. Writing $g(k, \rho)$ for the multiplier for block size $k^* = k(k-1)/2$, it turns out that the usual C-matrix for the correlated model under consideration is given by $C_{V,k,\rho} = g(k, \rho) C_{V,k,0}$ where the notations have their usual significance. This we have established for $k = 3, 4, 5$ along with derivation of the expressions $g(3, \rho) = (1-\rho)^{-1}, g(4, \rho) = 2, g(5, \rho) = 3(1+\rho)^{-1}$. It is clear that $g(3, \rho) \gtreqless g(3, 0)$ and $g(5, \rho) \lesseqgtr g(5, 0)$ according as $\rho \gtreqless 0$ while $g(4, \rho) = g(4, 0)$.

Remark 4. We have also examined this phenomenon for a correlated model arising out of triallel crosses involving 5 parental lines within each block, there being 10 triplets and we have assumed the following correlation structure: for one common treatment, correlation is ρ_1; for two common treatments, it is ρ_2. Thus writing the parental line combinations as

$(3,4,5);(2,4,5);(2,3,5);(1,4,5);(1,3,5);(1,2,5),(2,3,4);(1,3,4);(1,2,4);(1,2,3)$

the dispersion matrix Σ of order 10×10 is given by

$$\Sigma = \sigma^2[(1,\rho_2,\rho_2,\rho_2,\rho_2,\rho_1;\rho_2,\rho_2,\rho_1,\rho_1);...;(......,1)]$$

which has an interesting representation as $\Sigma = \sigma^2[(\Sigma_{11},\Sigma_{12});(\Sigma_{21},\Sigma_{22})]$ where $\Sigma_{11} = ((a,b,c))$, $\Sigma_{12} = [(\rho_2,\rho_2,\rho_1,\rho_1);(\rho_2,\rho_1,\rho_2,\rho_1);(\rho_2,\rho_1,\rho_1,\rho_2);(\rho_1,\rho_2,\rho_2,\rho_1);$ $(\rho_1,\rho_2,\rho_1,\rho_2);(\rho_1,\rho_1,\rho_2,\rho_2)]$ and $\Sigma_{22} = (1-\rho_2)I + \rho_2 J$. In the above, ((a,b,c)) has the same interpretation of a 6×6 matrix having "a" along the diagonals, "b" along all other positions except the anti-diagonals and "c" along the anti-diagonals. Further, we have here $a = 1, b = \rho_2, c = \rho_1$.

It is interesting to note the reason for specific arrangement of the triplets as we have indicated above. Since we may assume $\tau_1 + \tau_2 + ... + \tau_5 = 0$, under the above representation, the model expectation of $Y_{(345)}$ is given by $\mu + \beta + \tau_3 + \tau_4 + \tau_5 = \beta^* - \tau_1 - \tau_2$. Therefore, we end up virtually with the same arrangement as in the case of diallel crosses involving 5 parental lines, except for a different form of Σ.

We have seen that even under this general set-up, the information matrix for treatment effects is yet again proportional to $(I - J/5)$. We omit the details as the derivations are extremely lengthy.

Remark 5. We did not discuss explicitly the data analysis aspect of data arising out of such models. It follows that the **Q**-vector under the correlated case is also a multiple of the usual vector in the uncorrelated case and it has the *same* multiplier. Therefore, data analysis does not pose any additional problem. We need to pretend that we are in the uncorrelated case and go ahead with the usual data analysis. See Gupta and Kageyama (1994) and Mukerjee (1997), for example.

5. DCDs in Blocks of size 6

In a restricted setting, suppose there are b blocks, each of size 6. As usual, there are V "treatments" to be compared in respect of their combining abilities, by forming diallel crosses involving them. We start with 5 treatments and 5

blocks, each set being labeled as 1, 2, 3, 4, 5. Consider the following two options for allocation of the 5 treatments in this set of 5 blocks:
Option I : Use all but treatment number 6-i in block i and utilize all 6 pairwise crossings within each block;
Option II : Use all treatments in each block but select 6 suitably matched crossings for the blocks one by one.

For Option I, the data analysis is already indicated in Section 3. We know the form of the Information Matrix therein and it turns out that it is completely symmetric and that it is independent of the correlation structure imposed by our model between pairs of crossings. For ready reference, we note that the information matrix for the above reference set of 5 treatments in 5 blocks is $I_1(\tau) = (15/2)(I - J/5)$.

As to the Option II, we will consider the following design layout:

$$\begin{aligned}
&Block1 : (1,2); (1,3); (1,4); (1,5); (2,3); (4,5);\\
&Block2 : (2,1); (2,3); (2,4); (2,5); (1,4); (3,5);\\
&Block3 : (3,1); (3,2); (3,4); (3,5); (1,5); (2,4);\\
&Block4 : (4,1); (4,2); (4,3); (4,5); (1,3); (2,5);\\
&Block5 : (5,1); (5,2); (5,3); (5,4); (1,2); (3,4).
\end{aligned} \tag{5.1}$$

So far as uncorrelated model is concerned, our study reveals that the information matrix under the second option is given by $I_2(\tau) = (25/3)(I - J/5)$, while that for Option I is given by $I_1(\tau) = (15/2)(I - J/5)$. Therefore, we come across a situation wherein there are two completely symmetric information matrices with unequal multipliers under a block design set-up based on two [naturally non-isomorphic] DCDs with an additive model! This is in sharp contrast with the usual block design results wherein we may only come across non-isomorphic BIBDs possessing the same completely symmetric information matrices.

We now proceed to extend this result for the correlation structure involving ρ. We may summarize our result as follows:
Result 4. $I_1(\tau) = (15/2)[I - J/5]$ and $I_2(\tau) = a(\rho)[I - J/5]$ where $a(\rho) = 5(5 - 2\rho - \rho^2)/(1 + \rho)(3 - \rho)(1 - \rho)$ and $a(\rho) > 15/2$, uniformly in ρ.

To deduce the form of $I_1(\tau)$, we recall the result for one block under *Option I*, say Block 1. Under a correlated model, we have already examined the nature of the information matrix involving 4 treatments and it is given by $2[I - J/4]$ which has diagonal elements $3/2$ and off-diagonal elements $-1/2$ and which is of order 4×4. Since we are now in the frame work of 5 treatments, we have to *blow* this matrix. Moreover, each treatment under *Option I* appears in 4 blocks and hence the diagonal elements in the combined information matrix for the 5 treatments will be $12/2 = 6$ each while the off-diagonal elements will each be $-6/4 = -3/2$. Therefore, the information matrix has the representation $I\tau = 15/2I - 3/2J = 15/2[I - J/5]$, independent of ρ.

To deduce the form of $I_2(\tau)$ in the uncorrelated case, we may proceed as usual and show that it is a multiple of $I - J/5$ and the multiplier is $25/3 > 15/2$. Our contention here is that the same phenomenon continues to hold even when we impose a natural correlation structure in the model of the type we have been discussing in this paper. This we demonstrate in the appendix. The steps are quite routine but we need to encounter complicated expressions.

5. Conclusion

In this paper we have studied some aspects of diallel cross designs in a blocked situation with correlated observations within blocks. It is observed that when the information matrix for parental line effects possesses a completely symmetric structure for uncorrelated observations, the same structure continues to hold even in the correlated set-up, though the computation of the constant is non-trivial. We have resolved this computation for small block sizes.

Further, for block size 6 and correlated observations within blocks, we have provided examples of two competing designs, each with a completely symmetric information matrix, the constants of proportionality being different. In the context of a block design, however, this result does not hold. This is because of the difference in the contemplated model for diallel cross designs as against the one for block designs, irrespective of the presence or absence of correlation among observations within the blocks.

6. Appendix

Proof of Result 2.

Denoting by $\bar{I}$ a square matrix (of appropriate order) having 1's along the *anti-diagonal*, we may write the above matrix as $\Sigma(\rho) = (1-\rho)I + \rho J - \rho\bar{I}$. In the above matrix, replacing 1 by a, ρ by b and 0 by c and denoting the resultant matrix by $((a, b, c)) = (a - b)I + bJ + (c - b)\bar{I}$, it turns out that the inverse of ((a,b,c)) is given by $((d, e, f)) = (d - e)I + eJ + (f - e)\bar{I}$ where

$$d = [a(a + c) + 2b(a - 2b)/\Delta; \tag{6.1}$$

$$e = b(c - a)/\Delta;$$

$$f = [2b(2b - c) - c(a + c)]/\Delta; \tag{6.2}$$

$$\Delta = (a - c)(a + 4b + c)(a - 2b + c). \tag{6.3}$$

Also it is easy to write down the form of the matrix X based on the observation-vector. Therefore, the information matrix for θ is given by $X'\Sigma^{-1}(\rho)X$ and this simplifies to

$$I(\boldsymbol{\tau}) = \begin{pmatrix} 6(d^* + f^*) & 3(d^* + f^*) & 3(d^* + f^*) & 3(d^* + f^*) & 3(d^* + f^*) \\ 3(d^* + f^*) & 3d^* & d^* + 2f^* & d^* + 2f^* & d^* + 2f^* \\ 3(d^* + f^*) & d^* + 2f^* & 3d^* & d^* + 2f^* & d^* + 2f^* \\ 3(d^* + f^*) & d^* + 2f^* & d^* + 2f^* & 3d^* & d^* + 2f^* \\ 3(d^* + f^*) & d^* + 2f^* & d^* + 2f^* & d^* + 2f^* & 3d^* \end{pmatrix} \tag{6.4}$$

where $d^* = d + 2e, f^* = f + 2e$. From the above representation, we can deduce the form of the information matrix of the combining ability parameters $\tau_1,\ \tau_2, \tau_3,\ \tau_4$ of the 4 parental lines, by eliminating the general mean and the specific block effect (which are combined together and regarded as one single parameter in the representation of θ above). It follows that $I(\tau_1,\ \tau_2,\ \tau_3,\ \tau_4)$ is proportional to $I - J/4$ and the constant of proportionality is given by $2(d^* - f^*) = 2(d - f)$. Substituting the values of $a,\ b,\ c$ as $1,\ \rho,\ 0$ respectively, we obtain, upon simplification, $I(\tau_1,\ \tau_2,\ \tau_3,\ \tau_4) = 2[I - J/4]$, independent of ρ. This is because, $\Delta = (1 + 4\rho)(1 - 2\rho); d = (1 + 2\rho - 4\rho^2)/\Delta; e = -\rho/\Delta; f = 4\rho^2/\Delta$.

Proof of Result 3.

We will need to compute $\Sigma_{1,1}^{-1}(\rho), \Sigma_{2,2}^{-1}(\rho), [\Sigma_{1,1}(\rho) - \Sigma_{1,2}(\rho)\Sigma_{2,2}^{-1}(\rho)\Sigma_{2,1}(\rho)]^{-1}$ and

$[\Sigma_{2,2}(\rho) - \Sigma_{2,1}(\rho)\Sigma_{1,1}^{-1}(\rho)\Sigma_{1,2}(\rho)]^{-1}$ in the sequel.

We have already obtained a formal expression for an inverse of a matrix of order 6 of the form ((a,b,c)). Also we already have an expression for $\Sigma_{1,1}^{-1}(\rho)$. Below we display necessary matrix product computations.

$\Sigma_{1,2}(\rho)\Sigma_{2,1}(\rho) = \rho^2[I + J - \bar{I}]$;

$\Sigma_{2,2}^{-1}(\rho) = (1-\rho)^{-1}I - \rho(1-\rho)^{-1}(1+3\rho)^{-1}J;$
$\Sigma_{1,2}(\rho)\Sigma_{2,2}^{-1}(\rho)\Sigma_{2,1}(\rho) = \rho^2(1-\rho)^{-1}[I + J - \bar{I}] - 4\rho^3(1-\rho)^{-1}(1+3\rho)^{-1}J.$

Since $\Sigma(\rho)$ is viewed as a partitioned matrix, its inverse can also be written in the form of a partitioned matrix as $[(A, B); (B', C)]$ where $A^{-1} = \Sigma_{1,1} - \Sigma_{1,2}\Sigma_{2,2}^{-1}\Sigma_{2,1}; C^{-1} = \Sigma_{2,2} - \Sigma_{2,1}\Sigma_{1,1}^{-1}\Sigma_{1,2}; B = -A\Sigma_{1,2}\Sigma_{2,2}^{-1}$.

Using the above matrix product computations, we derive, upon simplification,

$$A^{-1} = [(1-\rho)I + \rho(J) - \rho(\bar{I})] - \rho^2(1-\rho)^{-1}[I + J - \bar{I}] + 4\rho^3(1-\rho)^{-1}(1+3\rho)^{-1}J = ((\tilde{a}, \tilde{b}, \tilde{c})),$$

where $((\tilde{a}, \tilde{b}, \tilde{c}))$ has exactly the *same* form as ((a,b,c)) defined above and

$$\tilde{a} = [(1+2\rho-5\rho^2-2\rho^3)](1-\rho)^{-1}(1+3\rho)^{-1}; \tilde{b} = \rho(1+2\rho)(1+3\rho)^{-1}; \tilde{c} = 4\rho^3(1-\rho)^{-1}(1+3\rho)^{-1}.$$

Therefore, A can be computed as say $A = ((\tilde{d}, \tilde{e}, \tilde{f}))$ with $\tilde{d}$ etc defined analogously.

Again, $C^{-1} = \Sigma_{2,2} - \Sigma_{2,1}\Sigma_{1,1}^{-1}\Sigma_{1,2}$ simplifies to $a^*I + b^*J$ where

$$a^* = (1 - \rho - 2p\rho^2 - 2r\rho^2); b^* = \rho - p\rho^2 - 9q\rho^2 + 2r\rho^2; p = d - e, q = e, r = e - f. \tag{6.5}$$

Herein we have used the representation of $\Sigma_{1,1}$ as ((a,b,c)) as indicated above which further suggests as to its inverse the matrix ((d,e,f)); see (3.2)-(3.4). The expressions for p, q, r are ultimately derived as functions of d, e, f. Note that for $\Sigma_{1,1}$, there are specific choices of a,b,c.

We now simplify a^*, b^* as

$$a^* = (1+\rho)(1-2\rho); b^* = \rho(1+\rho)(1+2\rho)(1+4\rho)^{-1}. \tag{6.6}$$

Therefore, we may compute C as

$$C = [a^*I + b^*J]^{-1} = (1/a^*)(I - J/4) + (1/[a^* + 4b^*])J/4 = a^{**}I + b^{**}J;$$
$$a^{**} = 1/a^* = (1+\rho)^{-1}(1-2\rho)^{-1};$$
$$b^{**} = -b^*/4a^*(a^* + 4b^*) = -\rho(1+2\rho)(1-2\rho)^{-1}(1+\rho)^{-1}(1+6\rho)^{-1}. \tag{6.7}$$

Next we compute B as

$$\begin{aligned} B &= -A\Sigma_{1,2}\Sigma_{2,2}^{-1} \\ &= -((\tilde{d}, \tilde{e}, \tilde{f}))\Sigma_{1,2}[(1-\rho)^{-1}I - \rho(1-\rho)^{-1}(1+3\rho)^{-1}J] \\ &= -(1-\rho)^{-1}((\tilde{d}, \tilde{e}, \tilde{f}))\Sigma_{1,2} + \rho(1-\rho)^{-1}(1+3\rho)^{-1}((\tilde{d}, \tilde{e}, \tilde{f}))\Sigma_{1,2}J. \end{aligned} \tag{6.8}$$

It follows that

$$((\tilde{d}, \tilde{e}, \tilde{f}))\Sigma_{1,2} = \rho W;$$

$$W = 2eJ + W^*;$$

$$W^* = \begin{pmatrix} \tilde{d} & \tilde{d} & \tilde{f} & \tilde{f} \\ \tilde{d} & \tilde{f} & \tilde{d} & \tilde{f} \\ \tilde{d} & \tilde{f} & \tilde{f} & \tilde{d} \\ \tilde{f} & \tilde{d} & \tilde{d} & \tilde{f} \\ \tilde{f} & \tilde{d} & \tilde{f} & \tilde{d} \\ \tilde{f} & \tilde{f} & \tilde{d} & \tilde{d} \end{pmatrix} \tag{6.9}$$

Thus, finally, B assumes the form

$$B = -\rho(1-\rho)^{-1}W + 2\rho^2(\tilde{d} + 4\tilde{e} + \tilde{f})(1-\rho)^{-1}(1+3\rho)^{-1}J. \tag{6.10}$$

Thus we have, by now, sorted out expressions for the components A, B, C in the partitioned form of the inverse of $\Sigma(\rho)$. Define $\tilde{d}^* = \tilde{d} + 2\tilde{e}; \tilde{f}^* = \tilde{f} + 2\tilde{e}$. Further, we write, as before X as the design matrix of order 10×6 and form a natural partition into $X^{(1)}, X^{(2)}$ of orders $6 \times 6, 4 \times 6$ respectively. This facilitates computation of

$$X^{'}\Sigma^{-1}X \;=\; X^{(1)'}AX^{(1)} + X^{(2)'}CX^{(2)} + [X^{(1)'}BX^{(2)} + X^{(2)'}B^{'}X^{(1)}]$$

as sum of three separate terms. We obtain $X^{(1)'}AX^{(1)}$ as

$$\begin{pmatrix} 6(\tilde{d}^* + \tilde{f}^*) & 3(\tilde{d}^* + \tilde{f}^*) & 3(\tilde{d}^* + \tilde{f}^*) & 3(\tilde{d}^* + \tilde{f}^*) & 3(\tilde{d}^* + \tilde{f}^*) & 0 \\ 3(\tilde{d}^* + \tilde{f}^*) & 3\tilde{d}^* & \tilde{d}^* + 2\tilde{f}^* & \tilde{d}^* + 2\tilde{f}^* & \tilde{d}^* + 2\tilde{f}^* & 0 \\ 3(\tilde{d}^* + \tilde{f}^*) & \tilde{d}^* + 2\tilde{f}^* & 3\tilde{d}^* & \tilde{d}^* + 2\tilde{f}^* & \tilde{d}^* + 2\tilde{f}^* & 0 \\ 3(\tilde{d}^* + \tilde{f}^*) & \tilde{d}^* + 2\tilde{f}^* & \tilde{d}^* + 2\tilde{f}^* & 3\tilde{d}^* & \tilde{d}^* + 2\tilde{f}^* & 0 \\ 3(\tilde{d}^* + \tilde{f}^*) & \tilde{d}^* + 2\tilde{f}^* & \tilde{d}^* + 2\tilde{f}^* & \tilde{d}^* + 2\tilde{f}^* & 3\tilde{d}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{6.11}$$

Next, we obtain

$$X^{(2)'}CX^{(2)} = a^{**}\begin{pmatrix} 4 & 1 & 1 & 1 & 1 & 4 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 4 \end{pmatrix} + b^{**}(4,1,1,1,1,4)^{'}(4,1,1,1,1,4) \tag{6.12}$$

Again, recalling the form of B from above, we derive the expression for $X^{(1)'}BX^{(2)} + X^{(2)'}B'X^{(1)}$ as

$$X^{(1)'}BX^{(2)} + X^{(2)'}B'X^{(1)} = -\rho(1-\rho)^{-1}[X^{(1)'}W^{*}X^{(2)} + X^{(2)'}W^{*'}X^{(1)}]$$
$$+[(-2\rho\tilde{e}(1-\rho)^{-1} + 2\rho^2(\tilde{d}+4\tilde{e}+\tilde{f})(1-\rho)^{-1}(1+3\rho)^{-1}][X^{(1)'}JX^{(2)} + X^{(2)'}JX^{(1)}] \quad (6.13)$$

Next, we derive the expression for $[X^{(1)'}W^{*}X^{(2)} + X^{(2)'}W^{*'}X^{(1)}]$ as

$$\begin{pmatrix} 24(\tilde{d}+\tilde{f}) & 9(\tilde{d}+\tilde{f}) & 9(\tilde{d}+\tilde{f}) & 9(\tilde{d}+\tilde{f}) & 9(\tilde{d}+\tilde{f}) & 12(\tilde{d}+\tilde{f}) \\ 9(\tilde{d}+\tilde{f}) & 6\tilde{d} & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 6(\tilde{d}+\tilde{f}) \\ 9(\tilde{d}+\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 6\tilde{d} & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 6(\tilde{d}+\tilde{f}) \\ 9(\tilde{d}+\tilde{f}) & 9(\tilde{d}+\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 6\tilde{d} & 2(\tilde{d}+2\tilde{f}) \\ 6(\tilde{d}+\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 6\tilde{d} & 2(\tilde{d}+2\tilde{f}) & 6(\tilde{d}+\tilde{f}) \\ 9(\tilde{d}+\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 2(\tilde{d}+2\tilde{f}) & 6\tilde{d} & 6(\tilde{d}+\tilde{f}) \\ 12(\tilde{d}+\tilde{f}) & 6(\tilde{d}+\tilde{f}) & 6(\tilde{d}+\tilde{f}) & 6(\tilde{d}+\tilde{f}) & 6(\tilde{d}+\tilde{f}) & 0 \end{pmatrix} \quad (6.14)$$

Also we need

$$[X^{(1)'}JX^{(2)} + X^{(2)'}JX^{(1)}] = \begin{pmatrix} 48 & 18 & 18 & 18 & 18 & 24 \\ 18 & 6 & 6 & 6 & 6 & 12 \\ 18 & 6 & 6 & 6 & 6 & 12 \\ 18 & 6 & 6 & 6 & 6 & 12 \\ 18 & 6 & 6 & 6 & 6 & 12 \\ 24 & 12 & 12 & 12 & 12 & 0 \end{pmatrix}$$

We are now in a position to visualize the form of the information matrix $I(\theta)$. Note that θ involves the combined effect of general mean and the specific block effect as one parameter, apart from the combining ability parameters of the 5 parental lines, viz., the τ_i's. Therefore, to get to the form of $I(\tau_1, ..., \tau_5)$, we need to partition $I(\theta)$ in the usual manner. At this stage, we will demonstrate that in the representation of $I(\theta)$, in the partitioned form, $I_{2,2}$ has all diagonal elements equal [**Claim 1**], that all the off-diagonal elements in the first row [column] of $I(\theta)$ are equal [**Claim 2**] and, further that, $I_{2,2}$ also has all its off-diagonal elements equal [**Claim 3**]. Once these claims are settled, it follows trivially that $I(\tau)$ is completely symmetric in respect of the combining ability parameters of the parental lines appearing in every block. Combining them over all the blocks is a routine task. We need to "blow up" every component information matrix into a full $V \times V$ matrix and then add all such information matrices as they are additive. Incidentally this also avoids computation of the information matrix for the general mean, all b block effects parameters and all V parental combining ability parameters.

Below we settle the claims one by one. It is readily seen that the structure is *nice* for the first 4 treatments. We have to ascertain *similar* feature for the 5^{th} treatment. We will keep this in view.

Claim 1. We have to show $LHS = RHS$ where

$$\begin{aligned} LHS &= 3\tilde{d}^* + a^{**} + b^{**} - 6\rho(1-\rho)^{-1}\tilde{d} \\ &+ 6[-2\rho(1-rho)^{-1}\tilde{e} + 2\rho^2(\tilde{d} + 4\tilde{e} + \tilde{f})(1-\rho)^{-1}(1+3\rho)^{-1}]; \\ RHS &= 4a^{**} + 16b^{**}. \end{aligned} \tag{6.15}$$

Recall, $\tilde{d}^* = \tilde{d} + 2\tilde{e}, \tilde{f}^* = \tilde{f} + 2\tilde{e}$. Therefore, we may simplify the expressions in LHS and RHS above and restate as

$$\begin{aligned} LHS &= 3\tilde{d}^*(1-3\rho)(1-\rho)^{-1} + 12\rho^2(\tilde{d}^* + \tilde{f}^*)(1-\rho)^{-1}(1+3\rho)^{-1}; \\ RHS &= 3[a^{**} + 5b^{**}]. \end{aligned} \tag{6.16}$$

Next, recall the formal expression for $((d, e, f))$ which represents the inverse of the 6×6 matrix ((a,b,c)). Using the expressions for $\tilde{a}, \tilde{b}, \tilde{c}$, we are in a position to evaluate the explicit expressions for $\tilde{d}, \tilde{e}, \tilde{f}$, and hence for $\tilde{d}^*, \tilde{f}^*$. Thus we obtain

$$\begin{aligned} \tilde{d}* &= \tilde{d} + 2\tilde{e} = (\tilde{a} + 2\tilde{b})/[(\tilde{a} - \tilde{c})(\tilde{a} + 4\tilde{b} + \tilde{c})]; \\ \tilde{f}* &= \tilde{f} + 2\tilde{e} = -(2\tilde{b} + \tilde{c})/[(\tilde{a} - \tilde{c})(\tilde{a} + 4\tilde{b} + \tilde{c})]. \end{aligned} \tag{6.17}$$

Upon substituting the expressions for $\tilde{a}, \tilde{b}, \tilde{c}$ from above (immediately before (9)), we derive

$$\begin{aligned} \tilde{a} - \tilde{c} &= (1-\rho)(1+\rho)(1+3\rho)^2(1-2\rho); \\ \tilde{a} + 2\tilde{b} &= (1-\rho)(1+3\rho)(1+\rho)(1+3\rho-6\rho^2); \\ \tilde{a} + 4\tilde{b} + \tilde{c} &= (1-\rho)(1+3\rho)(1-\rho^2)(1+6\rho). \end{aligned} \tag{6.18}$$

Therefore, LHS in the claim simplifies to

$$\begin{aligned} LHS &= 3[(1-3\rho)(1+3\rho-6\rho^2) + 4\rho^2(1-2\rho)](1-2\rho)^{-1}(1-\rho^2)^{-1}(1+6\rho)^{-1} \\ &= 3(1+\rho-10\rho^2)(1-2\rho)^{-1}(1+\rho)^{-1}(1+6\rho)^{-1}. \end{aligned} \tag{6.19}$$

Again, the RHS in the claim can be simplified as

$$\begin{aligned} RHS &= 3[a^{**}+5b^{**}] \\ &= 3[(1+\rho)^{-1}(1-2\rho)^{-1} - 5\rho(1+2\rho)(1-2\rho)^{-1}(1+\rho)^{-1}(1+6\rho)^{-1}] \\ &= 3(1+\rho-10\rho^2)(1-2\rho)^{-1}(1+\rho)^{-1}(1+6\rho)^{-1}. \end{aligned} \tag{6.20}$$

Hence Claim 1 is settled.

As for **Claim 2**, we express it into an equivalent form as

$$\begin{aligned} LHS &= 3(\tilde{d}^{*}+\tilde{f}^{*}) + 3(\tilde{d}+\tilde{f})\rho(1-\rho)^{-1} \\ &- 6[-2\rho\tilde{e}(1-\rho)^{-1} + 2\rho^2(\tilde{d}+4\tilde{e}+\tilde{f})(1-\rho)^{-1}(1+3\rho)^{-1}]; \\ RHS &= 3[a^{**}+4b^{**}]. \end{aligned} \tag{6.21}$$

We readily see that the LHS simplifies to [except for the multipier 3),

$$\begin{aligned} LHS &= (\tilde{d}^{*}+\tilde{f}^{*})[1+\rho(1-\rho)^{-1} - 4\rho^2(1-\rho)^{-1}(1+3\rho)^{-1}] \\ &= (1+4\rho)(1+\rho)^{-1}(1+6\rho)^{-1}, \end{aligned} \tag{6.22}$$

upon simplification. Finally, without the multiplier 3, it is readily seen that $RHS = a^{**}+4b^{**}$ simplifies to $(1+4\rho)(1+\rho)^{-1}(1+6\rho)^{-1}$ which is the same as the LHS. Hence the claim.

We now take up **Claim 3**. It can be equivalently expressed as

$$\begin{aligned} LHS &= \tilde{d}^{*}+2\tilde{f}^{*}+4(\tilde{d}+2\tilde{f})\rho(1-\rho)^{-1} - 6Q; \\ Q &= -2\rho(1-\rho)^{-1}\tilde{e} + 2\rho^2(1-\rho)^{-1}(1+3\rho)^{-1}(\tilde{d}+4\tilde{e}+\tilde{f}); \\ RHS &= a^{**}+3b^{**}. \end{aligned} \tag{6.23}$$

We find that the LHS can be expressed as

$$\begin{aligned} LHS = (1-\rho)^{-1}[(\tilde{d}+4\tilde{e}+\tilde{f}) + (\tilde{f}+2\tilde{e}) + 3\rho(\tilde{d}+2\tilde{e})] \\ -12\rho^2(1-\rho)^{-1}(1+3\rho)^{-1}(\tilde{d}+4\tilde{e}+\tilde{f}) \\ = (1+3\rho-6\rho^2)(1+\rho)^{-1}(1+6\rho)^{-1}(1-2\rho)^{-1}, \end{aligned} \tag{6.24}$$

upon simplification.

Again, the RHS simplifies to $(1+3\rho-6\rho^2)(1+\rho)^{-1}(1+6\rho)^{-1}(1-2\rho)^{-1}$ which is the same as the LHS. Thus we have also settled the Claim 3.

It thus follows that the information matrix for the τ parameters will be a multiple of $I - J/5$ in every single block involving 5 parental lines. To find the multiplier, it is enough to compute the expression for the common element in one diagonal position. This is obtained as $\alpha - \frac{\beta^2}{\gamma}$, where

$$\alpha = 3\tilde{d}^{*}+a^{**}+b^{**}+6\tilde{d}(-\rho/(1-\rho))+6[-2\rho\tilde{e}/(1-\rho)+2\rho^2(\tilde{d}+4\tilde{e}+\tilde{f})/(1-\rho)(1+3\rho)]; \tag{6.25}$$

$$\beta = 3(\tilde{d}^{*}+\tilde{f}^{*})+a^{**}+4b^{**}+9(\tilde{d}+\tilde{f})(-\rho/(1-\rho))+18[-2\rho\tilde{e}/(1-\rho)+2\rho^2(\tilde{d}+4\tilde{e}+\tilde{f})/(1-\rho)(1+3\rho)]; \tag{6.26}$$

$$\gamma = 6(\tilde{d}^{*}+\tilde{f}^{*})+4a^{**}+16b^{**}+24(\tilde{d}+\tilde{f})(-\rho/(1-\rho))+48[-2\rho\tilde{e}/(1-\rho)+2\rho^2(\tilde{d}+4\tilde{e}+\tilde{f})/(1-\rho)(1+3\rho)]; \tag{6.27}$$

Recalling the expressions for a^{**}, b^{**}, upon simplification, we obtain

$$\alpha = 4(1+4\rho)/(1+\rho)(1+6\rho).$$

Next, we simplify β as $\beta = 4/(1+6\rho)$. Finally, γ simplifies to $10/(1+6\rho)$. Therefore,

$$\alpha - \frac{\beta^2}{\gamma} = 12/5(1+\rho). \tag{6.28}$$

Hence, $I(\tau_1, \ldots, \tau_5)$ reads as $I(\tau) = [3/(1+\rho)][I - J/5]$.

Proof of Result 4.

We start with Block 1 under Option II. The 6X6 dispersion matrix is given by

$$\Sigma = \left(\begin{array}{cccc|cc} 1 & \rho & \rho & \rho & \rho & 0 \\ \rho & 1 & \rho & \rho & \rho & 0 \\ \rho & \rho & 1 & \rho & 0 & \rho \\ \rho & \rho & \rho & 1 & 0 & \rho \\ \rho & \rho & 0 & 0 & 1 & 0 \\ 0 & 0 & \rho & \rho & 0 & 1 \end{array}\right) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \tag{6.29}$$

Note that $\Sigma_{11}^{-1} = [(1-\rho)^{-1}]I - [\rho(1-\rho)^{-1}(1+3\rho)^{-1}]J$ and $\Sigma_{22}^{-1} = I$.

Let

$$\Sigma^{-1} = \begin{pmatrix} A & B \\ B' & C \end{pmatrix} \tag{6.30}$$

Then $A^{-1} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} = \begin{pmatrix} 1-\rho^2 & \rho-\rho^2 & \rho & \rho \\ \rho-\rho^2 & 1-\rho^2 & \rho & \rho \\ \rho & \rho & 1-\rho^2 & \rho-\rho^2 \\ \rho & \rho & \rho-\rho^2 & 1-\rho^2 \end{pmatrix}$

Hence,

$$A = \begin{pmatrix} p & q & r & r \\ q & p & r & r \\ r & r & p & q \\ r & r & q & p \end{pmatrix} \tag{6.31}$$

where $p + q + 2r = (1 + 3\rho - 2\rho^2)^{-1}, p = [(1-\rho)^2(1+\rho)(1+2\rho) - 2\rho^2]/(1 - \rho)(1 + 3\rho - 2\rho^2)(1 - 2\rho)(1 + \rho)$ and $q = p - (1 - \rho)^{-1}$.

Also, $C^{-1} = \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12} = \begin{pmatrix} \theta_1 & \theta_2 \\ \theta_2 & \theta_1 \end{pmatrix}$
where $\theta_1 = [1+2\rho-5\rho^2-2\rho^3]/[(1-\rho)(1+3\rho)]$ and $\theta_2 = [4\rho^3]/[(1-\rho)(1+3\rho)]$. This yields

$$C = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} \tag{6.32}$$

where $\alpha = [1 + 2\rho - 5\rho^2 - 2\rho^3]/[(1 + 3\rho - 2\rho^2)(1 + \rho)(1 - 2\rho)]$ and $\alpha + \beta = (1_3\rho)/(1 + 3\rho - 2\rho^2)$.

Further,

$$B = -A\Sigma_{12}\Sigma_{22}^{-1} = (-\rho)\begin{pmatrix} p+q & 2r \\ p+q & 2r \\ 2r & p+q \\ 2r & p+q \end{pmatrix} \tag{6.33}$$

Using the expressions (6.31), (6.32) and (6.33) in (6.30), we deduce

$$\Sigma^{-1} = \left(\begin{array}{cccc|cc} p & q & r & r & -w_{23} & -2r\rho) \\ q & p & r & r & -w_{23} & -2r\rho) \\ r & r & p & q & -2r\rho & -w_{23} \\ r & r & q & p & -2r\rho & w_{23} \\ -w_{23} & -w_{23} & -2r\rho & -2r\rho & \alpha & \beta \\ -2r\rho & -2r\rho & -w_{23} & -w_{23} & \beta & \alpha \end{array}\right) \tag{6.34}$$

where $w_{23} = \rho(p+q)$.

In Block 1, apart from the 5 parental lines effects, the block effect is a nuisance parameter. Below, starting with the information matrix of the parental line effects and this particular block effect, we first deduce the expression for the information matrix for the parental lines, eliminating the effect of the nuisance parameter. Towards this, using the model $[Y, X\theta, \Sigma]$ for Block 1, we obtain

$$\begin{aligned} X'\Sigma^{-1}X &= X \begin{pmatrix} w_1 & w_1 & w_1 & w_1 & w_2 & w_2 \\ w_3 & w_3 & w_3 & w_3 & w_{21} & w_{21} \\ p - w_{23} & q - w_{23} & r(1-2\rho) & r(1-2\rho) & \alpha - w_{23} & \beta - w_{23} \\ q - w_{23} & p - w_{23} & r(1-2\rho) & r(1-2\rho) & \alpha - w_{23} & \beta - w_{23} \\ r(1-2\rho) & r(1-2\rho) & p - w_{23} & q - w_{23} & \beta - 2r\rho & \alpha - w_{23} \\ r(1-2\rho) & r(1-2\rho) & q - w_{23} & p - w_{23} & \beta - 2r\rho & \alpha - w_{23} \end{pmatrix} X \\ &= \left(\begin{array}{c|c|cccc} 4w_1 + 2w_2 & 4w_1 & w_1 + w_2 & w_1 + w_2 & w_1 + w_2 & w_1 + w_2 \\ 4w_3 + 2w_{21} & 4w_3 & w_3 + w_{21} & w_3 + w_{21} & w_3 + w_{21} & w_3 + w_{21} \\ w_1 + w_2 & w_3 + w_{21} & \alpha + p - 2w_{23} & \alpha + q - 2w_{23} & \beta + r - 4r\rho & \beta + r - 4r\rho \\ w_1 + w_2 & w_3 + w_{21} & \alpha + q - 2w_{23} & \alpha + p - 2w_{23} & \beta + r - 4r\rho & \beta + r - 4r\rho \\ w_1 + w_2 & w_3 + w_{21} & \beta + r - 4r\rho & \beta + r - 4r\rho & \alpha + p - 2w_{23} & \alpha + q - 2w_{23} \\ w + 1 + w + 2 & w_3 + w_{21} & \beta + r - 4r\rho & \beta + r - 4r\rho & \alpha + q - 2w_{23} & \alpha + p - 2w_{23} \end{array}\right) \end{aligned} \tag{6.35}$$

where $w_1 = (1-\rho)(p+q+2r), w_2 = -2\rho(p+q+2r)+(\alpha+\beta), w_3 = p+q+2r, w_{21} = -2\rho(p+q+2r)$, and $w_{22} = \alpha + \beta$.

Therefore,

$$I(\tau) = \begin{pmatrix} 4a & -a & -a & -a & -a \\ -a & b & c & d & d \\ -a & c & b & d & d \\ -a & d & d & b & c \\ -a & d & d & c & b \end{pmatrix} \tag{6.36}$$

where $a = w_3 - 4w_1^2/[4w_1 + 2w_2], b = [(\alpha + p) - 2w_{23}] - [w_1 + w_2]^2/[4w_1 + 2w_2], 2w_1 + w_2 = 2(1-2\rho)(p+q+2r) + (\alpha+\beta), w_1 + w_2 = (1-3\rho)(p+q+2r) + (\alpha+\beta), a + b = w_3 + (\alpha+p) - 2w_{23} - [4w_1^2 + (w_1+w_2)^2]/[4w_1 + 2w_2] = (p+q+2r) + \alpha + (1-4\rho)p + 2\rho/(1-\rho) - \Delta$ and

$$\begin{aligned}\Delta &= [4w_1^2 + (w_1 + w_2)^2]/[4w_1 + 2w_2] \\ &= [4(1-\rho)^2(p+q+2r)^2 + [\alpha+\beta+(1-3\rho)(p+q+2r)]^2]/2[2(1-2\rho)(p+q+2r)+(\alpha+\beta)] \\ &= 2(2-2\rho+\rho^2)/(3-\rho)(1+3\rho-2\rho^2);\end{aligned}$$

From the above, we deduce that

$$\begin{aligned}a+b &= (1+3\rho-2\rho^2)^{-1} + \frac{1+2\rho-5\rho^2-2\rho^3}{(1+3\rho-2\rho^2)(1+\rho)(1-2\rho)} + (1-4\rho)p + 2\rho/(1-\rho) - \Delta \\ &= \frac{(5+3\rho-43\rho^2+35\rho^3-4\rho^5)}{(1+\rho)(1-\rho)(1-2\rho)(3-\rho)(1+3\rho-2\rho^2)} \\ &= \frac{(5-12\rho+3\rho^2+2\rho^3)}{(1+\rho)(1-\rho)(1-2\rho)(3-\rho)} = \frac{(5-2\rho-\rho^2)}{(1+\rho)(1-\rho)(3-\rho)} \qquad (6.37)\end{aligned}$$

We now combine the information matrices for the parental lines from the different blocks by identifying their forms from (6.36). It follows that the combined information matrix will have, for its diagonal elements, $4(a+b)$ in each position. And each off-diagonal element will be $-(a+b)$. Therefore, it will be a completely symmetric matrix of the type $a(\rho)[I - J/5]$ where $a(\rho) = 5(a+b) = 5(5-2\rho-\rho^2)/(1+\rho)(1-\rho)(3-\rho)$ which is the expression given in Result 4.

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