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An Application of a Mixed-Effects Location Scale Model for Analysis of Ecological Momentary Assessment (EMA) Data

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SUMMARY

For longitudinal data, mixed models include random subject effects to indicate how subjects influence their responses over the repeated assessments. The error variance and the variance of the random effects are usually considered to be homogeneous. These variance terms characterize the within-subjects (*i.e.*, error variance) and between-subjects (*i.e.*, random-effects variance) variation in the data. In this article, we describe how covariates can influence these variances, and also extend the standard mixed model by adding a subject-level random effect to the within-subject variance specification. This permits subjects to have influence on the mean, or location, and variability, or (square of the) scale, of their responses. Additionally, we allow the random effects to be correlated. We illustrate application of these models using Ecological Momentary Assessment (EMA) data, or intensive longitudinal data, from an adolescent smoking study. These mixed-effects location scale models have useful applications in many research areas where interest centers on the joint modeling of the mean and variance structure.

Key words: complex variation, heteroscedasticity, log-linear variance, multilevel, variance modeling

1 Introduction

Mixed-effects regression models (MRMs) have become a primary method for analysis of longitudinal data [Verbeke and Molenberghs, 2000, Hedeker and Gibbons, 2006]. A basic characteristic of these models is the inclusion of random subject effects into regression models in order to account for the influence of subjects on their repeated observations. These random effects reflect each person's growth or development across time, and the variance of these random effects indicate the degree of variation that exists in the population of subjects. Typically, the error variance, which characterizes the within-subjects variance, and the variance of the random effects, which characterizes the between-subjects variance, are treated as being homogeneous across subject groups or levels of covariates. However, these homogeneity of variance assumptions can be relaxed by modeling differences in variances, both between and within, across subject groups. The study of intra-individual variability has received increasing attention [Fleeson, 2004, Hertzog and Nesselroade, 2003, Martin and Hofer, 2004, Nesselroade, 2004]; these articles describe many of the conceptual issues and some traditional statistical approaches for examining such variation. MRMs can be used to broaden this notion by assessing the determinants of both intra-individual (within-subjects, WS) and inter-individual (between-subjects, BS) variation [Wolfinger and Tobias, 1998].

To reliably estimate and model variances, it is beneficial to have a fair amount of both WS and BS data. Modern data collection procedures, such as ecological momentary assessments (EMA) and/or real-time data captures, provide this opportunity. These procedures yield relatively large numbers of subjects and observations per subject, and data from such designs are sometimes referred to as intensive longitudinal data [Walls and Schafer, 2006]. Such designs are in keeping with the "bursts of measurement" approach described by Nesselroade and McCollam [2000], who called for such an approach in order to assess intra-individual variability. As noted by Nesselroade and McCollam [2000], such bursts of measurement increase the research burden in several ways; however, they are necessary for studying intra-individual variation.

Mixed model analysis of EMA data is well-described in Schwartz and Stone [2007]. Additionally, a few articles have described approaches for examining determinants of between and within subjects variance from EMA studies. Penner et al. [1994] used basic descriptive statistical methods to examine relationships among WS variation in several mood variables. More recently, Hedeker et al. [2006] and Hedeker and Mermelstein [2007] have described mixed model approaches incorporating a log-linear structure for determinants of the WS variance. In this article, we extend these approaches in several ways. First, we include log-linear models for both the WS and BS variance, allowing covariates to potentially influence both sources of variation. More importantly, we also allow the inclusion of a random subject effect to the WS variance specification. This permits the WS variance to vary at the subject level, above and beyond the influence of covariates on this variance.

Models with random WS variance effects have been considered to some extent in the statistical literature. In these models, there are one or more random effects that characterize an individual's mean response or location, and an additional random effect that characterizes the variability around an individual's mean response. The latter is typically specified in terms of the standard deviation, and dubbed a random scale parameter. The technical report by Cleveland et al. [2002] provides a detailed description of this general class of models and summarizes much of the relevant work. The first instances of these models were primarily Bayesian [Lindley, 1971, Leonard, 1975], with frequentist approaches being described more recently [James et al., 1994, Chinchilli et al., 1995, Lin et al., 1997]. Most of these authors take the random scale distribution to be square root inverse gamma, though Leonard [1975] considers the log normal distribution. In this regard, we also specify the log normal distribution for the (square of the) random scale effects. Furthermore, unlike the above approaches, we allow the random scale effect to be correlated with the random location effect. This allows a more general and realistic specification for the random effects. Additionally, because we assume the normal distribution for the (square of the) random scale effects, standard software (*i.e.*, SAS PROC NLMIXED) can be used to estimate these models, and therefore broaden the potential application of this approach. A syntax example is provided in the Appendix to facilitate this.

2 Mixed Location Scale Model

To begin, consider the following mixed-effects regression model for the measurement y of individual i ($i = 1, 2, \ldots, N$ subjects) on occasion j ($j = 1, 2, \ldots, n_i$ occasions):

$$y_{ij} = \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \upsilon_i + \epsilon_{ij} \ , \tag{1}$$

where $\boldsymbol{x}_{ij}$ is the $p \times 1$ vector of regressors (typically including a "1" for the intercept as the first element) and $\boldsymbol{\beta}$ is the corresponding $p \times 1$ vector of regression coefficients. The random subject effect υ_i indicates the influence of individual i on his/her repeated observations. The population distribution of these random effects is usually assumed to be a normal distribution with zero mean and variance σ^2_υ. The errors ϵ_{ij} are also assumed to be normally distributed in the population with zero mean and variance σ^2_ϵ, and independent of the random effects. Here, σ^2_υ represents the between-subjects (BS) variance and σ^2_ϵ is the within-subjects (WS) variance. To allow variables to influence these variances, we can utilize a log-linear representation, as has been described in the context of heteroscedastic (fixed-effects) regression models [Harvey, 1976, Aitkin, 1987], namely,

$$\sigma^2_{\upsilon_i} \ = \ \exp(\boldsymbol{u}'_i\boldsymbol{\alpha}) \ , \tag{2}$$

$$\sigma^2_{\epsilon_{ij}} = \exp(\boldsymbol{w}'_{ij}\boldsymbol{\tau}) . \tag{3}$$

The variances are subscripted by i and j to indicate that their values change depending on the values of the covariates $\boldsymbol{u}_i$ and $\boldsymbol{w}_{ij}$ (and their coefficients). The number of parameters associated with these variances does not vary with i or j. Both $\boldsymbol{u}_i$ and $\boldsymbol{w}_{ij}$ would normally include a (first) column of ones for the reference BS and WS variances (α_0 and τ_0), respectively. Thus, the BS variance equals $\exp \alpha_0$ when the subject-level covariates $\boldsymbol{u}_i$ equal 0, and is increased or decreased as a function of these covariates and their coefficients $\boldsymbol{\alpha}$. Specifically, for a particular covariate u^*, if $\alpha^* > 0$, then the BS variance increases as u^* increases (and vice versa if $\alpha^* < 0$). Note that the exponential function ensures a positive multiplicative factor for any finite value of α, and so the resulting variance is guaranteed to be positive. The WS variance is modeled in the same way, with the exception that both time-varying and subject-varying covariates can influence the WS variance. For this reason, the covariate vector is indicated as $\boldsymbol{w}_{ij}$ for the WS variance. Thus, this model allows both subject-varying and time-varying covariates to influence the WS variance, but only subject-varying variables to influence the BS variance. The coefficients in $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ indicate the degree of influence on the BS and WS variances, respectively, and the ordinary random-intercepts regression model is obtained as a special case if $\boldsymbol{\alpha} = \boldsymbol{\tau} = 0$ for all covariates in $\boldsymbol{u}_i$ and $\boldsymbol{w}_{ij}$ (*i.e.*, excluding the reference variances α_0 and τ_0).

We can further allow the WS variance to vary across individuals, above and beyond the contribution of covariates, namely,

$$\sigma^2_{\epsilon_{ij}} = \exp(\boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i) , \tag{4}$$

where the random subject effects ω_i are distributed in the population of subjects with mean

0 and variance σ^2_ω. Notice that taking logs yields,

$$\log(\sigma^2_{\epsilon_{ij}}) \;=\; \boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i \;, \tag{5}$$

which indicates that if the distribution of ω_i is specified as normal, then the random effects serve as log normal subject-specific perturbations of the WS variance. In other words, the WS variances follow a log normal distribution at the individual level. The skewed, nonnegative nature of the log normal distribution makes it a reasonable choice for representing variances, and it has been used in many diverse research areas for this purpose [Fowler and Whitlock, 1999, Leonard, 1975, Renò and Rizza, 2003, Shenk and Burnhamb, 1998, Vasseur, 1999].

In this model, υ_i is a random effect which influences an individual's mean, or location, and ω_i is a random effect which influences an individual's variance, or (square of the) scale. Thus, we dub the model with both types of random effects as a mixed-effects location scale model. These two random effects are correlated with covariance parameter $\sigma_{\upsilon\omega}$. This covariance parameter indicates the degree to which the random location and scale effects are associated with each other. As we will see in the examples, this parameter is particularly useful in accounting for ceiling and floor effects of measurement.

It is convenient to represent the random effects in standardized form (*i.e.*, as standard normals). For this, we can use the Cholesky factorization [Bock, 1975].

$$\begin{bmatrix} \upsilon_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} s_{1i} & 0 \\ s_{2i} & s_{3i} \end{bmatrix} \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \end{bmatrix} \tag{6}$$

where, the elements of the Cholesky factor are

$$\begin{bmatrix} s_{1i} & 0 \\ s_{2i} & s_{3i} \end{bmatrix} = \begin{bmatrix} \sigma_{\upsilon_i} & 0 \\ \sigma_{\upsilon\omega}/\sigma_{\upsilon_i} & \sqrt{\sigma^2_\omega - \sigma^2_{\upsilon\omega}/\sigma^2_{\upsilon_i}} \end{bmatrix} \tag{7}$$

Here, we include the subscript i on the Cholesky elements because the BS variance $\sigma^2_{\upsilon_i}$ varies with subjects. The model can now be written as

$$y_{ij} = \boldsymbol{x}'_{ij}\boldsymbol{\beta} + s_{1i}\theta_{1i} + \epsilon_{ij} \tag{8}$$

where

$$s_{1i} \ = \ \sigma_{\upsilon_i} \ = \ \sqrt{\exp(\boldsymbol{u}'_i\boldsymbol{\alpha})} \ = \ \exp\left(\frac{1}{2}\,\boldsymbol{u}'_i\boldsymbol{\alpha}\right) \ , \tag{9}$$

and the errors ϵ_{ij} have variance given by

$$\sigma^2_{\epsilon_{ij}} \ = \ \exp(\boldsymbol{w}'_{ij}\boldsymbol{\tau} + s_{2i}\theta_{1i} + s_{3i}\theta_{2i}) \ . \tag{10}$$

Here, the standardized random effects θ_{1i} and θ_{2i} are both normally distributed with mean 0 and variance 1, and are independent of each other. The expectation of y_{ij}, $E(y_{ij})$, is simply $\boldsymbol{x}'_{ij}\boldsymbol{\beta}$. Additionally, because $s^2_{2i} + s^2_{3i} = \sigma^2_\omega$, the variance of y_{ij} is given by

$$V(y_{ij}) \ = \ \exp(\boldsymbol{u}'_i\boldsymbol{\alpha}) \ + \ \exp\left(\boldsymbol{w}'_{ij}\boldsymbol{\tau} + \frac{1}{2}\sigma^2_\omega\right) \ . \tag{11}$$

The covariance for any two observations nested within the same individual i equals

$$C(y_{ij}, y_{ij'}) \ = \ \sigma^2_{\upsilon_i} \ = \ \exp\left(\boldsymbol{u}'_i\boldsymbol{\alpha}\right) \ \text{ for } j \neq j' \ . \tag{12}$$

This covariance can be expressed as a correlation, in which case it yields the intraclass correlation (ICC), denoted as r_{ij},

$$r_{ij} = \frac{\exp\left(\boldsymbol{u}'_i\boldsymbol{\alpha}\right)}{\exp\left(\boldsymbol{u}'_i\boldsymbol{\alpha}\right) \ + \ \exp\left(\boldsymbol{w}'_{ij}\boldsymbol{\tau} + \frac{1}{2}\sigma^2_\omega\right)} \ . \tag{13}$$

Note that the ICC, which represents the proportion of total *unexplained* variation that is at the subject level, can be obtained for specific values of the covariates $\boldsymbol{u}_i$ and $\boldsymbol{w}_{ij}$. Thus, based on the current model, the ICC is allowed to vary as a function of both time-varying and time-invariant covariates.

3 Estimation

The model can be written in terms of the $n_i \times 1$ vector of responses, $\boldsymbol{y}_i$, of subject i as

$$\boldsymbol{y}_i = \boldsymbol{X}_i\boldsymbol{\beta} + \mathbf{1}_i s_{1i}\theta_{1i} + \exp\left(\frac{1}{2}\left[\boldsymbol{W}_i\boldsymbol{\tau} + \mathbf{1}_i s_{2i}\theta_{1i} + \mathbf{1}_i s_{3i}\theta_{2i}\right]\right)\boldsymbol{e}_i \ , \tag{14}$$

where $\boldsymbol{X}_i$ is the $n_i \times p$ (location) covariate matrix influencing the mean of $\boldsymbol{y}_i$, $\boldsymbol{W}_i$ is the $n_i \times r$ (scale) covariate matrix influencing the WS variance of $\boldsymbol{y}_i$, and $\mathbf{1}_i$ is a $n_i \times 1$ vector of ones. Similar to the standardized random effects θ_{1i} and θ_{2i}, the errors, which comprise the vector $\boldsymbol{e}_i$, are each standard normals in this representation of the model.

Then marginally, the $\boldsymbol{y}_i$ are distributed as independent normals with mean $\boldsymbol{X}_i\boldsymbol{\beta}$ and variance-covariance matrix $\mathbf{1}_i\mathbf{1}_i'\sigma^2_{\upsilon_i} + \sigma^2_{e_{ij}}\boldsymbol{I}_i$, where these variances are given in (2) and (10), respectively. The marginal density of $\boldsymbol{y}_i$ in the population can be expressed as

$$h(\boldsymbol{y}_i) = \int_{\boldsymbol{\theta}} f(\boldsymbol{y}_i \mid \boldsymbol{\theta}_i)\, g(\boldsymbol{\theta})\, d\boldsymbol{\theta} \ , \tag{15}$$

where $f(\boldsymbol{y}_i \mid \boldsymbol{\theta}_i)$ represents the normal distribution of $\boldsymbol{y}_i$, given the random effects θ_i and θ_2, and $g(\boldsymbol{\theta})$ represents the distribution of the random effects, namely the standard bivariate normal density. The marginal log-likelihood from the sample of N subjects is then obtained as $\log L = \sum_i^N \log h(\boldsymbol{y}_i)$. Maximizing this log-likelihood yields maximum likelihood (ML) estimates, which are sometimes referred to as maximum marginal likelihood estimates [Bock, 1989] because integrating the joint likelihood of random effects and responses over the distribution of random effects translates to marginalization of the data distribution. SAS PROC NLMIXED can be used to readily obtain the ML estimates for this model. An example of the syntax necessary to run this procedure is provided in the Appendix.

4 Illustration: Adolescent Smoking Study

To illustrate model application, we analyzed data from a natural history study of adolescent smoking. Students included in this study were either in 9th or 10th grade at baseline, 55.1% female, and self-reported on a screening questionnaire 6-8 weeks prior to baseline that they had smoked at least one cigarette in their lifetime. The majority (57.6%) had smoked at least one cigarette in the past month at baseline. Written parental consent and student assent were required for participation. A total of 461 students completed the baseline measurement wave. The study utilized a multi-method approach to assess adolescents in terms of self-report questionnaires, a week-long time/event sampling method via palmtop computers (Ecological Momentary Assessments, EMA), and in-depth interviews.

Here, we focus on the EMA data. Adolescents carried the hand held computers with them at all times during a seven consecutive day data collection period and were trained to both respond to random prompts from the computers and to event record (initiate a data collection interview) smoking episodes. Questions included ones about place, activity, companionship, mood, and other subjective items. The hand held computers date and time-stamped each entry. For the analyses reported, we treated the responses obtained from the random prompts. In all, there were 14,105 random prompts obtained from the 461 students with an approximate average of 30 prompts per student (range = 7 to 71).

Two outcomes were considered: measures of the subject's negative and positive affect (NA and PA) at each random prompt. Both of these measures consisted of the average of several individual mood items, each rated from 1 to 10, that were identified via factor analysis. Specifically, PA consisted of the following items that reflected a subject's assessment of their positive mood before the prompt signal: I felt happy, I felt relaxed, I felt cheerful, I felt confident, and I felt accepted by others. Similarly, NA consisted of the following items assessing pre-prompt negative mood: I felt sad, I felt stressed, I felt angry, I felt frustrated, and I felt irritable. Over all prompts, and ignoring the clustering of the data, the marginal mean of PA was 6.797 (sd=1.935), while the NA marginal mean was 3.455 (sd=2.253). Table 1 lists the estimates of the location scale model of both outcomes, without any covariates.

Insert Table 1 here

As can be seen from the results in Table 1, there is considerable heterogeneity of scale between subjects. In other words, subjects differ in terms of their positive and negative affect variation. For both PA and NA, the estimates of σ_ω^2 greatly exceed their standard error estimates. Also, the covariance estimates are also relatively large and of opposite sign for PA and NA. For NA, the positive covariance indicates that subjects who are high in terms of their negative affect mean also exhibit greater variation in negative affect. Thus, subjects with relatively poor moods (higher NA) fluctuate more in their mood. This could reflect the notion that better moods (less NA) may be more "trait-like" and not as reactive to different situational cues. Alternatively, this positive association might reflect a floor effect of measurement. Namely, as noted, the marginal mean of negative affect is about 3.5, or relatively low and towards the minimum of this scale. Thus, subjects who are lower than average have relatively less room to vary downward than those who are above average. Conversely, the negative covariance for PA indicates that subjects who are relatively high in terms of their mean positive affect are less varied across prompts in their positive affect responses. Again, this could suggest that more positive moods reflect trait-like positivity, or this could be a ceiling effect of measurement, since the marginal mean of PA is about 6.8, or towards the maximum value for this scale. Thus, one might argue that subjects with greater than average positive affect have relatively less room for upward movement, and so less variation. Finally, though the estimated ICCs are similar for both variables, both the

BS and WS variances are larger for NA, relative to PA.

Based on these models without covariates, the empirical Bayes estimates of the random effects υ_i and ω_i were obtained. For PA, Table 2 presents a comparison of the empirical Bayes estimates of the location parameters (specifically, $\hat{\beta}_0 + \hat{\upsilon}_i$) with observed subject means, and also a similar comparison of the empirical Bayes scale parameter estimates (*i.e.*, $\sqrt{\exp[\hat{\tau}_0 + \hat{\omega}]}$) with observed subject standard deviations.

Insert Table 2 here

Results are presented for the entire sample, for subjects with relatively few and many prompts, and for subjects with large estimates of scale $\hat{\omega}_i$ (erratic subjects) and small estimates of scale $\hat{\omega}_i$ (consistent subjects). The table lists the means and standard deviations of the sample statistics and model-based estimates, as well as the slope obtained from regressing the model-based estimates on the sample statistics. As can be seen from Table 2, overall, the model-based estimates of location are near-identical to the sample means, with a small degree of shrinkage (*i.e.*, smaller sd for the model-based estimates and slope < 1). This is to be expected given the empirical Bayes nature of these estimates. Table 2 also shows that the shrinkage is greater for subjects with less observations and more erratic subjects, and less for subjects with many observations and more consistent subjects. Similar observations apply to the scale estimates, although the degree of shrinkage is greater in all cases.

Next, we included several covariates into the models. Subject-level covariates included `Smoker` (an indicator of whether the student is a current smoker, coded no=0 or yes=1; this was determined based on whether or not the subject provided at least one smoking event during the week-long data collection period), `Male` (coded 0=female or 1=male), `Grade10` (coded 0=9th or 1=10th grade), `NovSeek` (a measure of novelty seeking), and `NegMoodReg` (a measure of negative mood regulation). It is worth noting that these subject-level covariates were relatively uncorrelated with each other. All pairwise correlations among these five variables were below .10 with the exception of `Male` with `NegMoodReg`, where a positive correlation of .23 was observed (males were higher in terms of negative mood regulation). In terms of prompt-level covariates, we considered the day of the week and whether the subject was alone or not (coded 0=not alone or 1=alone) at the time of the prompt. For the latter, we created both a between-subjects and within-subjects version (`AloneBS` and `AloneWS`) as described in Neuhaus and Kalbfleisch [1998], namely,

$$X_{ij} = \bar{X}_i + (X_{ij} - \bar{X}_i) \; .$$

Notice that `AloneBS`, the first term on the right-hand side, equals the proportion of random prompts in which a subject was alone, and `AloneWS`, the latter term on the right-hand side, is the prompt-specific deviation relative to this proportion (*i.e.*, it equals either 0−`AloneBS` or 1−`AloneBS` if the subject was not alone or was alone, respectively, for the given random prompt). For day of week, we created six indicator variables using Monday as the reference.

Tables 3 and 4 present the results for negative and positive affect, respectively. The first column of these tables lists the estimated regression coefficients ($\hat{\boldsymbol{\beta}}$) and their standard errors. The second column lists the estimated effects pertaining to the BS variance, namely $\hat{\alpha}_0$ (*i.e.*, the estimate of the reference BS variance, which is listed in the row labeled "Intercept") and the covariate effects in $\hat{\boldsymbol{\alpha}}$. These estimates are on the natural log scale. We modeled the BS variance in terms of subject-level variables to examine how subject heterogeneity varies as a function of these specific subject characteristics. The final column lists the estimates of the WS variance, both $\hat{\tau}_0$ (*i.e.*, the estimate of the reference WS variance, listed in the row labeled "Intercept") and the covariate effects in $\hat{\boldsymbol{\tau}}$. Again, these estimates are on the natural log scale. For the WS variance, we included both subject- and prompt-varying covariates. Finally, estimates of the random WS variance (σ_ω^2) and covariance ($\sigma_{\upsilon\omega}$) are listed in this column towards the bottom.

Insert Tables 3 and 4 here

In terms of negative affect, the results in Table 3 show that several variables significantly increase (`Smoker`, `NovSeek`, `AloneBS`, `AloneWS`, `Wednesday`, `Thursday`, and `Friday`), and decrease (`NegMoodReg` and `Male`) the mean level of this variable. Thus, being a smoker and novelty seeker increase negative affect, as does being a loner (*i.e.*, higher on `AloneBS`) as well as being alone (*i.e.*, `AloneWS`). Conversely, being a male and having better negative mood regulation lower negative affect. In terms of day of week effects, negative affect is increased in the middle and towards the end of the week. In terms of BS heterogeneity, those with better negative mood regulation (*i.e.*, higher on `NegMoodReg`) are less varied and more homogeneous. There are many significant determinants of WS variance including those that increase this variance (`Smoker`, `NovSeek`, `AloneBS`, `AloneWS`, `Thursday`, `Friday`, and `Saturday`), and those that diminish this variance (`NegMoodReg` and `Male`). Thus, the WS data are more varied from smokers, novelty seekers, and loners, and less varied from males and negative mood regulators. It is particularly interesting to note the opposite effect on WS variance of novelty seeking (positive) and negative mood regulation (negative). In terms of prompt-level variables, we see increased WS variation towards the end of the week and on Saturday, and also increased variation when one is alone.

As can be seen from Table 3, the random WS variance and covariance parameters are both highly significant. There is clear evidence that the WS variance varies across individuals, above and beyond the contribution of the many covariates in the WS variance model. In other words, subjects differ in terms of their negative affect variation. As noted, the covariance parameter $\sigma_{\upsilon\omega}$ is estimated to be positive (and is highly significant). This indicates that subjects who are high in terms of their negative affect mean also exhibit greater variation in negative affect, and possibly a floor effect of measurement.

Turning to the results for positive affect in Table 4, we see that, in terms of the mean, several variables have opposite effects to those observed for negative affect, either increasing (`NegMoodReg`) or decreasing (`AloneBS`, `AloneWS`, and `Thursday`) the average positive affect. Additionally, the indicators for the weekend days (`Saturday`, `Sunday`) are seen to increase the mean of positive affect. In terms of BS heterogeneity, loners (*i.e.*, `AloneBS`) are more varied in positive affect, while novelty seekers and 10th grade students are less varied. For the WS variance, nearly all of the results observed for negative affect are replicated for positive affect. Namely, several variables increase the heterogeneity of an individual's responses (`NovSeek`, `AloneWS`, `Thursday`, `Friday`, `Saturday`), while others diminish this variance (`NegMoodReg`, `Male`). Additionally, decreased WS variance is observed for 10th grade students and increased variance for the `Wednesday` indicator.

As was the case for negative affect, the the random WS variance and covariance parameters are both highly significant for positive affect. The significant variance of the random WS variance effects indicates that subjects vary in terms of their positive affect variation. Also, the random WS covariance parameter is estimated to be negative. Thus, subjects who are relatively high in terms of their mean positive affect are less varied across prompts in their positive affect responses. As mentioned, this could be due to a ceiling effect of measurement.

5 Discussion

This article has illustrated how mixed models for longitudinal data can be used to model differences in variances, and not just means, across subject- and time-varying covariates. As such, these models can help to identify predictors of both within-subjects and between-subjects variation, and to test psychological hypotheses about these variances. Additionally, by including a random subject effect on the WS variance, this model can examine the degree to which subjects are heterogeneous in terms of their variation on the outcome variable. Our examples with negative and positive affect clearly show that subjects are quite heterogeneous in terms of their mood variation, as one might expect.

More applications of this class of models clearly exist. For example, many questions of both normal development and the development of psychopathology address the issue of variability or stability in emotional responses to various situations and/or contexts. Often, a concern is with the range of responses an individual gives to a variety of stimuli or situations, and not just with the overall mean level of responsivity. These models also allow us to examine hypotheses about cross-situational consistency of responses as well.

In this paper, we have only considered the case of a single random subject effect for location. This could be generalized to allow multiple location random effects. For example, it is typical in longitudinal studies in which time is a factor, to consider a random subject intercept as well as random time trend parameters. However, for EMA data, there is not necessarily a notion that a person has some kind of systematic trend over the random prompts. In any case, the model could clearly be extended to allow multiple random location effects, and SAS PROC NLMIXED could still be used to estimate such a model.

This article has focused on continuous outcomes. Because ordinal data are often obtained in many research areas as well, we are currently extending these procedures for ordinal data. Admittedly, there is more information in continuous than ordinal responses, so the ability to model variances in ordinal data may not be as general as what is possible using the methods presented here. Thus, we hope to examine the degree to which these models of variation can be applied to ordinal outcomes. Thus far, we have described an ordinal model that allows covariates to influence the WS and BS variances [Hedeker et al., 2006], which can be extended to also allow for the WS variance random effect, as presented here.

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Appendix

Below is a sample of syntax necessary to run the mixed location scale model described in this article. In this syntax, uppercase letters are used for SAS specific syntax and lowercase letters are used for user defined entities. In terms of the variables used in this syntax, `y` denotes the outcome, `x1` denotes a level-1 (*i.e.*, prompt-level or time-varying) covariate, `x2` denotes a level-2 (*i.e.*, subject-level or time-invariant) covariate, and `id` is a level-2 identifier. The random location effect is named `u1` and the random scale effect is named `u2`. The model for the mean response is summarized by `z`, with the $\boldsymbol{\beta}$ regression coefficients named `b0`, `b1`, and `b2`. The model for the BS variance is given by `varu1`, with `lnvaru1` indicating the reference BS variance, in ln units, and `alp1` characterizing how this variance varies with `x2`. Similarly, for the model of the WS variance, `vare` is modeled in terms of a reference variance `lnvare`, in ln units, with coefficients `tau1` and `tau2` specified for the two WS variance influences `x1` and `x2`, respectively.

```
PROC NLMIXED;
PARMS b0=.25 b1=-.5 b2=.3 lnvaru1=1 varu2=.05 cov12=0
       alp1=0 lnvare=1 tau1=0 tau2=0;
z = b0 + b1*x1 + b2*x2 + u1;
varu1 = EXP(lnvaru1 + x2*alp1);
vare = EXP(lnvare + x1*tau1 + x2*tau2 + u2);
MODEL y ~ NORMAL(z,vare);
RANDOM u1 u2 ~ NORMAL([0,0], [varu1,cov12,varu2]) SUBJECT=id;
RUN;
```

Users must provide starting values for all parameters on the `PARMS` statement. To do so, it is beneficial to run the model in stages using estimates from a prior stage as starting values and setting the additional parameters to zero or some small value. For example, one can start by estimating a random-intercepts model with fixed effects ($\boldsymbol{\beta}$), BS variance (`lnvaru1`), and WS variance (`lnvare`). Estimates of these parameters can then be specified as starting values in a model that adds in the WS variance parameters $\boldsymbol{\tau}$, and then the BS variance parameters $\boldsymbol{\alpha}$ (or vice versa). Finally, the full model with the additional parameters σ_{ω}^2 (or `varu2`) and $\sigma_{\upsilon\omega}$ (or `cov12`) can be estimated. In practice, this approach works well with PROC NLMIXED, which sometimes has difficulties in converging to a solution for complex models. Also, in our experience, it seems that specifying a small starting value for the second random effect variance (σ_{ω}^2 or `varu2`) helps model convergence.

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Table 1 Positive and Negative Affect - ML estimates and standard errors (se)

	Positive Affect		Negative Affect	
parameter	estimate	se	estimate	se
mean β_0	6.779	.058	3.482	.071
WS variance τ_0	.622	.036	.741	.047
BS variance of location α_0	.367	.069	.793	.069
BS variance of scale σ_ω^2	.518	.039	.963	.069
covariance $\sigma_{\upsilon\omega}$	-.386	.048	.765	.080
BS variance = $\exp(\alpha_0)$	1.443		2.210	
WS variance = $\exp(\tau_0 + .5\sigma_\omega^2)$	2.413		3.396	
ICC	.374		.394	

Table 2 Observed vs model-based subject means, standard deviations (sds), and slopes of PA location and scale

		Location					Scale				
		mean of		sd of			mean of		sd of		
sample	N	$\bar{y}_i$	$\hat{y}_i$	$\bar{y}_i$	$\hat{y}_i$	slope	S_{y_i}	$\hat{\sigma}_{y_i}$	S_{y_i}	$\hat{\sigma}_{y_i}$	slope
overall	461	6.78	6.78	1.24	1.18	.95	1.44	1.40	.51	.44	.86
$n_i \leq 15$	21	6.63	6.63	1.04	.90	.86	1.49	1.41	.62	.43	.69
$n_i \geq 43$	17	6.50	6.49	1.39	1.35	.97	1.56	1.52	.62	.55	.89
erratic	13	5.63	5.66	.92	.74	.80	2.71	2.48	.21	.19	.81
consistent	16	8.36	8.36	1.11	1.10	.99	.49	.55	.08	.08	.95

$\bar{y}_i$ = observed mean of responses for subject i

$\hat{y}_i = \hat{\beta}_0 + \hat{v}_i$ = model-based mean for subject i

S_{y_i} = observed standard deviation of responses for subject i

$\hat{\sigma}_{y_i} = \sqrt{\exp[\hat{\tau}_0 + \hat{\omega}_i]}$ = model-based standard deviation for subject i

Table 3 Mixed location scale model of Negative Affect (NA), $N = 461$ and $\sum n_i = 14105$, maximum likelihood estimates (standard errors).

	mean	BS variance	WS variance
Intercept	4.310 *** (.382)	1.200** (.360)	.580 * (.269)
Smoker	.245 * (.124)	.118 (.119)	.210 * (.088)
NovSeek	.197 * (.097)	-.133 (.087)	.223 ** (.068)
NegMoodReg	-.762 *** (.095)	-.240 * (.095)	-.278 *** (.067)
Male	-.371 ** (.130)	-.208 (.126)	-.378 *** (.094)
Grade10	.097 (.124)	.030 (.120)	-.074 (.089)
AloneBS	.935 ** (.322)	.526 (.306)	.487 * (.231)
AloneWS	.172 *** (.021)		.070 * (.029)
Tuesday	.057 (.031)		.038 (.051)
Wednesday	.125 *** (.032)		.099 (.051)
Thursday	.122 *** (.032)		.181 *** (.051)
Friday	.082 * (.032)		.249 *** (.051)
Saturday	-.023 (.033)		.264 *** (.053)
Sunday	-.038 (.030)		.027 (.051)
Random WS variance σ_ω^2			.812 *** (.059)
Random WS covariance $\sigma_{\upsilon\omega}$			.526 *** (.061)

$^{***} = p < .001$, $^{**} = p < .01$, $^{*} = p < .05$

Table 4 Mixed location scale model of Positive Affect (PA), $N = 461$ and $\sum n_i = 14105$, maximum likelihood estimates (standard errors).

	mean	BS variance	WS variance
Intercept	5.850 *** (.319)	.688 (.371)	.541 * (.210)
Smoker	-.123 (.101)	.011 (.124)	.073 (.069)
NovSeek	.053 (.081)	-.270 ** (.092)	.130 * (.053)
NegMoodReg	.585 *** (.077)	-.096 (.098)	-.167 ** (.052)
Male	.180 (.106)	-.134 (.130)	-.229 ** (.073)
Grade10	-.015 (.103)	-.294 * (.122)	-.151 * (.069)
AloneBS	-1.279 *** (.268)	1.088 *** (.313)	.328 (.180)
AloneWS	-.364 *** (.023)		.070 * (.028)
Tuesday	-.035 (.036)		.039 (.050)
Wednesday	-.065 (.038)		.136 ** (.050)
Thursday	-.096 * (.039)		.227 *** (.050)
Friday	.003 (.039)		.259 *** (.050)
Saturday	.174 *** (.038)		.151 ** (.050)
Sunday	.149 *** (.036)		-.017 (.050)
Random WS variance σ^2_ω			.461 *** (.036)
Random WS covariance $\sigma_{\upsilon\omega}$			-.306 *** (.040)

$^{***} = p < .001$, $^{**} = p < .01$, $^{*} = p < .05$